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Transverse vibration of cracked graded shear beam with axial motion

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Abstract
This paper presents a study on the vibration characteristic of open edge cracks graded moving shear beam. The material property gradient is based on the distribution of the power-law in the direction of the beam thickness. The vibration equation is obtained depending on the precept of Hamilton principle and resolved by the extension of Galerkin's approach. To represent the cracking in the beam, a rotational spring is used. The effects of the axial velocity, gradient index, and cracking parameters on vibration characteristics are observed. Furthermore, the shapes of the model are determined for simply supported cracked moving graded shear beam. The results show that the natural frequencies decrease with the rise in the axial velocity, the crack depth, and the material property index. The percentage of decrease in the natural frequencies as a result of increase in crack depth ratio is 40%.

Keywords: Shear Model; Cracks; Functionally Graded Material; Beam; Axially Moving.

1. Introduction

The advanced type of composite materials produced by the combination of two materials are dynamically graduated materials and the properties of the gradient material differ from the properties of the main materials of the component. This material is used in many engineering applications, such as rocket engine components, aerospace structures, turbine blade structures, fusion energy devices, defense industries [1]. In the mid-eighties of the last century, Japanese scientists first used this form of material, and it became the object of researchers' attention later on, and many researchers covered it. The vibration of a graded beam, which differs in its properties in the direction of thickness by relying on the law of force, was demonstrated by S. A. Sina et al. [2]. The vibration equation was obtained by applying the precept of Hamilton and resolved by the analytical solution. The concern of moving load applied to a gradient beam whose power-law properties change during the thickness direction was discussed by K. Rajabi et al. [3]. Also, the power-law form was used by F. Q. Zhao and Z. M. Wang [4], to represent the change of material properties through-thickness direction for the system of deploying graded beam. The motion equation was modeled using Euler theory and Hamilton’s precept and disband by the Galerkin's technique. The results clear up the improvement in the initial length of the FGM beam deployed would diminish the frequency of vibration and the beam amplitude. The increase in the cross-sectional height of the FGM beam deployed will cause the frequency of vibration to boost but has no effect on the amplitude.

The properties of the gradient material may change in one or two directions, such as axial or thicknesses directions, and this resulting gradient can be expressed using the power, exponential, and sigmoid laws as Aydogdu and Taskin [5], the properties of the gradient material differ through-thickness direction relying on the laws of power and exponential. Hamilton's precept was used to derive the vibration equation of graded beam with the edge of simply supported. The characteristics of vibration rising with increasing the ratio of beam length to its height while decrease with the rise of gradient index. Huu-Tai Thai and Thus P. Vo [6], in order to study the vibration of the graded beam that its properties differ in the direction of beam thickness, counted on the power-law, the different theory of higher-order shear deformation was used. The results explained that increasing the power-law index would decrease the FG beam's stiffness, resulting in an increase in deflections and a decrease in natural frequencies. M. Simsek [7], the properties of graded material change during bi- directions as depth and axial counted on the law of power. The theories of Euler and Timoshenko were used to model the vibration equation by applying the precept of Hamilton to study the beam buckling. This equation was resolved by the method of Ritz. In order to achieve the desired objectives, the buckling behavior can be managed by selecting the relevant gradient indexes as shown by the results. Y. S. Al Rjoub and A. G. Hamad [8] studied the vibration of the gradient beam, using Euler and Timoshenko models with H-H, C-C, C-F, and C-H end support. They used the transfer matrix to resolve the vibration equation. The impact of porosity that decreases the natural non-dimensional frequencies are critical in the volume fraction index taller than 1as indicated by the results. The decrease in non-dimensional natural frequencies is as the value increases in the volume fraction index. A. E. Alshorbagy et al. [9] used the approach of the finite element to resolve the vibration equation of graded beam modeled by the Euler theorem and Hamilton's precept. The distribution of power-law was relying on to represent the gradient in the material properties through two directions as axial and thickness. Y Liu et al. [10], the gradient in the properties of material followed the law of exponential to represent it. Explained the impact of single delamination on graded beam. C. F. Lü et al. [11], the material properties differ in two axes (length and thickness) using exponential law. S. N. PADHI et al. [12] adopted the sigmoid law to explain the gradient in the material properties of the graded beam.

In addition, the cracked beam has been touched upon by several researchers. The effect of multiple cracking on the gradient beam was discussed by T. V. Lien et al. [13]. Modeling the kinematic equation was based on Timoshenko's theory. The results showed that an increase in the depth of the crack, as well as the amount of crack, causes the natural frequencies to decrease. H. Ozturk et al. [14] examined the impact of the moving load on the cracked beam. As the crack depth expands, the natural frequencies of the cracked beam get smaller. In addition, when positioned at the unique points (nodes in mode shapes) along the beam, the crack does not affect the natural frequencies. E.C. Yang et al. [15] used the Continuous Beam theory to model the cracked graded beam. The transfer matrix method was used to resolve the vibration equation. The increase in crack depth leads to a decrease in the natural frequency ratio, while the increase in the slenderness and elasticity ratio leads to a rise in natural frequencies. T. YAN and J. YANG [16], the transverse moving load and compressive force axially affected the cracked graded beam. Due to the presence of edge crack, the dynamic deflection grows as well as the axial compressive load, the axial compression is much more affected than the edge crack. Sometimes, the crack is represented as linear or rotational spring or both. N. T. Khiema et al. [17] used Timoshenko's model to set up equations governing the motion of the cracked gradient beam in which the crack was represented as two linear and rotational springs. Y. Cunedioglu and S. Shabani [18] used a finite element approach to analyze the vibration equation of cracked graded beam with multi-layers. In mode shapes, the effects of phase position are more profound than the depth of the crack and crack location modifications. The nonlinear vibration of the cracked graded beam based on the Timoshenko model and the precept of Hamilton appeared in S. Kitipornchai et al. [19]. Rotational spring models are used to model the crack. The linear frequency is greatly reduced with an increase in crack depth, but the nonlinear frequency ratio and mode shapes are much less impacted by the change in crack depth.

Muhannad Al-Waily [20] employed the analytical solution and finite element technique as numerical solution to resolve the vibration equation of pinned and clamped ends beam with crack which obtained utilizing Euler theory. The position of crack near the ends of beam has small influence on beam stiffness which make the beam frequency is high with compare the position of crack near the beam middle. In addition, the crack depth rising leads to drop in the beam stiffness which decrease the beam frequencies. Influences of flow velocity and angle of crack on the characteristics of pipe vibration and its flow with simple support end conditions were clarified by Muhannad Al-Waily et al. [21]. The results were obtained by experimentally and numerically using finite element approach. At certain crack depth, certain crack position, and certain crack angle, the frequencies of pipe were drop as a result to increase in flow velocity. The change of crack angle from crack parallel to pipe to perpendicular on pipe make the influence of crack on pipe stiffness is increase which leads to decrease in pipe frequencies.

