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Stochastic finite-time control for uncertain jump system with energy-storing electrical circuit simulation

Shuping He, Fei Liu

Institute of Automation, Jiangnan University, 214122 Wuxi, P. R. China.

Abstract
The stochastic finite-time H_∞ control problem is considered for a class of linear uncertain Markov jump systems that possess randomly jumping parameters. The transition of the jumping parameters is governed by a finite-state Markov process. A sufficient condition is provided to solve the above finite-time control problem and a stochastic finite-time H_∞ controller such that the resulting closed-loop system is stochastic finite-time boundedness and stochastic finite-time stabilization and has the disturbance attenuation γ for all admissible uncertainties. The control criterion is formulated in the form of linear matrix inequalities and the designed finite-time stabilization controller is described as an optimization one. Simulation results illustrate the effectiveness of the developed approaches.

Keywords: Markov jump systems, Finite-time H_∞ control, Stochastic finite-time boundedness, Finite-time stabilization, Uncertainties, Linear matrix inequalities.

1. Introduction
A lot of dynamical systems are highly relevant to processes whose parameters are subject to random abrupt changes due to, for example, sudden environment changes, subsystem switching, system noises, failures occurred in components or interconnections and executor faults, etc. As a special kind of hybrid systems with two components which are the mode and the state, Markov jump systems (MJSs) may be employed to model the above phenomena. In MJSs, the dynamics of jump modes and continuous states are respectively modeled by finite state Markov chains and differential equations. Since the pioneering work of Krasovskii and Lidskii on quadratic control [1] in the mid 1960s, MJSs has regained increasing interest owing to the arising subject on the study of hybrid systems which involve both time-evolving and event-driven mechanisms. In fact, the applications of these hybrid systems are more comprehensive, for instance, economic systems [2], solar thermal receiver systems [3], communication systems [4], electrical power systems [5], robot manipulator system [6] and circuit systems [7, 8], etc. In the past decades, the characterization of stochastic Lyapunov stability and control issues of MJSs has been widely investigated, and it is worth noticing that Rami and Ghaoui [9] started a new and prolific trend in the area of using LMIs. For more results on this topic, we refer readers to [1-14] and the references therein.
As we all know, most of the results relate to the robust Lyapunov stability and performance criteria of linear systems over an infinite-time interval and it always deals with the asymptotic property of system trajectories. But in some practical processes, the main attention is the behavior of the control dynamics over a fixed finite-time interval. That is because a Lyapunov asymptotically stable system over an

infinite-time interval does not mean that it has good transient characteristics, for instance, biochemistry reaction system, robot control system and communication network system, etc. Therefore, we need to check the unacceptable values to see whether the system states remain within the prescribed bound in a fixed finite-time interval.

Lyapunov stability deals with asymptotic pattern of system trajectories by concerning the steady-state behavior of control dynamics over an infinite-time interval. But in many practical applications, For example, large values of the state are not acceptable in the presence of saturations [11]. In these cases, we need to check the unacceptable values that the system state remains within prescribed bounds in the fixed finite-time interval by giving some initial conditions. To study these transient performances of control dynamics, Dorato [15] gave the concept of finite-time stability (or short-time stability) in the early 1960s. A system is said to be finite-time stable if, its state does not exceed a certain threshold for the given bound initial condition during a fixed finite-time interval. Then, some attempts on finite-time stability can be found in [16, 17]. For more results on this topic, we refer readers to [15-23] and the references therein. Towards each case above, it is worth noticing that Doroto et. al [17] started a new and prolific trend in the area of using linear matrix inequalities (LMIs) [23] techniques. However, more details are related to linear control dynamic systems, and very few literatures consider finite-time interval problem for Markov switching stochastic systems. In the work of [22], Yang et. al made some attempt to the finite-time stability and stabilization of impulsive Markov switching systems. But parameters uncertainties are not included and the feedback controller is designed based on the state partition of continuous parts of systems.

In this paper, we discuss the stochastic finite-time H_∞ control problem of continuous-time MJSs with uncertain parameters and norm bounded external disturbance. We aim at the dynamics of the uncertain MJSs of each system mode is stochastically finite-time stable and finite-time stabilizable via state feedback for all admissible uncertainties and has the disturbance attenuation γ for all admissible uncertainties. By selecting the appropriate Lyapunov-Krasovskii functions, it gives the sufficient conditions that the stochastic finite-time boundedness and finite-time stabilization problems can be tackled in the form of LMIs and the designed finite-time stabilization controller is described as an optimization one. At last, a numerical example is provided to illustrate the proposed results.

In the sequel, the following notion will be used: R^n and $R^{n\times m}$ denote n-dimensional Euclidean space, and the set of all the $n\times m$ real matrices, $\boldsymbol{A}^{\mathrm{T}}$ (or $\boldsymbol{x}^{\mathrm{T}}$) and $\boldsymbol{A}^{-1}$ denote the transpose and the inverse of matrix $\boldsymbol{A}$ or vector $\boldsymbol{x}$, $\sigma_{\max}(\boldsymbol{A})$ and $\sigma_{\min}(\boldsymbol{A})$ denote the maximal and minimal eigenvalue of a real matrix $\boldsymbol{A}$, $\|\boldsymbol{A}\|$ denotes the Euclidean norm of matrix $\boldsymbol{A}$, $E\{*\}$ denotes the mathematics statistical expectation of the stochastic process or vector, $L_2^n[-d \quad 0]$ is the space of n-dimensional square integrable function vector over $[-d \quad 0]$, $\boldsymbol{P}>0$ stands for a positive-definite matrix, $\boldsymbol{I}$ is the unit matrix with appropriate dimensions.

2. System description

Let us first consider an energy-storing electrical circuit illustrated by Figure 1. In this model, we assume that the position of the switch follows a continuous-time Markov process $\{r_t,\}$ with three states, $M=[1 \quad 2 \quad 3]$. This Markov process is the consequence of a random request that may result from the choice of an operator. The energy-storing elements in this circuit are the capacitor C_1, C_2, C_3 and the inductor L. $r_t=1$ means that the system goes with the circuit loop of capacitor C_1, and it is similar as $r_t=2$ and $r_t=3$. The switching between these three states is described by the following probability transitions,

$$\boldsymbol{P}_r=\boldsymbol{P}_{ij}(t)=P\{\boldsymbol{r}_{t+\Delta t}=j|\boldsymbol{r}_t=i\}=\begin{cases}\pi_{ij}\,\Delta t+\boldsymbol{o}(\Delta t), & i\neq j\\ 1+\pi_{ii}\,\Delta t+\boldsymbol{o}(\Delta t), & i=j\end{cases} \tag{1}$$

where $\Delta t > 0$ and $\lim_{\Delta t \downarrow 0} \boldsymbol{o}(\Delta t)/\Delta t \to 0$. $\pi_{ij} \geq 0$ is the transition probability rates from mode i at time t to mode $j(i \neq j)$ at time $t + \Delta t$, and $\sum_{j=1, j\neq i}^{N} \pi_{ij} = -\pi_{ii}$.

