



## **Ecological performance of a generalized irreversible Carnot heat engine with complex heat transfer law**

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### **Abstract**

The optimal ecological performance of a generalized irreversible Carnot heat engine with the losses of heat-resistance, heat leakage and internal irreversibility, in which the transfer between the working fluid and the heat reservoirs obeys a complex heat transfer law, including generalized convective heat transfer law and generalized radiative heat transfer law,  $Q \propto \Delta(T^n)^m$ , is derived by taking an ecological optimization criterion as the objective, which consists of maximizing a function representing the best compromise between the power and entropy production rate of the heat engine. The effects of heat transfer laws and various loss terms are analyzed. The obtained results include those obtained in many literatures.

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**Keywords:** Finite time thermodynamics, Irreversible Carnot heat engine, Ecological optimization, Heat transfer law.

### **1. Introduction**

In the last decades, most of the finite time thermodynamic works were concentrated on the performance limits of thermodynamic processes and optimization of thermodynamic cycles [1-20]. Different optimization objectives were adopted in the analysis and optimization of heat engine cycles, including power output, exergy output, efficiency, specific power output, power density, etc. In 1991, Angulo-Brown [21] proved that the product of the entropy generation rate  $\sigma$  and the temperature  $T_L$  of low-temperature heat reservoir reflects the dissipation of the power output  $P$  of the heat engine. So he investigated the optimal performance of heat engine by taking into account the function representing best compromise between  $P$  and  $T_L\sigma$ ,  $E' = P - T_L\sigma$  as the objective function. Since the objective function  $E'$  is similar to the ecological objective in some sense, it is called ecological objective function. However, Yan [22] considered the function is not reasonable because, if the cold reservoir temperature  $T_L$  is not equal to the environment temperature  $T_0$ , in the definition of  $E'$ , two different quantities, exergy output  $P$  and non-exergy  $T_L\sigma$ , were compared. And he brought forward a function  $E = P - T_0\sigma$  instead of  $E'$ . This criterion function is more reasonable than that presented by Angulo-Brown [21]. The optimization of the ecological function represents a compromise between the power output  $P$  and the lost power  $T_0\sigma$ , which is produced by entropy generation in the system and its surroundings.

In the analysis of many papers concerning ecological performance optimization were for endoreversible Carnot and Brayton heat engines [23-30], in which only the irreversibility of finite rate heat transfer is

considered. The endoreversible heat engine requires no internal irreversibility. However, real heat engines are usually devices with both internal and external irreversibilities. Besides the irreversibility of finite rate heat transfer, there are also other sources of irreversibilities, such as heat leakage, dissipation processes inside the working fluid, etc. [31, 32]. Based on the work of Refs. [31, 32], the optimal ecological performance of a Newton's law generalized irreversible Carnot engine with the losses of heat-resistance, heat leakage and internal irreversibility is derived by taking an ecological optimization criterion as the objective by Chen *et al.* [33]. Some authors studied the ecological performance of irreversible Stirling, Ericsson and Brayton heat-engines [34, 35].

In general, heat transfer is not necessarily linear. Heat transfer law has a strong effect on the performance of endoreversible and irreversible heat engines [18, 36-49]. Recently, Li *et al.* [50] and Chen *et al.* [51] obtained the fundamental optimal relationship of the endoreversible [50] and irreversible [51] Carnot heat engines by using a complex heat transfer law, including generalized convective heat transfer law [ $Q \propto (\Delta T)^n$ ] [18, 39-41, 47, 48] and generalized radiative heat transfer law [ $Q \propto (\Delta T^n)$ ] [42-46],  $Q \propto (\Delta T^n)^m$  in the heat transfer processes between the working fluid and the heat reservoirs of the heat engine. And they further obtained the optimal ecological performance of an endoreversible heat engine based on this heat transfer law [52]. Chen *et al.* [53, 54] investigated the finite time ecological optimal performance for endoreversible [53] and irreversible [54] Carnot heat engines by using linear phenomenological heat transfer law  $Q \propto (\Delta T^{-1})$ . Sogut *et al.* [55] studied the optimal ecological performance of a solar driven heat engine. Zhu *et al.* [56, 57] obtained the optimal ecological performance for irreversible Carnot heat engine by using generalized convective heat transfer law  $Q \propto (\Delta T)^m$  [56] and generalized radiative heat transfer law  $Q \propto (\Delta T^n)$  [57].