Alborz Mirzabeigy et al. [22] studied free vibration of simply supported double-beam with crack. Euler and Winkler models were used to obtain an equation to find the natural frequencies of the beam, but the resulting equation is algebraically meaning that it needs a numerical solution as it does not explain the effect of the parameters on the frequencies, so Rayleigh's method was used to derive an explicit equation to find the natural frequencies. The impacts of crack depth and crack position on the generation of heat in the beam under the periodic load with S-S, C-C, and C-F boundary conditions were illustrated by Diyaa H. J. Al-Zubaidi et al. [23]. Euler theory was used to obtain the motion equation which solved utilizing analytically and numerically solutions such as finite element technique. The results showed the generation of heat in the beam is rise due to crack depth increasing, additionally, the approaching the site of the crack than the instant higher of the beam make the heat generation of the beam is rise. The influences of crack parameters on the beam vibration characteristics under harmonic load effect with pinned, clamped, and cantilever ends were addressed by Diyaa H. J. Al-Zubaidi et al. [24]. Finite element approach was employed to solve the vibration equation which found using Euler beam theory. The crack depth increasing leads to decrease the beam stiffness and beam frequencies, but beam deflection increase. The results illustrated also that the action for frequency harmonic load applied rise with increment the crack depth.

There is also a lot of literature covering the subject of homogeneous and graded beam movement. In order to evaluate the motion equation of the graded beam with axially moving, Yusuf Yesilce [25], implemented the Differential Transform method. The results shows that the increase in the axial tensile force results in an increase in the natural frequency values under various boundary conditions. L. Q. Yao et al. [26], investigated the axially traveling inhomogeneous micro beam. Changjian Ji et al. [27] studied transverse vibration of graded moving Nano-beam. As seen in the results, the rise in axial velocity and the gradient indicator lead to a decrease in natural frequencies. In addition, Hu Ding et al. [28] used dynamic stiffness as a method to discuss the axially movement of homogeneous beams subjected to rotational and vertical springs at both ends of the beam. Jer-Rong Chang et al. [29] used the Rayleigh theory to model the axially moving beam and the equation of motion was solved using the process of finite elements.

In this paper, a study of vibration behavior cracked axially movement graded beam based on the shear theory and Hamilton principles adopt. To solve the system's vibration equation, Galerkin's technique is used. The effects of several variables on the characteristics of the vibration, including crack width, crack location, axial speed, and material property gradient index, are observed.

2. Theory

A cracked axially moving graded beam is shown in Figure 1. The co-ordinates of the axial and thickness axes are expressed by X and Z. The length of the beam is expressed by the symbol L, while the height is h and the width is b. The crack parameters are represented as depth and position by symbols ɑ and x_c, respectively. Pure ceramic is present on the upper surface of the beam, while pure metal is present on the lower surface.

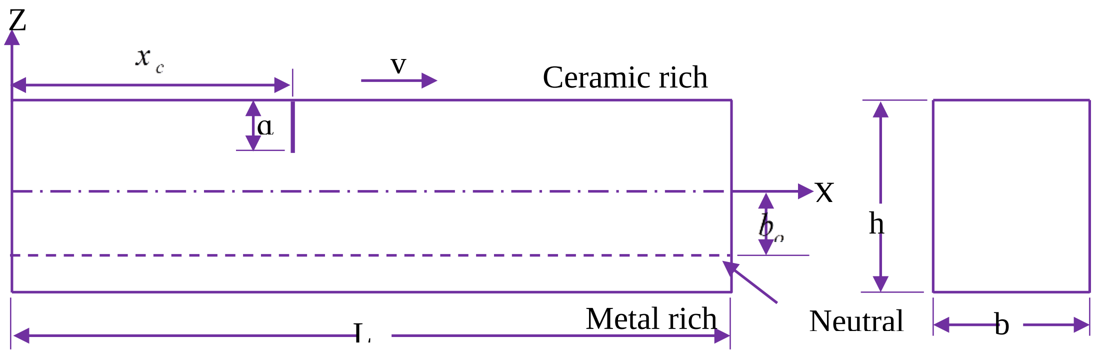


Figure 1. The cracked moving graded beam

During the beam thickness, the properties of the gradient materials differed and depended on the law of power, and this material consists of two basic materials, namely metal and ceramic.

A graded beam volume fraction can be expressed as [30]:

$$V_t = \left(\frac{z}{h} + \frac{1}{2}\right)^p \qquad 0 \le p \le \infty \tag{1}$$

$$V_t + V_b = 1 \tag{2}$$

Where z is the distance to the mid-plane of the FG beam $\left(-\frac{h}{2} \le z \le \frac{h}{2}\right)$, $-\frac{h}{2}$ and $\frac{h}{2}$ represented the thickness of beam at top and bottom, respectively, while p is defined as the index of gradient.
For power-law exponents, the effective modulus of elasticity and effective density are given as:

$$E(z) = (E_t - E_b)\left(\frac{z}{h} + \frac{1}{2}\right)^p + E_b$$
$$\rho(z) = (\rho_t - \rho_b)\left(\frac{z}{h} + \frac{1}{2}\right)^p + \rho_b \tag{3}$$

In which E_c and E_m represent the elasticity modulus of ceramic and metal, respectively.
The displacements of shear FG beams can be obtained as [27]

$$u_1(x,z,t) = -(z - b_o)\varphi(x,t)$$
$$u_2(x,z,t) = w(x,t) \tag{4}$$

Where $u_1(x,z,t)$ and $u_2(x,z,t)$ are axial and transverse displacements of beam, respectively. The transverse displacement of middle plane is $w(x,t)$ while $\varphi(x,t)$ represent cross section rotation angle.
The normal strain and stress of FG shear beam are given as:

$$\varepsilon_x = \frac{\partial u_1(x,t)}{\partial x} = -(z - b_o)\frac{\partial \varphi(x,t)}{\partial x}$$
$$\sigma_x = E(z)\varepsilon_x = -(z - b_o)E(z)\frac{\partial \varphi(x,t)}{\partial x} \tag{5}$$

The shear strain and stress of FG shear beam are given as:

$$\gamma_{xz} = \frac{\partial u_1(x,t)}{\partial z} + \frac{\partial u_2(x,t)}{\partial x} = -\varphi + \frac{\partial w}{\partial x}$$
$$\tau_{xz} = k_o G(z)\varepsilon_{xz} = k_o G(z)\left(-\varphi + \frac{\partial w}{\partial x}\right) \tag{6}$$

Where k_o represent the shear correction factor and its equal to 5/6
The potential energy of Shear FG beam can be expressed as [31]:

$$P_E = \int_0^l\int_A \varepsilon_x \sigma_x dAdx + \int_0^l\int_A \gamma_{xz}\tau_{xz} dAdx = \int_0^l\int_A (z - b_o)^2 E(z)\frac{\partial^2 \varphi}{\partial x^2} dAdx$$
$$+ \int_0^l\int_A k_o \frac{E(z)}{2(1+\mu)}\left(\frac{\partial w}{\partial x} - \varphi\right)^2 dAdx = I_1\int_0^l \frac{\partial^2 \varphi}{\partial x^2} dx + I_2\int_0^l \frac{k_o}{2(1+\mu)}\left(\frac{\partial w}{\partial x} - \varphi\right)^2 dx \tag{7}$$