Assuming the capacitor C_1, C_2, C_3 and the inductor L are linear and time invariant, we can model them as $a(r_t) i_C = \frac{dv_c}{dt}$ and $v_L = L\frac{di_L}{dt}$, where where i and v are the current through and the voltage across an element, with the subscript specifying the element with $a(r_t)$ satisfies

$$a(r_t) = \begin{cases} \frac{1}{C_1}, & if\ r_t = 1 \\ \frac{1}{C_2}, & if\ r_t = 2 \\ \frac{1}{C_3}, & if\ r_t = 3 \end{cases} \tag{2}$$

Let us take $x_1(t) = v_C(t)$ and $x_2(t) = i_L(t)$ as the state variables $u = E$ as the excitation and $z(t) = v_C(t) + 0.5E$ as the output, and use the basic electrical circuits laws, then we get

$$\begin{cases} \dot{x}_1(t) = \frac{1}{a(r_t)}\left[-\frac{x_1(t)}{R_2} + x_2(t)\right] \\ \dot{x}_2(t) = \frac{1}{L}\left[-x_1(t) - R_1 x_2(t) + u(t)\right] \\ z(t) = x_1(t) + 0.5u(t) \end{cases} \tag{3}$$

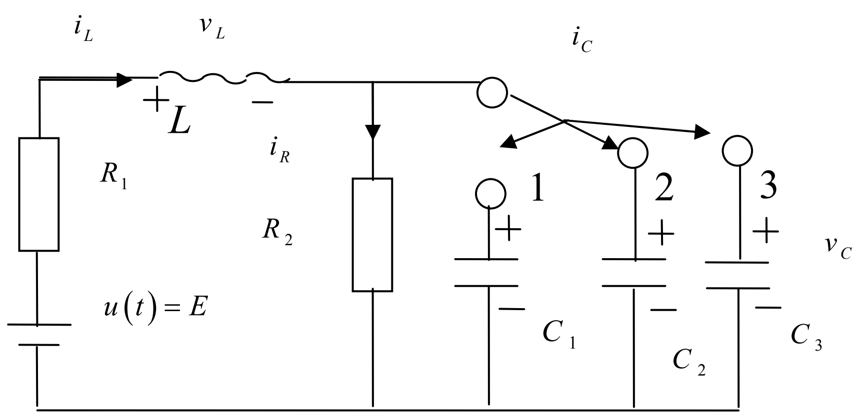

Figure 1. Energy-storing electrical circuit

Then, we can get

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} -\dfrac{1}{a(r_t)R_2} & \dfrac{1}{a(r_t)} \\ -\dfrac{1}{L} & -\dfrac{R_1}{L} \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ \dfrac{1}{L} \end{bmatrix} u(t) \\ z(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t) + 0.5u(t) \end{cases} \tag{4}$$

We will study the above system in the following section. Without loss of generality, we first consider the following MJSs with general form,

$$\begin{cases} \dot{\boldsymbol{x}}(t) = [\boldsymbol{A}(\boldsymbol{r}_t) + \Delta\boldsymbol{A}(\boldsymbol{r}_t, t)]\boldsymbol{x}(t) + [\boldsymbol{B}(\boldsymbol{r}_t) + \Delta\boldsymbol{B}(\boldsymbol{r}_t, t)]\boldsymbol{u}(t) + \boldsymbol{B}_d(\boldsymbol{r}_t)\boldsymbol{d}(t) \\ \boldsymbol{z}(t) = \boldsymbol{C}(\boldsymbol{r}_t)\boldsymbol{x}(t) + \boldsymbol{D}(\boldsymbol{r}_t)\boldsymbol{u}(t) + \boldsymbol{D}_d(\boldsymbol{r}_t)\boldsymbol{d}(t) \\ \boldsymbol{x}(t) = \boldsymbol{x}_0,\ \boldsymbol{r}_t = \boldsymbol{r}_0, t = 0 \end{cases} \tag{5}$$

where $\boldsymbol{x}(t) \in R^n$ is the state, $\boldsymbol{z}(t) \in R^l$ is the controlled output, $\boldsymbol{u}(t) \in R^m$ is the controlled input, $\boldsymbol{d}(t) \in L_2^p[0 \quad +\infty)$ is the external disturbances, $\boldsymbol{x}_0, \boldsymbol{r}_0$ respectively represent the initial state and initial mode, $\boldsymbol{A}(\boldsymbol{r}_t)$, $\boldsymbol{B}(\boldsymbol{r}_t)$, $\boldsymbol{B}_d(\boldsymbol{r}_t)$, $\boldsymbol{C}(\boldsymbol{r}_t)$, $\boldsymbol{D}(\boldsymbol{r}_t)$, $\boldsymbol{D}_d(\boldsymbol{r}_t)$ are known mode-dependent constant matrices with appropriate dimensions. For notational simplicity, when $\boldsymbol{r}_t = i$, $i \in \boldsymbol{M}$, $\boldsymbol{A}(\boldsymbol{r}_t)$, $\Delta\boldsymbol{A}(\boldsymbol{r}_t, t)$, $\boldsymbol{B}(\boldsymbol{r}_t)$, $\Delta\boldsymbol{B}(\boldsymbol{r}_t, t)$, $\boldsymbol{B}_d(\boldsymbol{r}_t)$, $\boldsymbol{C}(\boldsymbol{r}_t)$, $\boldsymbol{D}(\boldsymbol{r}_t)$, $\boldsymbol{D}_d(\boldsymbol{r}_t)$ are respectively denoted as $\boldsymbol{A}_i$, $\Delta\boldsymbol{A}_i$, $\boldsymbol{B}_i$, $\Delta\boldsymbol{B}_i$, $\boldsymbol{B}_{di}$, $\boldsymbol{C}_i$, $\boldsymbol{D}_i$, $\boldsymbol{D}_{di}$. $\Delta\boldsymbol{A}_i$ and $\Delta\boldsymbol{B}_i$ are the time-varying but norm bounded uncertainties satisfying

$$[\Delta\boldsymbol{A}_i \quad \Delta\boldsymbol{B}_i] = \boldsymbol{M}_i\boldsymbol{F}_i(t)[\boldsymbol{N}_{1i} \quad \boldsymbol{N}_{2i}] \tag{6}$$

where $\boldsymbol{M}_i$, $\boldsymbol{N}_{1i}$, $\boldsymbol{N}_{2i}$ are known mode-dependent matrices with appropriate dimensions and $\boldsymbol{F}_i(t)$ is the time-varying unknown matrix function with Lebesgue norm measurable elements satisfying $\boldsymbol{F}_i^{\mathrm{T}}(t)\boldsymbol{F}_i(t) \le \boldsymbol{I}$.