One of aims of finite time thermodynamics is to pursue generalized rules and results. In this paper, on the basis of Ref. [51], the optimal ecological performance of a generalized irreversible Carnot heat engine with the losses of heat resistance, heat leakage and internal irreversibility, in which the heat transfer between the working fluid and the heat reservoirs obeys a complex heat transfer law  $Q \propto (\Delta T^n)^m$ , is derived by taking an ecological optimization criterion as the objective. The effects of heat transfer laws and various loss terms are analyzed.

## 2. Generalized irreversible Carnot engine model

The generalized irreversible Carnot engine and its surroundings to be considered in this paper are shown in Figure 1. The following assumptions are made for this model [17, 31-33, 46, 47, 51, 54, 56, 57]:

(1) The working fluid flows through the system in a quasistatic-state fashion. The cycle consists of two isothermal processes and two adiabatic processes. All four processes are irreversible.

(2) There exist external irreversibilities due to heat transfer in the high- and low-temperature heat exchangers between the heat engine and its surrounding heat reservoirs. The working fluid temperatures ( $T_{HC}$  and  $T_{LC}$ ) are different from the reservoir temperatures ( $T_H$  and  $T_L$ ). These temperatures satisfy the following inequality:  $T_H > T_{HC} > T_{LC} > T_L$ . The heat-transfer surface areas ( $F_1$  and  $F_2$ ) of high- and low-temperature heat exchangers are finite. The total heat transfer surface area ( $F$ ) of the two heat exchangers is assumed to be a constant:  $F = F_1 + F_2$ .

(3) There exists a constant rate of bypass heat leakage ( $q$ ) from the heat source to the heat sink. Thus  $Q_H = Q_{HC} + q$  and  $Q_L = Q_{LC} + q$ , where  $Q_{HC}$  is the rate of heat flow from heat source to working fluid due to the deriving force of  $T_H - T_{HC}$ ,  $Q_{LC}$  is the rate of heat flow from working fluid to the heat sink due to the deriving force of  $T_{LC} - T_L$ ,  $Q_H$  is rate of heat transfer supplied by the heat source, and  $Q_L$  is rate of heat transfer released to the heat sink.

(4) There are irreversibilities in the system due to: (a) the heat resistance between the working fluid and the heat reservoirs, (b) the heat leakage between the heat reservoirs and (c) miscellaneous factors such as friction, turbulence and non-equilibrium activities inside the heat engine. Thus, the power output produced by the generalized irreversible Carnot engine is less than that of the endoreversible Carnot engine with the same heat input. In other words, the rate of heat flow ( $Q_{LC}$ ) from cold working fluid to the heat sink for the generalized irreversible Carnot engine is larger than that for the endoreversible Carnot engine. A constant coefficient  $\Phi$  is introduced, in the following expression, to characterize the additional internal miscellaneous irreversibility effects:  $\Phi = Q_{LC} / Q'_{LC} \geq 1$ , where  $Q'_{LC}$  is the rate of heat

flow from the cold working fluid to the heat sink for the Carnot engine with the only loss of heat resistance.

The model described above is a more general one than the endoreversible Carnot heat engine model. If  $q=0$  and  $\Phi=1$ , the model is reduced to the endoreversible Carnot engine [23-33, 36-40, 50, 53]. If  $q=0$  and  $\Phi > 1$ , the model is reduced to the irreversible Carnot engine with heat resistance and internal irreversibilities [58]. If  $q > 0$  and  $\Phi=1$ , the model is reduced to the irreversible Carnot engine with heat resistance and heat leakage losses [59, 60].