Where $(I_1, I_2) = \int_A (1, (z - b_o)^2) E(z) dA$ (8)

The actual position of neutral axis determined as [26]:

$$b_o = \frac{hp\alpha}{2(2+p)(1+p+\alpha)}, \ \alpha = (E_{ratio} - 1), \ E_{ratio} = \frac{E_c}{E_m} \tag{9}$$

The variation of potential energy of shear FG beam can be expressed as:

$$\delta P_E = I_1 \int_0^l \frac{\partial \varphi}{\partial x}\frac{\partial \delta\varphi}{\partial x} dx + \frac{k_o I_2}{2(1+\mu)} \int_0^l \left(\frac{\partial w}{\partial x} - \varphi\right)\left(\frac{\partial \delta w}{\partial x} - \delta\varphi\right) dx \tag{10}$$

The kinetic energy of shear FG beam can be expressed as [31]:

$$K_E = \frac{1}{2}\rho(z)\int_0^l\int_A \left(\frac{\partial w}{\partial t} + v\frac{\partial w}{\partial x}\right)^2 dA dx = n_1 \int_0^l \left(\frac{\partial^2 w}{\partial t^2} + 2v\frac{\partial^2 w}{\partial x \partial t} + v^2 \frac{\partial^2 w}{\partial x^2}\right) dx \tag{11}$$

Where

$$n_1 = \int_A \rho(z) dA \tag{12}$$

The variation of kinetic energy of shear FG beam can be expressed as:

$$\delta K_E = n_1 \int_0^l \left(\frac{\partial w}{\partial t}\frac{\partial \delta w}{\partial t} + v\frac{\partial w}{\partial x}\frac{\partial \delta w}{\partial t} + v\frac{\partial w}{\partial t}\frac{\partial \delta w}{\partial x} + v^2\frac{\partial w}{\partial x}\frac{\partial \delta w}{\partial x}\right) dx \tag{13}$$

2.1 Hamilton's principle

The general form of Hamilton's principle is given by [31]

$$\delta \int_{t_1}^{t_2} (P_E - K_E) dt = 0 \tag{14}$$

Substituting the energy terms equations (10) and (13) into Hamilton's principle, integrating by parts and setting the coefficients of equal to zero, the vibration equations of axially moving functionally graded shear beam are written as:

$$\frac{I_2 k_o}{2(1+\mu)}\left(\frac{\partial^2 w}{\partial x^2} - \frac{\partial \varphi}{\partial x}\right) = n_1\left(\frac{\partial^2 w}{\partial t^2} + 2v\frac{\partial^2 w}{\partial x \partial t} + v^2\frac{\partial^2 w}{\partial x^2}\right) \tag{15}$$

$$\frac{I_2 k_o}{2(1+\mu)}\left(\frac{\partial w}{\partial x} - \varphi\right) + I_1\frac{\partial^2 \varphi}{\partial x^2} = 0 \tag{16}$$

From equation (15)

$$\frac{\partial \varphi}{\partial x} = \frac{\partial^2 w}{\partial x^2} - \frac{2n_1(1+\mu)}{I_2 k_o}\left(\frac{\partial^2 w}{\partial t^2} + 2v\frac{\partial^2 w}{\partial x \partial t} + v^2\frac{\partial^2 w}{\partial x^2}\right) \tag{17}$$

Substituting equation (17) into equation (16), we get

$$n_1\left(\frac{\partial^2 w}{\partial t^2} + 2v\frac{\partial^2 w}{\partial x \partial t} + v^2\frac{\partial^2 w}{\partial x^2}\right) + I_1\left(\frac{\partial^4 w}{\partial x^4}\right) - \frac{2n_1 I_1(1+\mu)}{I_2 k_o}\left(\frac{\partial^4 w}{\partial t^2 \partial x^2} + 2v\frac{\partial^4 w}{\partial x^3 \partial t} + v^2\frac{\partial^4 w}{\partial x^4}\right) = 0 \tag{18}$$

The final governing equation of shear FG beam is given as:

$$\left(I_1 - \frac{2n_1 I_1(1+\mu)}{I_2 k_o}v^2\right)\frac{\partial^4 w}{\partial x^4} - 4v\frac{n_1 I_1(1+\mu)}{I_2 k_o}\frac{\partial^4 w}{\partial x^3 \partial t} - \frac{2n_1 I_1(1+\mu)}{I_2 k_o}\frac{\partial^4 w}{\partial x^2 \partial t^2} + n_1 v^2\frac{\partial^2 w}{\partial x^2} + 2vn_1\frac{\partial^2 w}{\partial x \partial t} + n_1\frac{\partial^2 w}{\partial t^2} = 0 \tag{19}$$

In this work, simply supported boundary conditions were used,

$$x = 0, w(0) = M(0) = 0 \qquad x = l, w(l) = M(l) = 0 \tag{20}$$

The following dimensionless parameters are used to simplify the calculation

$$\xi = \frac{x}{L}, \eta = \frac{w}{L}, V = v\sqrt{\frac{\rho_m L^2 A}{E_m I}}, T = t\sqrt{\frac{E_m I}{\rho_m L^4 A}}, \qquad \alpha_s = 2(1+\mu)\frac{I}{k_o A L^2} \tag{21}$$

By substituting the dimensionless parameters into the equation (19) and boundary conditions (20). The non-dimensional governing equation can be written as:

$$\left(\frac{\alpha_2}{\alpha_4}-\frac{\alpha_s V^2\alpha_2}{\alpha_3}\right)\frac{\partial^4\eta}{\partial\xi^4}+V^2\frac{\partial^2\eta}{\partial\xi^2}+2V\left(\frac{\partial\eta}{\partial\xi}-\frac{\alpha_s\alpha_2}{\alpha_3}\frac{\partial^3\eta}{\partial\xi^3}\right)\frac{\partial\eta}{\partial T}+\left(1-\frac{\alpha_s\alpha_2}{\alpha_3}\frac{\partial^2\eta}{\partial\xi^2}\right)\frac{\partial^2\eta}{\partial T^2}=0 \tag{22}$$

And boundary conditions in dimensional form written as:

At $\xi=0, \eta(0)=\eta''(0)=0$ (23-a)

At $\xi=1, \eta(1)=\eta''(1)=0$ (23-b)

2.2 Crack Modeling

We can derive the average strain energy via the theory of fracture mechanics by implementing a local flexibility model such as [32]

$$S=\frac{g^2}{E_{tip}} \tag{24}$$

Where

S: Average strain energy; g: Stress intensity factor; and E_{tip} : Modulus of elasticity at the crack tip of beam

$$S=-\frac{\partial U}{\partial a}=-\frac{d}{da}\left(\frac{\theta M(a)}{2}\right)=-\frac{1}{2}\theta\frac{dM}{da} \tag{25}$$

Where M, θ and ɑ are the bending moment, strain energy, deflection due to bending, and the depth of the crack, respectively.