Remark 1. The parameter uncertainties $\Delta\boldsymbol{A}_i$ and $\Delta\boldsymbol{B}_i$ are said to be admissible if both condition (6) and $\boldsymbol{F}_i^{\mathrm{T}}(t)\boldsymbol{F}_i(t) \le \boldsymbol{I}$ hold. The matrix $\boldsymbol{M}_i$ $(\forall i \in \boldsymbol{M})$ is chosen as full row rank matrix. The motivation for us to consider MJSs (5) containing uncertainties $\Delta\boldsymbol{A}_i$ and $\Delta\boldsymbol{B}_i$ stems from the fact that, it is always impossible to obtain the exact mathematical model of a practical dynamics due to the complexity process, the environmental noises, time-varying parameters and the difficulties if measuring various and uncertain parameters, etc. Thus, the model of a practical dynamic to be controlled almost contains some types of uncertainties. In fact, the uncertainties described in (6) have been widely used in the schemes of stochastic robust H_∞ control and stochastic robust filtering of uncertain MJSs. For more results on this topic, we refer readers to [5-8, 14, 17, 18] and the references therein.. Note that the unknown mode-dependent matrix $\boldsymbol{F}_i(t)$ in (6) can also be allowed to be state-dependent, i.e., $\boldsymbol{F}_i(t) = \boldsymbol{F}_i(t, \boldsymbol{x}(t))$, as long as $\|\boldsymbol{F}_i(t, \boldsymbol{x}(t))\| \le 1$ is satisfied.

Assumption 1. The external disturbance $\boldsymbol{d}(t)$ is time-varying and satisfies the following constraint condition

$$\int_0^T \boldsymbol{d}^{\mathrm{T}}(t)\boldsymbol{d}(t)dt \le d,\ d \ge 0 \tag{7}$$

Concerning MJSs (5), we construct the following state-feedback controller:

$$\boldsymbol{u}(t) = \boldsymbol{K}_i \boldsymbol{x}(t) \tag{8}$$

where $\boldsymbol{K}_i$ is state feedback gain to be designed. Then, the resulting closed-loop MJSs follows that

$$\begin{cases} \dot{\boldsymbol{x}}(t) = \left(\overline{\boldsymbol{A}}_i + \Delta\overline{\boldsymbol{A}}_i\right)\boldsymbol{x}(t) + \boldsymbol{B}_{di}\boldsymbol{d}(t) \\ \boldsymbol{z}(t) = \overline{\boldsymbol{C}}_i \boldsymbol{x}(t) + \boldsymbol{D}_{di}\boldsymbol{d}(t) \end{cases} \tag{9}$$

where $\overline{\boldsymbol{A}}_i = \boldsymbol{A}_i + \boldsymbol{B}_i \boldsymbol{K}_i$, $\Delta\overline{\boldsymbol{A}}_i = \Delta\boldsymbol{A}_i + \Delta\boldsymbol{B}_i \boldsymbol{K}_i$, $\overline{\boldsymbol{C}}_i = \boldsymbol{C}_i + \boldsymbol{D}_i \boldsymbol{K}_i$.

The aim of this paper is to analyze the finite-time control problem of uncertain continuous-time MJSs (5). By using the stochastic Lyapunov-Krasovskii functional method, the main results will be given in the form of LMIs. In the work of [15-22], the following definitions over a finite-time interval for some given initial conditions can be formalized.

Definition 1. MJSs (5) with $\boldsymbol{u}(t) = 0$, $\boldsymbol{w}(t) = 0$ is stochastically finite-time stable (FTS) with respect to $\begin{pmatrix} c_1 & c_2 & T & R_i \end{pmatrix}$, where $R_i > 0$, $c_1 < c_2$, if for a given time-constant $T > 0$, the following relation holds.

$$E\left\{x_0^{\mathrm{T}} R_i x_0\right\} \le c_1 \Rightarrow E\left\{x^{\mathrm{T}}(t) R_i x(t)\right\} < c_2,\ \forall t \in \begin{bmatrix} 0 & T \end{bmatrix} \tag{10}$$

Definition 2. MJSs (5) with $\boldsymbol{u}(t) = 0$ is stochastically finite-time bounded (FTB) with respect to $\begin{pmatrix} c_1 & c_2 & T & R_i \end{pmatrix}$, where $R_i > 0$, $c_1 < c_2$, if for a given time-constant $T > 0$, condition (10) holds.

Definition 3. (Stochastic finite-time stabilization via state feedback). Given a time-constant $T > 0$, positive scalars c_1, c_2, $c_1 < c_2$, and mode-dependent positive definite matrix $R_i > 0$, MJSs (5) exists a state feedback controller in form (8), such that the closed-loop system (9) is stochastically FTB with respect to $\begin{pmatrix} c_1 & c_2 & T & R_i & d \end{pmatrix}$.

Remark 2. It is necessary to point out that there is a great difference between Lyapunov stochastic stability and stochastic finite-time stability. The concept of Lyapunov stochastic stability (or Lyapunov almost asymptotic stability [7]) is largely known to the control community, but a stochastic MJSs is FTS if, once we fix a finite time-interval, its state does not exceeds some bounds during this time-interval. Moreover, a MJSs system which is stochastic FTS may not be Lyapunov stochastic stable; conversely, a Lyapunov stochastic stable MJSs could not be FTS if its states exceed the prescribed bounds during the transients. For linear systems with Markov jump, the mode jumping is included and it will influence the designing of FTS controller. A key assumption in definition 1-3 is the initial jump instants are known in advance. The concept of FTS also differs from invariant set, which pays more attention to the control properties in an infinite time-interval. The definition of FTS can be interpreted in terms of ellipsoidal domains. The set defined by $E\left\{x_0^{\mathrm{T}} R_i x_0\right\} \le c_1$ contains all the admissible initial states. Instead, the inequality $E\left\{x^{\mathrm{T}}(t) R_i x(t)\right\} < c_2$ defines a time-varying ellipsoid which bounds the state trajectory over the finite-time interval $t \in \begin{bmatrix} 0 & T \end{bmatrix}$.

Remark 3. If there is no external disturbances in system (2), i.e., $d = 0$, then finite-time boundedness can be recovered as finite-time stability. In the presence of external disturbances, finite-time stability leads to the concept of finite-time boundedness. That is to say, a system is FTB if, given a bound initial condition and a characterization of the set of admissible inputs, the system states remain below the prescribed limit for all inputs in the bound set. FTB and FTS are open-loop concepts. But stochastic finite-time control problem concerns the design of a stochastic finite-time controller which guarantees the stochastic finite-time boundedness and finite-time stabilization of closed-loop system via state feedback.