The resulting beam deflection due to the bending can be expressed as the following relationship:

$\theta=cM$

The beam compliance at the crack position is symbolized by the symbol c, which can be derived from compliance equation as:

$$\int_0^a Sda=-\frac{\theta M}{2}=-\frac{\theta^2}{2c} \tag{26}$$

The magnitudes of the crack can be written as [33]:

$$c=6\pi(1-\nu^2)h.\alpha_2.f(z) \tag{27}$$

Where c is depend on gradient index and crack depth ratio and $f(z)$ is a function of the crack depth ratio (z=a/h) and defined as below:

$$f(z)=0.6272z^2-1.04533z^3+4.5948z^4-9.9736z^5+20.2948z^6-33.0351z^7 +47.1063z^8-40.7556z^9+19.6z^{10} \tag{28}$$

In case of cracking, the beam is composed of two segments, as shown in the below Figure 2.

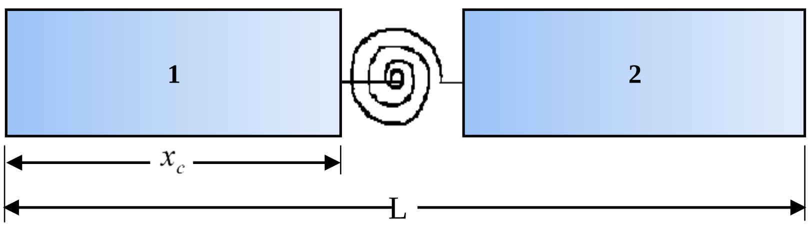


Figure 2. The crack representation as a rotational spring.

After dividing the beam into two parts, the first part is integrated from 0 to x_c while the second part is integrated from x_c to L. Thus, the governing equations of cracked FG beam with axial motion are:

At $0\le x\le x_c$

$$\left(I_1-\frac{2n_1I_1(1+\mu)}{I_2k_o}v^2\right)\frac{\partial^4 w_1}{\partial x^4}-4v\frac{n_1I_1(1+\mu)}{I_2k_o}\frac{\partial^4 w_1}{\partial x^3\partial t}-\frac{2n_1I_1(1+\mu)}{I_2k_o}\frac{\partial^4 w_1}{\partial x^2\partial t^2}+n_1v^2\frac{\partial^2 w_1}{\partial x^2}$$
$$+2vn_1\frac{\partial^2 w_1}{\partial x\partial t}+n_1\frac{\partial^2 w_1}{\partial t^2}=0 \tag{29-a}$$

And at $x_c \le x \le L$

$$\left(I_1-\frac{2n_1I_1(1+\mu)}{I_2k_o}v^2\right)\frac{\partial^4 w_2}{\partial x^4}-4v\frac{n_1I_1(1+\mu)}{I_2k_o}\frac{\partial^4 w_2}{\partial x^3\partial t}-\frac{2n_1I_1(1+\mu)}{I_2k_o}\frac{\partial^4 w_2}{\partial x^2\partial t^2}+n_1v^2\frac{\partial^2 w_2}{\partial x^2}$$
$$+2vn_1\frac{\partial^2 w_2}{\partial x\partial t}+n_1\frac{\partial^2 w_2}{\partial t^2}=0 \tag{29-b}$$

The dimensionless governing equations of cracked FG beam with axial motion are:

$0 \le \xi \le \xi_c$

$$\left(\frac{\alpha_2}{\alpha_4}-\frac{\alpha_s V^2\alpha_2}{\alpha_3}\right)\frac{\partial^4\eta_1}{\partial\xi^4}+V^2\frac{\partial^2\eta_1}{\partial\xi^2}+2V\left(\frac{\partial\eta_1}{\partial\xi}-\frac{\alpha_s\alpha_2}{\alpha_3}\frac{\partial^3\eta_1}{\partial\xi^3}\right)\frac{\partial\eta_1}{\partial T}+\left(1-\frac{\alpha_s\alpha_2}{\alpha_3}\frac{\partial^2\eta_1}{\partial\xi^2}\right)\frac{\partial^2\eta_1}{\partial T^2}=0 \tag{30-a}$$

$\xi_c \le \xi \le 1$

$$\left(\frac{\alpha_2}{\alpha_4}-\frac{\alpha_s V^2\alpha_2}{\alpha_3}\right)\frac{\partial^4\eta_2}{\partial\xi^4}+V^2\frac{\partial^2\eta_2}{\partial\xi^2}+2V\left(\frac{\partial\eta_2}{\partial\xi}-\frac{\alpha_s\alpha_2}{\alpha_3}\frac{\partial^3\eta_2}{\partial\xi^3}\right)\frac{\partial\eta_2}{\partial T}+\left(1-\frac{\alpha_s\alpha_2}{\alpha_3}\frac{\partial^2\eta_2}{\partial\xi^2}\right)\frac{\partial^2\eta_2}{\partial T^2}=0 \tag{30-b}$$

2.3 Solution Method

The general form of Galerkin's equation is given as [34]:

$$\eta(\xi,T)=\sum_{r=1}^{n}\phi_r(\xi)q_r(T) \tag{31}$$

Where $\phi_r(\xi)$ and $q_r(T)$ represents the shape function and generalized coordinates, respectively.

The mode shape function of the cracked FG beam can be expressed as the sum of the formation function of the un-cracked FG beam and the polynomial of (ξ) [35, 36]:

$$\phi_{i1}=\phi(\xi)+A_o+A_1\xi+A_2\xi^2+A_3\xi^3;\ for\,0\le\xi\le\xi_c$$
$$\phi_{i2}=\phi(\xi)+B_o+B_1\xi+B_2\xi^2+B_3\xi^3;\ for\,\xi_c\le\xi\le 1 \tag{32}$$

The continuity conditions and compatibility of the cracked FG beam at $\xi=\xi_c$ are given by

$$\phi_{i1}(\xi_c)=\phi_{i2}(\xi_c);\quad \phi''_{i1}(\xi_c)=\phi''_{i2}(\xi_c)\ ;\ \phi'''_{i1}(\xi_c)=\phi'''_{i2}(\xi_c);$$
$$\phi'_{i2}(\xi_c)-\phi'_{i1}(\xi_c)=c\phi''_{i2}(\xi_c) \tag{33}$$

By coupling the boundary in equations (23-a) & (23-b) and compatibility conditions in eq. (32) and equation (33), we get

$$\phi_{i1}=\phi(\xi)-\left(\frac{\xi_c-1}{\xi_c}\right)\left[c\xi_c(n\pi)^2\sin(n\pi\xi_c)\right]\xi \quad \text{for}\quad (0\le\xi\le\xi_c) \tag{34-a}$$

$$\phi_{i2}=\phi(\xi)+(1-\xi)\left(c\xi_c(n\pi)^2\sin(n\pi\xi_c)\right) \quad \text{for}\quad (\xi_c\le\xi\le 1) \tag{34-b}$$

By substituting equation (31) into equations (30-a) and (30-b), integration this equations and multiplying by ϕ_s , the resultant vibration equation in matrix form is given as

$$[M]\ddot{q}+[C]\dot{q}+[K]q=0 \tag{35}$$

Where $[M]$ is the mass matrix, $[C]$ is the damping matrix and $[K]$ is stiffness matrix of cracked axially moving shear FGM beam and their elements are given as follows:

$$M_{r,s} = \int_0^{\xi_c} \left(\phi_{r1}(\xi) - \frac{\alpha_s \alpha_4}{\alpha_3} \phi''_{r1}(\xi) \right) \phi_{s1}(\xi) d\xi + \int_{\xi_c}^{1} \left(\phi_{r2}(\xi) - \frac{\alpha_s \alpha_4}{\alpha_3} \phi''_{r2}(\xi) \right) \phi_{s2}(\xi) d\xi \tag{36}$$

$$C_{r,s} = \int_0^{\xi_c} \left(2V\phi'_{r1}(\xi) - 2V \frac{\alpha_r \alpha_4}{\alpha_3} \phi'''_{r1}(\xi) \right) \phi_{s1}(\xi) d\xi + \int_{\xi_c}^{1} \left(2V\phi'_{r2}(\xi) - 2V \frac{\alpha_r \alpha_4}{\alpha_3} \phi'''_{r2}(\xi) \right) \phi_{s2}(\xi) d\xi \tag{37}$$

$$\begin{aligned} K_{r,s} &= \int_0^{\xi_c} \left[\left(\frac{\alpha_2}{\alpha_3} - \frac{\alpha_s \alpha_4 V^2}{\alpha_3} \right) \phi_{r1}{}^{(4)}(\xi) \phi_{s1}(\xi) + V^2 \phi''_{r1}(\xi) \phi_{s1}(\xi) \right] d\xi \\ &+ \int_{\xi_c}^{1} \left[\left(\frac{\alpha_2}{\alpha_3} - \frac{\alpha_s \alpha_4 V^2}{\alpha_3} \right) \phi_{r2}{}^{(4)}(\xi) \phi_{s2}(\xi) + V^2 \phi''_{r2}(\xi) \phi_{s2}(\xi) \right] d\xi \end{aligned} \tag{38}$$

By solving the eigenvalue of equation (35), the frequencies of the system can be observed. Obviously, by letting the parameter of α_s to zero in the Equation (22), the vibration equation of the FGM Euler-Bernoulli beam will be obtained. Furthermore, by neglecting the shear factor, and axial motion in equation (35), the vibration equation of cracked FG cracked Euler-Bernoulli beam will be obtained.

3. Results

The beam used in this paper is made of functionally graded material that it is composed of ceramic and metal. Pure ceramic sits on the upper surface of the beam, while pure metal sits on the lower surface. The relevant parameters and the material properties are given as L=1m, b=0.1m, h=0.1m, as in Table 1.

Table 1. The graded material properties.

Material	Elasticity modulus	Density
Steel	210GPa	$7800 kg/m^3$
Alumina	390GPa	$3960 kg/m^3$

4. Validation

In this sub-section, we will compare the results obtained through this work at V=0, L/h=100, $E_{ratio} = \frac{E_c}{E_m} = 4$, $\rho_{ratio} = \frac{\rho_c}{\rho_m} = 1$, $\alpha_s = 0$ with a result for graded Euler beam in ref. [37], as shown in Table 2.

Table 2. Validity test of the results of this work.

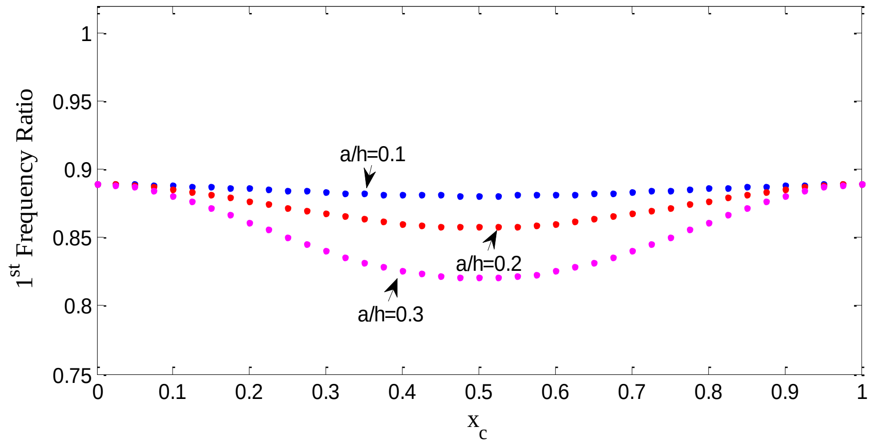
	ω_1		ω_2		ω_3	
p	Present	Ref. [37]	Present	Ref. [37]	Present	Ref. [37]
0	19.738	19.738	78.944	78.943	177.59	177.58
0.1	18.825	18.825	75.290	75.290	169.37	169.36
1	14.638	14.638	58.546	58.545	131.70	131.70
10	11.948	11.948	47.787	47.787	107.50	107.50

Table 3 shows the influences of the crack depth ratio and the location of crack on the first three mode shapes, as it is observed that the deeper the crack, the lower the natural frequencies. As the crack location is shifted from the edges to the center of the beam, the natural frequency varies and gives its maximum value at the center of the beam, i.e. 50% of the complete length. The beam is simply supported on both ends, so a symmetrical behavior is observed on both sides of the beam with respect to center of the beam.

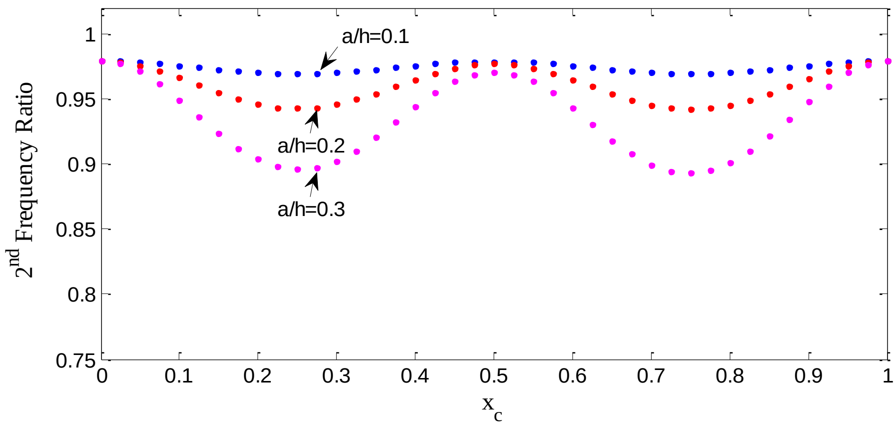
Table 3. the effect of crack depth & crack location on the natural frequencies.