Definition 4. (Mao [11]) In the Euclidean space $\{R^n \times \boldsymbol{M} \times R_+\}$, we introduce the stochastic Lyapunov-Krasovskii function of uncertain continuous-time MJSs (2) as $\boldsymbol{V}(\boldsymbol{x}(t), \boldsymbol{r}_t = i, t > 0)$, the weak infinitesimal operator of which satisfies

$$\begin{aligned}\Im V\left(x(t),i\right) &= \lim_{\Delta t \to 0} \frac{1}{\Delta t}\left[E\left\{V\left(x(t+\Delta t), r_{t+\Delta t}, t+\Delta t\right) \middle| x(t)=x, r_t = i\right\} - V\left(x(t),i,t\right)\right] \\ &= \frac{\partial}{\partial t} V\left(x(t),i,t\right) + \frac{\partial}{\partial x} V\left(x(t),i,t\right)\dot{x}(t,i) + \sum_{j=1}^{N} \pi_{ij} V\left(x(t),j,t\right)\end{aligned} \tag{11}$$

Definition 5. For MJSs (5) and (9), if there exist a state-feedback controller in form (8), such that the resulting closed-loop dynamic MJSs (9) is stochastically FTB and under the zero-valued initial condition, the system output satisfies the following cost function inequality for $T > 0$ with attenuation $\gamma > 0$ and for all admissible $\boldsymbol{d}(t)$ with constraint condition (7),

$$J = E\left\{\int_0^T \boldsymbol{z}^{\mathrm{T}}(t)\boldsymbol{z}(t)dt\right\} - \gamma^2 \int_0^T \boldsymbol{d}^{\mathrm{T}}(t)\boldsymbol{d}(t)dt < 0 \tag{12}$$

Then the state-feedback controller (8) is called as the robust stochastic finite-time H_∞ controller of closed-loop dynamic MJSs (9) with γ–disturbance attenuation.

Remark 4. In the design of robust stochastic finite-time H_∞ controller, the unknown disturbance $\boldsymbol{d}(t)$ is assumed to be arbitrary deterministic signal of bounded energy, and the problem of this paper is to design a controller that guarantees a prescribed bounded for the finite-time interval induced L_2 norm of the operator from unknown disturbance $\boldsymbol{d}(t)$ to system output $\boldsymbol{z}(t)$, i. e., the designed robust stochastic finite-time H_∞ controller is supposed to satisfy relation (9) with disturbance attenuation level γ.

3. Robust stochastic finite-time H_∞ control for uncertain continuous-time MJSs

In this section, we will first consider the robust stochastic finite-time H_∞ control problem for uncertain continuous-time MJSs (5). Before proceeding with the study, the following lemmas presented will be useful.

Lemma 1. (Wang et. al [24]) Let $\boldsymbol{T}$, $\boldsymbol{M}$, $\boldsymbol{F}$ and $\boldsymbol{N}$ be real matrices of appropriate dimension with $\boldsymbol{F}^{\mathrm{T}}\boldsymbol{F} \le \boldsymbol{I}$, then for a positive scalar $\alpha > 0$, such that

$$\boldsymbol{T} + \boldsymbol{MFN} + \boldsymbol{N}^{\mathrm{T}}\boldsymbol{F}^{\mathrm{T}}\boldsymbol{M}^{\mathrm{T}} \le \boldsymbol{T} + \alpha\boldsymbol{MM}^{\mathrm{T}} + \alpha^{-1}\boldsymbol{N}^{\mathrm{T}}\boldsymbol{N} \tag{13}$$

Lemma 2. For given time-constant $T > 0$, the uncertain continuous-time MJSs (5) is stochastic finite-time stabilization via state feedback with respect to $(c_1 \quad c_2 \quad T \quad \boldsymbol{R}_i \quad d)$, if there exist positive constant $\alpha > 0$, mode-dependent symmetric positive-definite matrix $\boldsymbol{P}_i \in R^{n\times n}$, $i \in \boldsymbol{M}$, symmetric positive-definite matrix $\boldsymbol{Q} \in R^{p\times p}$, such that

$$\begin{bmatrix} \boldsymbol{\Xi}_i & \boldsymbol{P}_i\boldsymbol{B}_{di} \\ \boldsymbol{B}_{di}^{\mathrm{T}}\boldsymbol{P}_i & -\alpha\boldsymbol{Q} \end{bmatrix} < 0 \tag{14}$$

$$\frac{c_1\sigma_P + d\sigma_Q\left(1 - e^{-\alpha T}\right)}{\sigma_p} < e^{-\alpha T}c_2 \tag{15}$$

where $\boldsymbol{\Xi}_i = \left(\overline{\boldsymbol{A}}_i + \Delta\overline{\boldsymbol{A}}_i\right)^{\mathrm{T}} \boldsymbol{P}_i + \boldsymbol{P}_i\left(\overline{\boldsymbol{A}}_i + \Delta\overline{\boldsymbol{A}}_i\right) + \sum_{j=1}^{N} \pi_{ij}\boldsymbol{P}_j - \alpha\boldsymbol{P}_i$, $\sigma_P = \max_{i\in\boldsymbol{M}} \sigma_{\max}\left(\widetilde{\boldsymbol{P}}_i\right)$, $\sigma_Q = \sigma_{\max}(\boldsymbol{Q})$, $\sigma_p = \min_{i\in\boldsymbol{M}} \sigma_{\min}\left(\widetilde{\boldsymbol{P}}_i\right)$, $\widetilde{\boldsymbol{P}}_i = \boldsymbol{R}_i^{-1/2}\boldsymbol{P}_i\boldsymbol{R}_i^{-1/2}$.

Proof: For the given mode-dependent symmetric positive-definite matrix $\boldsymbol{P}_i$ $(i \in \boldsymbol{M})$, we define the following Lyapunov-Krasovskii function $\boldsymbol{V}(\boldsymbol{x}(t), i) = \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}_i\boldsymbol{x}(t)$. Along the trajectories of system (9), the corresponding time derivative of $\boldsymbol{V}(\boldsymbol{x}(t), i)$ is given by

$$\begin{aligned}\Im V\left(x(t),i\right) &= \lim_{\Delta t\to 0}\frac{1}{\Delta t}\left[E\left\{V\left(x(t+\Delta t), r_{t+\Delta t}, t+\Delta t\right)\middle| x(t)=x, r_t=i\right\} - V\left(x(t),i,t\right)\right]\\ &= x^{\mathrm{T}}(t)\left[\left(\overline{A}_i+\Delta\overline{A}_i\right)^{\mathrm{T}}P_i + P_i\left(\overline{A}_i+\Delta\overline{A}_i\right)+\sum_{j=1}^{N}\pi_{ij}P_j\right]x(t) + 2x^{\mathrm{T}}(t)P_iB_{di}d(t)\end{aligned}$$

From condition (14), we have

$$\Im V\left(x(t),i\right) < \alpha V\left(x(t),i\right) + \alpha d^{\mathrm{T}}(t)Qd(t)$$

Multiplying the above inequality by $e^{-\alpha t}$, we can get

$$\Im\left[e^{-\alpha t}V\left(x(t),i\right)\right] < \alpha e^{-\alpha t}d^{\mathrm{T}}(t)Qd(t)$$

By integrating the above inequality between 0 to t, it follows that

$$e^{-\alpha t}V\left(x(t),i\right) - V\left(x_0, r_0\right) < \alpha\int_0^T e^{-\alpha t}d^{\mathrm{T}}(t)Qd(t)\,dt$$

Denote $\widetilde{\boldsymbol{P}}_i = \boldsymbol{R}_i^{-1/2}\boldsymbol{P}_i\boldsymbol{R}_i^{-1/2}$, $\sigma_P = \max_{i\in M}\sigma_{\max}\left(\widetilde{\boldsymbol{P}}_i\right)$, $\sigma_p = \min_{i\in M}\sigma_{\min}\left(\widetilde{\boldsymbol{P}}_i\right)$, $\sigma_Q = \sigma_{\max}(\boldsymbol{Q})$. Note that $\alpha, t \in [0 \quad T]$, we can obtain the following relation

$$\begin{aligned}&\boldsymbol{V}(\boldsymbol{x}(t), i) = \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}_i\boldsymbol{x}(t) < \alpha d\sigma_Q e^{\alpha t}\int_0^T e^{-\alpha t}dt + e^{\alpha t}\boldsymbol{V}(\boldsymbol{x}_0, \boldsymbol{r}_0)\\ &< e^{\alpha T}\left[\boldsymbol{V}(\boldsymbol{x}_0, \boldsymbol{r}_0) + d\sigma_Q\left(1-e^{-\alpha T}\right)\right] < e^{\alpha T}\left[c_1\sigma_P + d\sigma_Q\left(1-e^{-\alpha T}\right)\right]\end{aligned}$$

On the other hand, we have

$$\boldsymbol{V}(\boldsymbol{x}(t), i) = \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}_i\boldsymbol{x}(t) \geq \sigma_p\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{R}_i\boldsymbol{x}(t)$$

Then we can get

$$E\left\{x^{\mathrm{T}}(t)R_ix(t)\right\} < \frac{e^{\alpha T}\left[c_1\sigma_P + d\sigma_Q\left(1-e^{-\alpha T}\right)\right]}{\sigma_p}$$

Condition (15) implies that for $\forall t \in [0 \quad T]$, $E\left\{x^{\mathrm{T}}(t)R_ix(t)\right\} < c_2$. This completes the proof.

Lemma 3. For given positive scalars $\alpha, T > 0$, the uncertain continuous-time MJSs (5) is said to be stochastic finite-time stabilization via state feedback with respect to $(c_1 \quad c_2 \quad T \quad \boldsymbol{R}_i \quad d)$, if there exist positive scalar $\gamma > 0$, mode-dependent symmetric positive-definite matrix $\boldsymbol{P}_i \in R^{n\times n}$, $i \in \boldsymbol{M}$, such that

$$\begin{bmatrix}\boldsymbol{\Xi}_i & \boldsymbol{P}_i\boldsymbol{B}_{di}\\ \boldsymbol{B}_{di}^{\mathrm{T}}\boldsymbol{P}_i & -\gamma^2\boldsymbol{I}\end{bmatrix} < 0 \qquad (16)$$

$$c_1\sigma_P + \frac{\gamma^2 d}{\alpha}\left(1 - e^{-\alpha T}\right) < e^{-\alpha T} c_2 \sigma_p \tag{17}$$

where $\sigma_P = \max_{i\in \boldsymbol{M}} \sigma_{\max}\left(\widetilde{\boldsymbol{P}}_i\right)$, $\sigma_Q = \sigma_{\max}(\boldsymbol{Q})$, $\sigma_p = \min_{i\in \boldsymbol{M}} \sigma_{\min}\left(\widetilde{\boldsymbol{P}}_i\right)$, $\widetilde{\boldsymbol{P}}_i = \boldsymbol{R}_i^{-1/2}\boldsymbol{P}_i\boldsymbol{R}_i^{-1/2}$.

Proof: Consider the similar Lyapunov-Krasovskii function $\boldsymbol{V}(\boldsymbol{x}(t), i) = \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}_i\boldsymbol{x}(t)$. Along the trajectories of system (9), and recall condition (13), we have

$$\Im V\left(x(t), i\right) < \alpha V\left(x(t), i\right) + \gamma^2 d^{\mathrm{T}}(t) d(t)$$

Then following the similar proof of Lemma 2, inequalities (16) (17) can be easily obtained. This completes the proof.

Theorem 1. For given positive scalars $\alpha, T > 0$, the uncertain continuous-time MJSs (5) is stochastic finite-time stabilization via state feedback with respect to $\left(c_1 \quad c_2 \quad T \quad \boldsymbol{R}_i \quad d\right)$ and satisfies the cost function inequality (12) for all admissible $\boldsymbol{d}(t)$ with the constraint condition (4), if there exist positive scalar $\gamma > 0$, mode-dependent symmetric positive-definite matrix $\boldsymbol{P}_i \in R^{n\times n}$, $i \in \boldsymbol{M}$, such that condition (17) and the following inequality hold,

$$\begin{bmatrix} \boldsymbol{\Xi}_i + \overline{\boldsymbol{C}}_i^{\mathrm{T}}\overline{\boldsymbol{C}}_i & \boldsymbol{P}_i\boldsymbol{B}_{di} + \overline{\boldsymbol{C}}_i^{\mathrm{T}}\boldsymbol{D}_{di} \\ \boldsymbol{B}_{di}^{\mathrm{T}}\boldsymbol{P}_i & -\gamma^2\boldsymbol{I} + \boldsymbol{D}_{di}^{\mathrm{T}}\boldsymbol{D}_{di} \end{bmatrix} < 0 \tag{18}$$

Proof: Considering Lemma 3 and the overall closed-loop dynamic MJSs (9), we introduce the following inequality by defining the similar Lyapunov-Krasovskii function $\boldsymbol{V}(\boldsymbol{x}(t), i) = \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}_i\boldsymbol{x}(t)$,

$$\Im V\left(x(t), i\right) < \alpha V\left(x(t), i\right) + \gamma^2 d^{\mathrm{T}}(t) d(t) - z^{\mathrm{T}}(t) z(t)$$

Obviously, the above relation can be guaranteed by condition (15). On the other hand, multiplying the above inequality by $e^{-\alpha t}$, it follows that

$$\Im\left[e^{-\alpha t} V\left(x(t), i\right)\right] < e^{-\alpha t}\left[\gamma^2 d^{\mathrm{T}}(t) d(t) - z^{\mathrm{T}}(t) z(t)\right]$$

In zero initial condition, by integrating the above inequality between 0 to T , we can get

$$e^{-\alpha T} V\left(x(t), i\right) < \int_0^T e^{-\alpha t}\left[\gamma^2 d^{\mathrm{T}}(t) d(t) - z^{\mathrm{T}}(t) z(t)\right] dt$$

Thus, the following condition holds,

$$\int_0^T e^{-\alpha t} z^{\mathrm{T}}(t) z(t)\, dt < \gamma^2 \int_0^T e^{-\alpha t} d^{\mathrm{T}}(t) d(t)\, dt$$

Note that $t \in \left[0 \quad T\right]$, then yields

$$\int_0^T z^{\mathrm{T}}(t) z(t)\, dt < e^{\alpha T}\gamma^2 \int_0^T d^{\mathrm{T}}(t) d(t)\, dt$$

Therefore, condition (9) can be guaranteed by letting $\overline{\gamma} = \sqrt{e^{\alpha T}}\gamma$. This completes the proof.