x_c	ωi	a/h 0.1	0.2	0.3	0.4	0.5	0.6
0	ω_1	12.8402	12.8402	12.8402	12.8402	12.8402	12.8402
	ω_2	51.8834	51.8834	51.8834	51.8834	51.8834	51.8834
	ω_3	113.8202	113.8202	113.8202	113.8202	113.8202	113.8202
0.3	ω_1	12.7772	12.5634	12.0788	11.0913	9.3232	6.7796
	ω_2	51.2493	49.4895	46.7398	43.4715	40.6811	39.4220
	ω_3	112.4290	109.2212	105.6989	103.3529	103.1309	105.2710
0.5	ω_1	12.6692	12.1659	11.2626	9.8633	7.9403	5.6574
	ω_2	51.8845	51.8841	51.8751	51.8194	51.6230	51.1060
	ω_3	112.2847	108.7890	105.1198	103.4017	106.0681	116.2855

Figure 3 illustrates the first and second natural frequencies ratios of simple support cracked FG beam with axial motion at u=2, p=1, L/h=10. It turns out that the shapes in (a) & (b) are symmetrical. In (a), when the crack location increases, the natural frequencies are decreased until reach xc= 0.5, then the frequencies are increases in the second part of the beam until reaches the end of the beam. In (b), also the frequency ratios decrease with the increase in the location of the crack and then increase to the maximum value at xc= 0.5, after that, it begins to decrease. Besides, it is obtained that increasing the slit depth leads to a decrease in the natural frequency ratios due to the decrease in the beam stiffness, which leads to an increase in the beam deflection and thus reduces the natural frequencies.



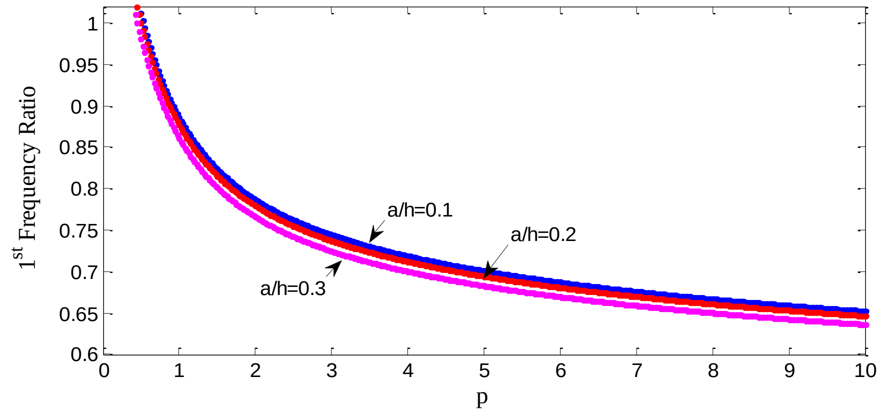
(a)



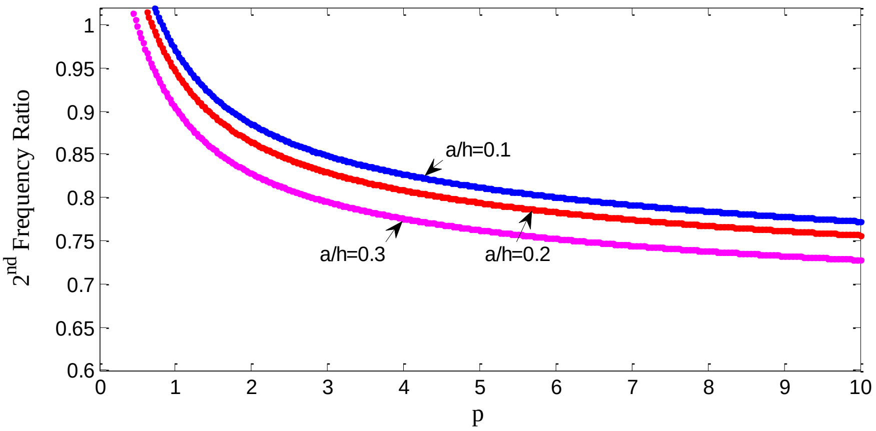
(b)

Figure 3. The relationship between the natural frequencies ratios of FG beam and the crack location (xc) with different crack depth (a/h).

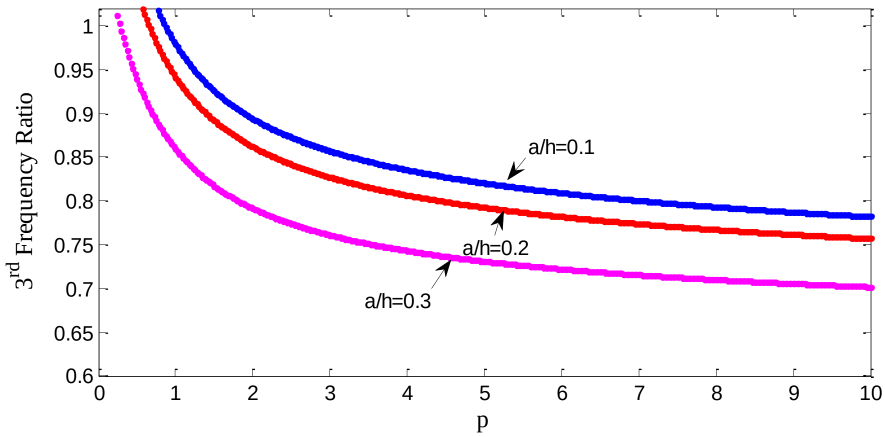
In Figure 4, the relationship between the frequencies ratio against the power index. We see that increasing the gradient indicator causes the reduction in the fourth frequency ratios when the axial speed u=2, slenderness ratio δ=0.1, and the crack location xc=0.2 for simple support ends. Also, the crack depth increases lead to a decrease in the frequency ratios.



(a)