Theorem 2. For given positive scalars $\alpha, T > 0$, the uncertain continuous-time MJSs (5) is stochastic finite-time stabilization via state feedback with respect to $\left(c_1 \quad c_2 \quad T \quad \boldsymbol{R}_i \quad d\right)$, if there exist a state feedback controller $\boldsymbol{K}_i = \boldsymbol{Y}_i\boldsymbol{X}_i^{-1}$, and satisfies the cost function inequality (12) for all admissible $\boldsymbol{d}(t)$

with the constraint condition (7), if there exists positive scalar $\gamma > 0$, mode-dependent symmetric positive-definite matrix $\boldsymbol{X}_i \in R^{n\times n}, i \in \boldsymbol{M}$, mode-dependent matrix $\boldsymbol{Y}_i \in R^{m\times n}, i \in \boldsymbol{M}$ and a sequence $\{\beta_i > 0, i \in \boldsymbol{M}\}$, such that

$$\begin{bmatrix} \boldsymbol{L}(\boldsymbol{X}_i, \boldsymbol{Y}_i) & \boldsymbol{B}_{di} & \boldsymbol{X}_i\boldsymbol{C}_i^{\mathrm{T}} + \boldsymbol{Y}_i^{\mathrm{T}}\boldsymbol{D}_i^{\mathrm{T}} & \boldsymbol{X}_i\boldsymbol{N}_{1i}^{\mathrm{T}} + \boldsymbol{Y}_i^{\mathrm{T}}\boldsymbol{N}_{2i}^{\mathrm{T}} & \boldsymbol{M}(\boldsymbol{X}_i) \\ \boldsymbol{B}_{di}^{\mathrm{T}} & -\gamma^2\boldsymbol{I} & \boldsymbol{D}_{di}^{\mathrm{T}} & 0 & 0 \\ \boldsymbol{C}_i\boldsymbol{X}_i + \boldsymbol{D}_i\boldsymbol{Y}_i & \boldsymbol{D}_{di} & -\boldsymbol{I} & 0 & 0 \\ \boldsymbol{N}_{1i}\boldsymbol{X}_i + \boldsymbol{N}_{2i}\boldsymbol{Y} & 0 & 0 & -\beta_i\boldsymbol{I} & 0 \\ \boldsymbol{M}^{\mathrm{T}}(\boldsymbol{X}_i) & 0 & 0 & 0 & \boldsymbol{N}(\boldsymbol{X}_i) \end{bmatrix} < 0 \tag{19}$$

$$\sigma_1\boldsymbol{R}_i^{-1} < \boldsymbol{X}_i < \boldsymbol{R}_i^{-1} \tag{20}$$

$$\begin{bmatrix} -e^{-\alpha T}c_2 + \dfrac{\gamma^2 d}{\alpha}\left(1 - e^{-\alpha T}\right) & \sqrt{c_1} \\ \sqrt{c_1} & -\sigma_1 \end{bmatrix} < 0 \tag{21}$$

where $\boldsymbol{L}(\boldsymbol{X}_i, \boldsymbol{Y}_i) = \boldsymbol{X}_i\boldsymbol{A}_i^{\mathrm{T}} + \boldsymbol{A}_i\boldsymbol{X}_i + \boldsymbol{B}_i\boldsymbol{Y}_i + \boldsymbol{Y}_i^{\mathrm{T}}\boldsymbol{B}_i + \pi_{ii}\boldsymbol{X}_i + \beta_i\boldsymbol{M}_i\boldsymbol{M}_i^{\mathrm{T}} - \alpha\boldsymbol{X}_i$,
$\boldsymbol{M}(\boldsymbol{X}_i) = \left[\sqrt{\pi_{i1}}\boldsymbol{X}_i \quad \cdots \quad \sqrt{\pi_{i(i-1)}}\boldsymbol{X}_i \quad \sqrt{\pi_{i(i+1)}}\boldsymbol{X}_i \quad \cdots \quad \sqrt{\pi_{iN}}\boldsymbol{X}_i\right]$,
$\boldsymbol{N}(\boldsymbol{X}_i) = -\mathrm{diag}\{\boldsymbol{X}_1 \quad \cdots \quad \boldsymbol{X}_{i-1} \quad \boldsymbol{X}_{i+1} \quad \cdots \quad \boldsymbol{X}_N\}$.

Proof: Note that inequality (18) is equivalent the following relation

$$\boldsymbol{S}_i = \begin{bmatrix} \boldsymbol{\Xi}_i & \boldsymbol{P}_i\boldsymbol{B}_{di} & \overline{\boldsymbol{C}}_i^{\mathrm{T}} \\ \boldsymbol{B}_{di}^{\mathrm{T}}\boldsymbol{P}_i & -\gamma^2\boldsymbol{I} & \boldsymbol{D}_{di}^{\mathrm{T}} \\ \overline{\boldsymbol{C}}_i & \boldsymbol{D}_{di} & -\boldsymbol{I} \end{bmatrix} < 0$$

In order to dealt with the uncertainties described as the form in equation (5), we can use the following approach
$\boldsymbol{S}_i = \boldsymbol{T}_i + \Delta\boldsymbol{T}_i < 0$

where

$$\boldsymbol{T}_i = \begin{bmatrix} \boldsymbol{\Lambda}_i - \alpha\boldsymbol{P}_i & \boldsymbol{P}_i\boldsymbol{B}_{di} & (\boldsymbol{C}_i + \boldsymbol{D}_i\boldsymbol{K}_i)^{\mathrm{T}} \\ \boldsymbol{B}_{di}^{\mathrm{T}}\boldsymbol{P}_i & -\gamma^2\boldsymbol{I} & \boldsymbol{D}_{di}^{\mathrm{T}} \\ \boldsymbol{C}_i + \boldsymbol{D}_i\boldsymbol{K}_i & \boldsymbol{D}_{di} & -\boldsymbol{I} \end{bmatrix},\ \Delta\boldsymbol{T}_i = \begin{bmatrix} (\Delta\boldsymbol{A}_i + \Delta\boldsymbol{B}_i\boldsymbol{K}_i)^{\mathrm{T}}\boldsymbol{P}_i + \boldsymbol{P}_i(\Delta\boldsymbol{A}_i + \Delta\boldsymbol{B}_i\boldsymbol{K}_i) & 0 \\ 0 & 0 \end{bmatrix}$$

in which $\boldsymbol{\Lambda}_i = (\boldsymbol{A}_i + \boldsymbol{B}_i\boldsymbol{K}_i)^{\mathrm{T}}\boldsymbol{P}_i + \boldsymbol{P}_i(\boldsymbol{A}_i + \boldsymbol{B}_i\boldsymbol{K}_i) + \sum_{j=1}^{N}\pi_{ij}\boldsymbol{P}_j$.
According to Lemma 1, $\Delta\boldsymbol{T}_i$ can be presented as the following form