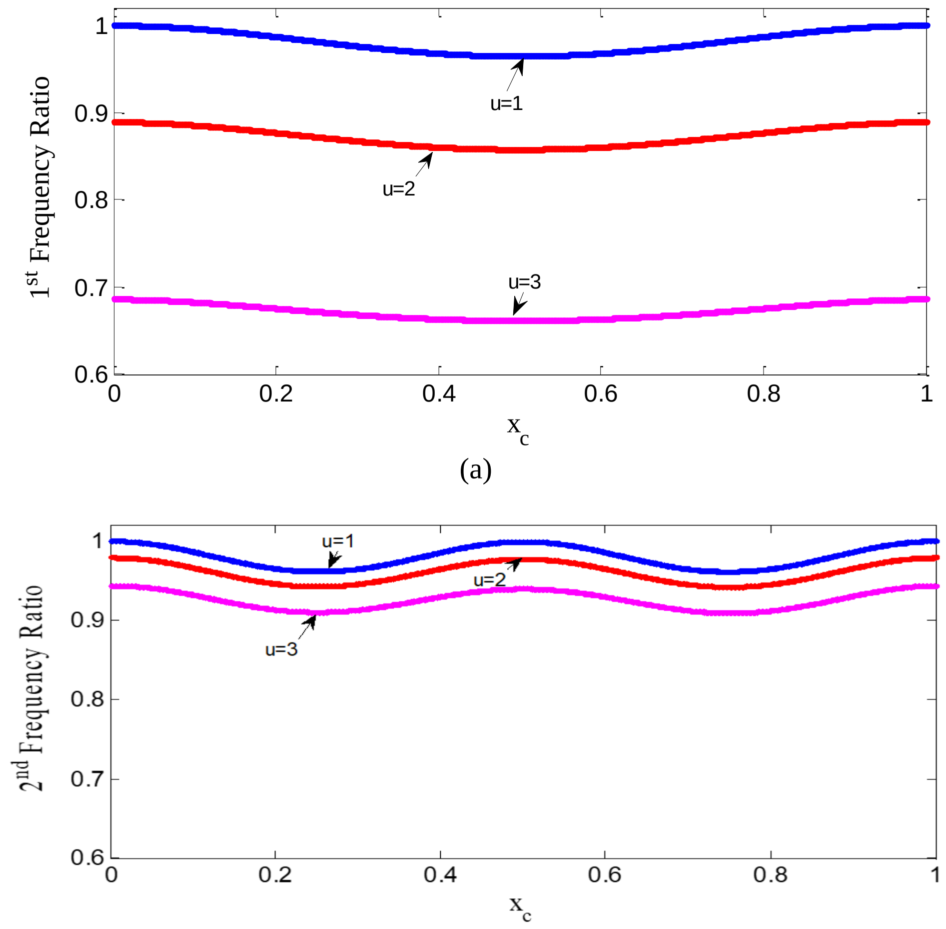
(b)



(c)

Figure 4. The relationship between the natural frequencies ratios of S-S FG beam and the gradient indicator (p) with different crack depth ratio (a/h) at u=2, L/h=10, xc = 0.2.

The effect of the axial velocity on the ratios of the three frequencies is observed in Figure 5. From these figures, it is evident that the ratios of the three frequencies have decreased due to the increase in the axial velocity.



(a)

(b)

Figure 5. The relationship between the natural frequencies ratios of S-S FG beam and the crack location with different axial speed at p=1, L/h=10, a/h=0.2.

5. Conclusions

In this work based on the shear model, the cracked moving simply supported graded beam is investigated. Hamilton's principle is used to derive the equation of motion and then solved by the method of Galerkin's. Crack depth, crack position, axial velocity and gradient index effects are discussed. In this study, the main conclusions are summarized as follows:

1. The increasing of the crack depth causes a decrease in the natural frequencies for different crack positions, the minimum first natural frequency is in the middle of the beam and the maximum occur at the ends of the beam, while the minimum second natural frequency is at ¼ L and ¾ L of the beam and the maximum occur at the middle and ends of the beam.
2. Increasing the speed of the moving beam leads to a decrease in the natural frequencies.
3. Increasing the graded material's gradient index reduces the natural frequencies for various crack locations.

Parameters	Meaning
a	Crack depth
b	Beam width
h	Beam thickness
L	Beam length
x_c	Crack position
(S-S)	Simply supported boundary condition
(C-C)	Clamped-clamped boundary condition
(C-F)	Clamped-free boundary condition
FGM	Functionally graded material

ρ_c , ρ_m	Density of ceramic and metal, respectively
V	Axial velocity
E_c , E_m	Elastic modulus of ceramic and metal, respectively
p	Property gradient index
b_o	Actual position of neutral axis
k_o	Shear correction factor
α_s	Shear factor

References

[1] N. Wattanasakulpong and V. Ungbhakorn, "Free Vibration Analysis of Functionally Graded Beams with General Elastically End Constraints by DTM," World J. Mech., vol. 02, no. 06, pp. 297–310, 2012, doi: 10.4236/wjm.2012.26036.

[2] S. A. Sina, H. M. Navazi, and H. Haddadpour, "An analytical method for free vibration analysis of functionally graded beams," Mater. Des., vol. 30, no. 3, pp. 741–747, 2009, doi: 10.1016/j.matdes.2008.05.015.

[3] K. Rajabi, M. H. Kargarnovin, and M. Gharini, "Dynamic analysis of a functionally graded simply supported Euler-Bernoulli beam subjected to a moving oscillator," Acta Mech., vol. 224, no. 2, pp. 425–446, 2013, doi: 10.1007/s00707-012-0769-y.

[4] F. Q. Zhao and Z. M. Wang, "Vibration of deploying rectangular cross-sectional beam made of functionally graded materials," J. Low Freq. Noise Vib. Act. Control, vol. 37, no. 4, pp. 748–761, 2018, doi: 10.1177/1461348418765957.

[5] M. Aydogdu and V. Taskin, "Free vibration analysis of functionally graded beams with simply supported edges," Mater. Des., vol. 28, no. 5, pp. 1651–1656, 2007, doi: 10.1016/j.matdes.2006.02.007.

[6] H. T. Thai and T. P. Vo, "Bending and free vibration of functionally graded beams using various higher-order shear deformation beam theories," Int. J. Mech. Sci., vol. 62, no. 1, pp. 57–66, 2012, doi: 10.1016/j.ijmecsci.2012.05.014.

[7] M. Şimşek, "Buckling of Timoshenko beams composed of two-dimensional functionally graded material (2D-FGM) having different boundary conditions," Compos. Struct., vol. 149, pp. 304–314, 2016, doi: 10.1016/j.compstruct.2016.04.034.

[8] Y. S. Al Rjoub and A. G. Hamad, "Free vibration of functionally Euler-Bernoulli and Timoshenko graded porous beams using the transfer matrix method," KSCE J. Civ. Eng., vol. 21, no. 3, pp. 792–806, 2017, doi: 10.1007/s12205-016-0149-6.

[9] A. E. Alshorbagy, M. A. Eltaher, and F. F. Mahmoud, "Free vibration characteristics of a functionally graded beam by finite element method," Appl. Math. Model., vol. 35, no. 1, pp. 412–425, 2011, doi: 10.1016/j.apm.2010.07.006.

[10] Y. Liu, J. Xiao, and D. Shu, "Free vibration of exponential functionally graded beams with single delamination," Procedia Eng., vol. 75, pp. 164–168, 2014, doi: 10.1016/j.proeng.2013.11.041.

[11] C. F. Lü, W. Q. Chen, R. Q. Xu, and C. W. Lim, "Semi-analytical elasticity solutions for bi-directional functionally graded beams," Int. J. Solids Struct., vol. 45, no. 1, pp. 258–275, 2008, doi: 10.1016/j.ijsolstr.2007.07.018.

[12] S. N. Padhi, S. S. Choudhury, and T. Rout, "Static and Dynamic Stability Analysis of a Sigmoid," no. 5, pp. 18–23, 2015.

[13] T. Van Lien, N. T. Duc, and N. T. Khiem, "Free vibration analysis of multiple cracked functionally graded Timoshenko beams," Lat. Am. J. Solids Struct., vol. 14, no. 9, pp. 1752–1766, 2017, doi: 10.1590/1679-78253693.

[14] H. Ozturk, Z. Kiral, and B. G. Kiral, "Dynamic analysis of elastically supported cracked beam subjected to a concentrated moving load," Lat. Am. J. Solids Struct., vol. 13, no. 1, pp. 175–200, 2016, doi: 10.1590/1679-78252195.

[15] E. C. Yang, X. Zhao, and Y. H. Li, "Free Vibration Analysis for Cracked FGM Beams by Means of a Continuous Beam Model," Shock Vib., vol. 2015, no. October, 2015, doi: 10.1155/2015/197049.

[16] T. Yan and J. Yang, "Forced vibration of edge-cracked functionally graded beams due to a transverse moving load," Procedia Eng., vol. 14, pp. 3293–3300, 2011, doi: 10.1016/j.proeng.2011.07.416.

[17] N. T. Khiem, H. T. Tran, and D. Nam, "Modal analysis of cracked continuous Timoshenko beam made of functionally graded material," Mech. Based Des. Struct. Mach., vol. 48, no. 4, pp. 459–479, 2020, doi: 10.1080/15397734.2019.1639518.

[18] Y. Cunedioglu and S. Shabani, "Free vibration analysis of a single edge cracked symmetric functionally graded stepped beams," Adv. Struct. Eng., pp. 1–14, 2020, doi: 10.1177/1369433220939214.

[19] S. Kitipornchai, L. L. Ke, J. Yang, and Y. Xiang, "Nonlinear vibration of edge cracked functionally graded Timoshenko beams," J. Sound Vib., vol. 324, no. 3–5, pp. 962–982, 2009, doi: 10.1016/j.jsv.2009.02.023.

[20] M. Al-Waily, "Theoretical and Numerical Vibration Study of Continuous Beam with Crack Size and Location Effect," Int. J. Innov. Res. Sci. Eng. Technol., vol. 2, no. 9, pp. 4166–4177, 2013.

[21] M. Al-Waily, M. A. R. S. Al-Baghdadi, and R. H. Al-Khayat, "Flow velocity and crack angle effect on vibration and flow characterization for pipe induce vibration," Int. J. Mech. Mechatronics Eng., vol. 17, no. 5, pp. 19–27, 2017.

[22] A. Mirzabeigy, R. Madoliat, and C. Surace, "Explicit formula to estimate natural frequencies of a double-beam system with crack," J. Brazilian Soc. Mech. Sci. Eng., vol. 41, no. 5, 2019, doi: 10.1007/s40430-019-1714-y.

[23] D. H. J. Al-Zubaidi, M. Al-Waily, and E. Q. Hussein, "Analytical heat generation investigation for forced vibration beam with different crack characterizations influence," Int. J. ENERGY ENVIROMENT, vol. 10, no. 1, pp. 33–48, 2019.

[24] D. H. J. Al-Zubaidi, M. Al-Waily, and E. Q. Hussein, "Analytical investigation of crack depth and position effect onto beam force vibration responce with various harmonic frequency influence," Int. J. ENERGY ENVIROMENT, vol. 11, no. 1, pp. 1–28, 2020.

[25] Y. Yesilce, "Determination of natural frequencies and mode shapes of axially moving timoshenko beams with different boundary conditions using differential transform method," Adv. Vib. Eng., vol. 12, no. 1, pp. 89–108, 2013.

[26] L. Q. Yao, C. J. Ji, J. P. Shen, and C. Li, "Free vibration and wave propagation of axially moving functionally graded Timoshenko microbeams," J. Brazilian Soc. Mech. Sci. Eng., vol. 42, no. 3, 2020, doi: 10.1007/s40430-020-2206-9.

[27] C. Ji, L. Yao, and C. Li, "Transverse Vibration and Wave Propagation of Functionally Graded Nanobeams with Axial Motion," J. Vib. Eng. Technol., vol. 8, no. 1, pp. 257–266, 2020, doi: 10.1007/s42417-019-00130-3.

[28] H. Ding, M. Zhu, and L. Chen, "Dynamic stiffness method for free vibration of an axially moving beam with generalized boundary conditions," Appl. Math. Mech. (English Ed., vol. 40, no. 7, pp. 911–924, 2019, doi: 10.1007/s10483-019-2493-8.

[29] J. R. Chang, W. J. Lin, C. J. Huang, and S. T. Choi, "Vibration and stability of an axially moving Rayleigh beam," Appl. Math. Model., vol. 34, no. 6, pp. 1482–1497, 2010, doi: 10.1016/j.apm.2009.08.022.

[30] J. W. Lee and J. Y. Lee, "Free vibration analysis of functionally graded Bernoulli-Euler beams using an exact transfer matrix expression," Int. J. Mech. Sci., vol. 122, no. January, pp. 1–17, 2017, doi: 10.1016/j.ijmecsci.2017.01.011.

[31] Anderson Soares and Simone dos Santos Hoefel, "Analysis of Rotatory Inertia and Shear Deformation on Transverse Vibration of Beams," An. do IX Congr. Nac. Eng. Mecânica, vol. 550, no. 1966, 2016, doi: 10.20906/cps/con-2016-1101.

[32] H. Z. Ferezqi, M. Tahani, and H. E. Toussi, "Analytical approach to free vibrations of cracked timoshenko beams made of functionally graded materials," Mech. Adv. Mater. Struct., vol. 17, no. 5, pp. 353–365, 2010, doi: 10.1080/15376494.2010.488608.

[33] T. Van Lien, N. T. Duc, and N. T. Khiem, "A New Form of Frequency Equation for Functionally Graded Timoshenko Beams with Arbitrary Number of Open Transverse Cracks," Iran. J. Sci. Technol. - Trans. Mech. Eng., vol. 43, no. February, pp. 235–250, 2019, doi: 10.1007/s40997-018-0152-2.

[34] A. Shariati, D. won Jung, H. Mohammad-Sedighi, K. K. Zur, M. Habibi, and M. Safa, "Stability and dynamics of viscoelastic moving rayleigh beams with an asymmetrical distribution of material parameters," Symmetry (Basel)., vol. 12, no. 4, pp. 1–23, 2020, doi: 10.3390/SYM12040586.

[35] J. Fernández-Sáez, L. Rubio, and C. Navarro, “Approximate calculation of the fundamental frequency for bending vibrations of cracked beams,” J. Sound Vib., vol. 225, no. 2, pp. 345–352, 1999, doi: 10.1006/jsvi.1999.2251.
[36] Talib. EH. Elaikh, S. M. Hmod, and E. R. Bustan, “Dynamic behavior of cracked functionally graded (FG) material pipe conveying fluid,” Int. J. ENERGY ENVIROMENT, vol. 9, no. 6, pp. 591–606, 2018.
[37] S. R. Li, D. F. Cao, and Z. Q. Wan, “Bending solutions of FGM Timoshenko beams from those of the homogenous Euler-Bernoulli beams,” Appl. Math. Model., vol. 37, no. 10–11, pp. 7077–7085, 2013, doi: 10.1016/j.apm.2013.02.047.