$$\Delta\boldsymbol{T}_i = \boldsymbol{L}_{11}\boldsymbol{F}_i\boldsymbol{L}_{12} + \boldsymbol{L}_{12}^{\mathrm{T}}\boldsymbol{F}_i^{\mathrm{T}}\boldsymbol{L}_{11}^{\mathrm{T}} < \beta_i\boldsymbol{L}_{11}\boldsymbol{L}_{11}^{\mathrm{T}} + \beta_i^{-1}\boldsymbol{L}_{12}^{\mathrm{T}}\boldsymbol{L}_{12}$$

where $\boldsymbol{L}_{11}^{\mathrm{T}} = [\boldsymbol{M}_i^{\mathrm{T}}\boldsymbol{P}_i \quad 0]$, $\boldsymbol{L}_{12} = [\boldsymbol{N}_{1i} + \boldsymbol{N}_{2i}\boldsymbol{K}_i \quad 0]$.
Then we can get

$$\boldsymbol{S}_i = \begin{bmatrix} \boldsymbol{\Lambda}_i + \beta_i \boldsymbol{P}_i \boldsymbol{M}_i \boldsymbol{M}_i^{\mathrm{T}} \boldsymbol{P}_i - \alpha \boldsymbol{P}_i & \boldsymbol{P}_i \boldsymbol{B}_{di} & (\boldsymbol{C}_i + \boldsymbol{D}_i \boldsymbol{K}_i)^{\mathrm{T}} & \boldsymbol{N}_{1i}^{\mathrm{T}} + \boldsymbol{K}_i^{\mathrm{T}} \boldsymbol{N}_{2i}^{\mathrm{T}} \\ \boldsymbol{B}_{di}^{\mathrm{T}} \boldsymbol{P}_i & -\gamma^2 \boldsymbol{I} & \boldsymbol{D}_{di}^{\mathrm{T}} & 0 \\ \boldsymbol{C}_i + \boldsymbol{D}_i \boldsymbol{K}_i & \boldsymbol{D}_{di} & -\boldsymbol{I} & 0 \\ \boldsymbol{N}_{1i} + \boldsymbol{N}_{2i} \boldsymbol{K}_i & 0 & 0 & -\beta_i \boldsymbol{I} \end{bmatrix} < 0$$

Pre- and post-multiplying the inequality $\boldsymbol{S}_i < 0$ by block-diagonal matrix $diag\{\boldsymbol{P}_i^{-1} \quad \boldsymbol{I} \quad \boldsymbol{I} \quad \boldsymbol{I}\}$, letting $\boldsymbol{X}_i = \boldsymbol{P}_i^{-1}$ and $\boldsymbol{Y}_i = \boldsymbol{K}_i \boldsymbol{X}_i$ and applying Schur complement formula, then it leads to inequality (16).

On the other hand, we denote $\widetilde{\boldsymbol{P}}_i = \boldsymbol{R}_i^{1/2} \boldsymbol{X}_i \boldsymbol{R}_i^{1/2}$, $\sigma_P = \max_{i \in M} \sigma_{\max}(\widetilde{\boldsymbol{P}}_i)$, $\sigma_p = \min_{i \in M} \sigma_{\min}(\widetilde{\boldsymbol{P}}_i)$. Consider that $\boldsymbol{P}_i$ is mode-dependent positive defined and

$$\max_{i \in \boldsymbol{M}} \sigma_{\max}(\boldsymbol{X}_i) = \frac{1}{\min_{i \in \boldsymbol{M}} \sigma_{\min}(\boldsymbol{P}_i)}$$

It follows from condition (17) that

$$\frac{c_1}{\sigma_p} + \frac{\gamma^2 d}{\alpha}\left(1 - e^{-\alpha t}\right) < \frac{e^{-\alpha T} c_2}{\sigma_P} \tag{22}$$

Now we have that inequalities (22) implies

$$\sigma_1 < \sigma_p = \min_{i \in M} \sigma_{\min}(\widetilde{\boldsymbol{P}}_i),\ \sigma_P = \max_{i \in M} \sigma_{\max}(\widetilde{\boldsymbol{P}}_i) < 1.$$

Then inequality (21) holds by putting the above conditions into inequality (22). This completes the proof.

Remark 5. Theorem 2 has presented the sufficient condition of designing the stochastic finite-time stabilized controller for uncertain continuous-time MJSs with uncertain parameters. Note that the coupled LMIs (19-21) are respect to $\boldsymbol{X}_i$, $\boldsymbol{Y}_i$, β_i, c_1, c_2, σ_1, σ_2, t, α, d and γ^2. Therefore, for given scalars c_1, c_2, t, α and d, we can take γ^2 as optimized variable, i.e., to obtain an optimized finite-time stabilized controller, the attenuation lever γ^2 can be reduced to the minimum possible value such that LMIs (19-21) are satisfied. The optimization problem can be described as follows:

$$\begin{aligned} &\min_{\mathrm{X}_i, \mathrm{Y}_i, \sigma_1, \sigma_2, c_2, \rho} \rho \\ &\text{s.t. LMIs}(19-21)\, with\, \rho = \gamma^2 \end{aligned} \tag{23}$$

Remark 6. If condition (14) is satisfied $\alpha = 0$ and $\boldsymbol{d}(t) = 0$, we can get that $(\boldsymbol{A}_i + \Delta \boldsymbol{A}_i)^{\mathrm{T}} \boldsymbol{P}_i + \boldsymbol{P}_i (\boldsymbol{A}_i + \Delta \boldsymbol{A}_i) + \sum_{j=1}^{N} \pi_{ij} \boldsymbol{P}_j < 0$, i.e., the uncertain continuous-time MJSs (5) is Lyapunov stochastic stable (or almost asymptotically stable). If $\alpha < 0$, then it is globally exponentially stochastically stable. For more results regarding to the stability analysis of this class of systems, we refer the reader to [5-14].By using the MATLAB LMI Toolbox [23], it is straightforward to check the feasibility of Theorem 2 and Remark 5. In order to illustrate the effectiveness of the developed techniques, we will give a numerical example about uncertain continuous-time MJSs in the following Section 4.

4. Numerical examples

Consider the energy-storing electrical circuit illustrated by Figure 1. We assume that the circuit parameters are $C_1 = 2.0\times10^3\,\mu F$, $C_2 = 1.6\times10^3\,\mu F$, $C_3 = 1.0\times10^3\,\mu F$, $L = 0.1H$, $R_1 = 100\Omega$ and $R_2 = 1k\Omega = 1\times10^3\Omega$. Measuring time in seconds, the currents $x_2(t)$ in A and $u(t)$ in V, the state model can be given by

$$\begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) \\ z(t) = C_i x(t) + D_i u(t) \end{cases} \tag{24}$$

where $A_1 = \begin{bmatrix} -0.5 & 500 \\ -10 & -1000 \end{bmatrix}$, $A_2 = \begin{bmatrix} -0.625 & 625 \\ -10 & -1000 \end{bmatrix}$, $A_3 = \begin{bmatrix} -1 & 1000 \\ -10 & -1000 \end{bmatrix}$, $B_1 = B_2 = B_3 = \begin{bmatrix} 0 \\ 10 \end{bmatrix}$, $C_1 = C_2 = C_3 = \begin{bmatrix} 1 & 0 \end{bmatrix}$, $D_1 = D_2 = D_3 = 0.5$.

It has been recognized that the unknown disturbances and parameter uncertainties are inherent features of many physical process and often encountered in engineering systems, their presences must be considered in realistic controller design. For these, we let

$$\boldsymbol{B}_{d1} = \begin{bmatrix} -0.1 \\ 0.1 \end{bmatrix},\ \boldsymbol{B}_{d2} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix},\ B_{d3} = \begin{bmatrix} 0.1 \\ -0.2 \end{bmatrix},\ \boldsymbol{D}_{d1} = [-0.1],\ \boldsymbol{D}_{d2} = [0.1],\ D_{d3} = [-0.1].$$

And the transition rate matrix is defined by $\Pi = \begin{bmatrix} -0.3 & 0.25 & 0.05 \\ 0.1 & -0.2 & 0.1 \\ 0.03 & 0.07 & -0.1 \end{bmatrix}$.

The uncertain parameters and mode switching is governed by a Markov chain that has the following transition rate matrix:

$$M_1 = M_2 = M_3 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix},\ N_{11} = N_{12} = N_{13} = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix},\ N_{21} = N_{22} = N_{23} = 0.$$

With the initial value for $\alpha = 1.0$, $c_1 = 0.5$, $T = 2$, $\boldsymbol{R}_i = \boldsymbol{I}_2$, $d = 4$, $\tau = 0.9$, we solve LMIs (19-21) by Theorem 2 and optimization algorithm (21) and get the following optimal H_∞ controller

$$K_1 = \begin{bmatrix} -2.2134 & 1.6458 \end{bmatrix},\ K_2 = \begin{bmatrix} -1.2537 & 3.4522 \end{bmatrix},\ K_3 = \begin{bmatrix} -3.5859 & 2.9210 \end{bmatrix}.$$

guarantees the stochastic finite-time stabilization via state-feedback of desired close-loop properties with the optimization attenuation lever $\gamma = 0.1325$.

Subsequently, to demonstrate the effectiveness of the design in case 1, we assume the unknown inputs are unknown white noise with noise power 0.1 over a finite-time interval $t \in [0 \quad 10]$. With the initial condition $x(0) = \begin{bmatrix} 1 & 0.8 \end{bmatrix}^{\mathrm{T}}$, the jump mode and system states $\boldsymbol{x}(t)$ are shown in Figure.2-Figure.3 respectively. It can be seen that the system is robustly finite-time stabilized by the designed robust stochastic finite-time controller.

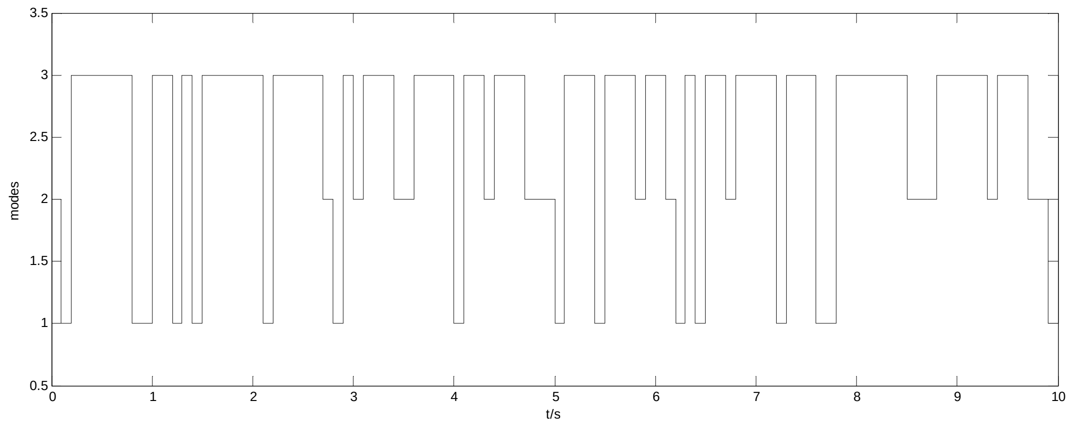

Figure 2. The jump mode

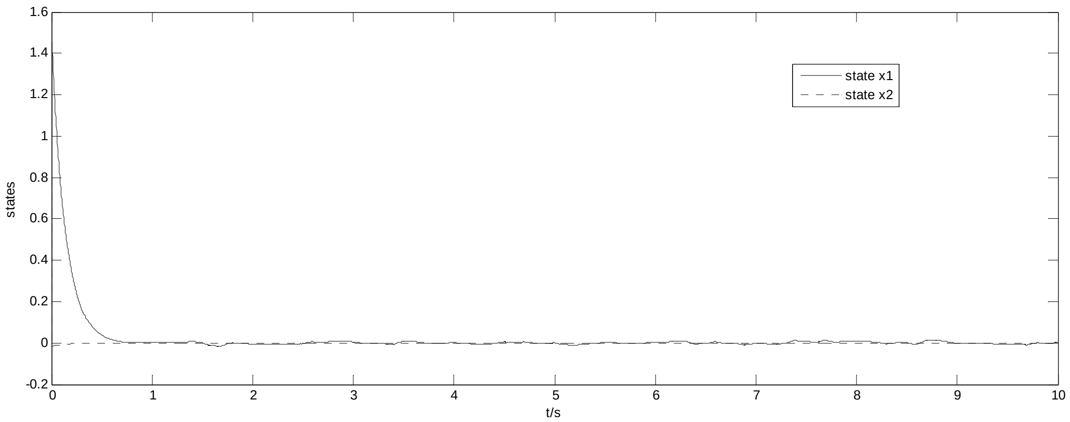

Figure 3. The system state $x(t)$

5. Conclusions

In this paper, the solution to the problem of stochastic finite-time H_∞ control for a class of uncertain MJSs has been presented. The uncertain parameters are assumed to unknown, but norm bounded. By reconstructing the overall closed-loop dynamic system, sufficient condition has been derived such that the uncertain MJSs is stochastic finite-time boundedness (SFTB) and stochastic finite-time stabilization and satisfies the given H_∞ control index. The main results are presented in the form of linear matrix inequalities (LMIs). Simulation example demonstrates the contribution of the main results and more details related to the stochastic finite-time control problem of stochastic MJSs will be studied in the further.

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Shuping He was born in 1983. He is with the Institute of Automation, Jiangnan University, Wuxi 214122, P. R. China, and now he is the visiting doctor in the Control System Center, University of Manchester Institute of Science and Technology, Manchester, U.K. His research interests include robust control, filtering and fault detection of stochastic system.
E-mail address: henyhappy@yahoo.com.cn

Fei Liu was born in 1965. He received the Ph. D degree in control science and control engineering from Zhejiang University. Now he is currently a professor of Jiangnan University in Institute of Automation. His research interests include advanced control theory and application, industrial process monitoring, and system fault detection and diagnosis, etc.
E-mail address: fliu@jiangnan.edu.cn