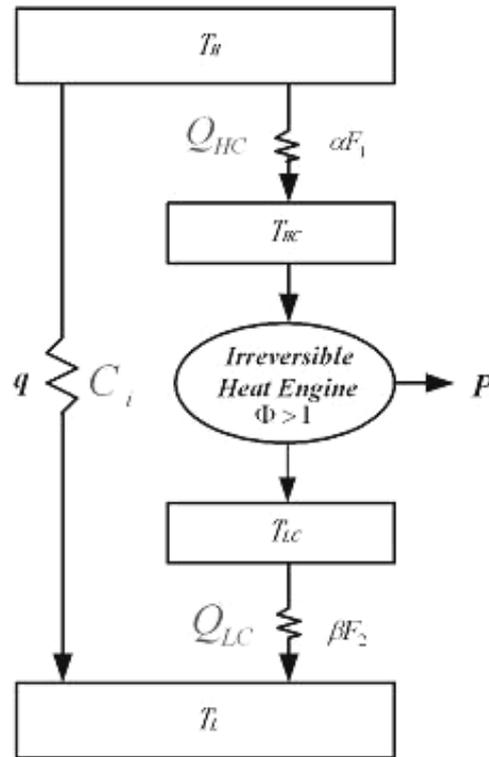


Figure 1. The model of a generalized irreversible Carnot heat engine

### 3. Generalized optimal characteristics

The second law of thermodynamics requires that  $Q_{LC}/Q_{HC} = \Phi T_{LC}/T_{HC}$ . The first law of thermodynamics gives that the power output ( $P$ ) of the engine is  $P = Q_H - Q_L = Q_{HC} - Q_{LC}$ , and the efficiency ( $\eta$ ) of the engine is  $\eta = P/Q_H = P/(Q_{HC} + q)$ .

Consider that the heat transfers between the engine and its surroundings follow the complex law  $Q \propto (\Delta T^n)^m$ . Then

$$Q_{HC} = \alpha F_1 (T_H^n - T_{HC}^n)^m, Q_{LC} = \beta F_2 (T_{LC}^n - T_L^n)^m \quad (1)$$

where  $\alpha$  is the overall heat transfer coefficient and  $F_1$  is the heat-transfer surface area of the high-temperature-side heat exchanger,  $\beta$  is the overall heat transfer coefficient and  $F_2$  is the heat-transfer surface area of the low-temperature-side heat exchanger.

Defining the heat transfer surface area ratio ( $f$ ) and the working fluid temperature ratio ( $x$ ) as follows:  $f = F_1/F_2$ ,  $x = T_{HC}/T_{LC}$ , where  $1 \leq x \leq T_H/T_L$ . Then one can obtain

$$P = \frac{\alpha F f (T_H^n x^{-n} - T_L^n)^m (x - \Phi)}{x(1+f)[x^{-n} + (\Phi f x^{-1})^{1/m}]^m} \quad (2)$$

$$\eta = \frac{\alpha F f (T_H^n x^{-n} - T_L^n)^m (x - \Phi)}{x \alpha F f (T_H^n x^{-n} - T_L^n)^m + q x (1 + f) [x^{-n} + (\Phi r f x^{-1})^{1/m}]^m} \quad (3)$$

where  $r = \alpha/\beta$ . Thus the entropy generation rate of the engine is as following

$$\sigma = \frac{\alpha f F (T_H^n x^{-n} - T_L^n)^m}{(1 + f) [x^{-n} + (\Phi r f x^{-1})^{1/m}]^m} \left( \frac{\Phi}{T_L x} - \frac{1}{T_H} \right) + q \left( \frac{1}{T_L} - \frac{1}{T_H} \right) \quad (4)$$

Substituting equations (2) and (4) into ecological function  $E = P - T_0 \sigma$  yields

$$E = \frac{\alpha f F (T_H^n x^{-n} - T_L^n)^m}{(1 + f) [x^{-n} + (\Phi r f x^{-1})^{1/m}]^m} \left[ \left(1 + \frac{T_0}{T_H}\right) - \frac{\Phi}{x} \left(1 + \frac{T_0}{T_L}\right) \right] - q T_0 \left( \frac{1}{T_L} - \frac{1}{T_H} \right) \quad (5)$$

Equations (2)-(5) indicate that power output ( $P$ ), efficiency ( $\eta$ ), entropy generation rate ( $\sigma$ ) and ecological function ( $E$ ) of the generalized irreversible Carnot heat engine are functions of the heat transfer surface area ratio ( $f$ ) for given  $T_H$ ,  $T_L$ ,  $T_0$ ,  $\alpha$ ,  $\beta$ ,  $n$ ,  $m$ ,  $\Phi$  and  $x$ . Taking the derivatives of  $P$ ,  $\eta$ ,  $\sigma$  and  $E$  with respect to  $f$  and setting them equal to zero yields the same optimum surface area ratio

$$f_a = (x^{1-nm} / \Phi r)^{1/(m+1)} \quad (6)$$

The corresponding optimal power, optimal efficiency, optimal entropy generation rate and optimal ecological function are as follows:

$$P = \frac{\alpha F (1 - \Phi/x) (T_H^n - T_L^n x^n)^m}{[1 + (\Phi r)^{1/(m+1)} x^{(nm-1)/(1+m)}]^{m+1}} \quad (7)$$

$$\eta = \frac{\alpha F (1 - \Phi/x) (T_H^n - x^n T_L^n)^m}{\alpha F (T_H^n - T_L^n x^n)^m + q [1 + (\Phi r)^{1/(m+1)} x^{(nm-1)/(1+m)}]^{m+1}} \quad (8)$$

$$\sigma = \frac{\alpha F (T_H^n - T_L^n x^n)^m}{[1 + (\Phi r x^{nm-1})^{1/(m+1)}]^{m+1}} \left( \frac{\Phi}{T_L x} - \frac{1}{T_H} \right) + q \left( \frac{1}{T_L} - \frac{1}{T_H} \right) \quad (9)$$

$$E = \frac{\alpha F (T_H^n - T_L^n x^n)^m}{[1 + (\Phi r x^{nm-1})^{1/(m+1)}]^{m+1}} \left[ \left(1 + \frac{T_0}{T_H}\right) - \frac{\Phi}{x} \left(1 + \frac{T_0}{T_L}\right) \right] - q T_0 \left( \frac{1}{T_L} - \frac{1}{T_H} \right) \quad (10)$$

Equations (9) and (10) are the major results of this paper. At the maximum ecological function condition ( $E_{\max}$ ), the corresponding efficiency, power output and entropy generation rate are  $\eta_E$ ,  $P_E$  and  $\sigma_E$ . At maximum power output condition ( $P_{\max}$ ), the corresponding efficiency, ecological function and entropy generation rate are  $\eta_P$ ,  $E_P$  and  $\sigma_P$ . Because of the complexity of equations (7)-(10), it is difficult to obtain the analytical expressions of  $\eta_E$ ,  $\eta_P$ ,  $P_{\max}$ ,  $P_E$ ,  $E_{\max}$ ,  $E_P$ ,  $\sigma_E$  and  $\sigma_P$ , they can be obtained by numerical calculations.

## 4. Discussions

### 4.1 Effect of different losses on the optimal characteristics

(1). If there is no bypass heat leakage in the cycle (i.e.,  $q = 0$ ), Equations (7)-(10) become

$$P = \frac{\alpha F (1 - \Phi/x) (T_H^n - T_L^n x^n)^m}{[1 + (\Phi r)^{1/(m+1)} x^{(nm-1)/(1+m)}]^{m+1}} \quad (11)$$

$$\eta = 1 - \Phi/x \quad (12)$$

$$\sigma = \frac{\alpha F (T_H^n - T_L^n x^n)^m}{[1 + (\Phi r x^{nm-1})^{1/(m+1)}]^{m+1}} \left( \frac{\Phi}{T_L x} - \frac{1}{T_H} \right) \quad (13)$$

$$E = \frac{\alpha F (T_H^n - T_L^n x^n)^m}{[1 + (\Phi r x^{nm-1})^{1/(m+1)}]^{m+1}} \left[ \left(1 + \frac{T_0}{T_H}\right) - \frac{\Phi}{x} \left(1 + \frac{T_0}{T_L}\right) \right] \quad (14)$$

The power output ( $P$ ), ecological function ( $E$ ) versus efficiency ( $\eta$ ) curves are parabolic-like ones, and the entropy generation rate ( $\sigma$ ) decreases with the increase of efficiency ( $\eta$ ).

(2). If there are only heat resistance and by pass heat leakage in the cycle (i.e.,  $\Phi = 1$ ), Equations (7) -(10) become

$$P = \frac{\alpha F (1 - 1/x) (T_H^n - T_L^n x^n)^m}{[1 + r^{1/(m+1)} x^{(nm-1)/(1+m)}]^{m+1}} \quad (15)$$

$$\eta = \frac{\alpha F (1 - 1/x) (T_H^n - x^n T_L^n)^m}{\alpha F (T_H^n - T_L^n x^n)^m + q [1 + (r x^{nm-1})^{1/(1+m)}]^{m+1}} \quad (16)$$

$$\sigma = \frac{\alpha F (T_H^n - T_L^n x^n)^m}{[1 + (r x^{nm-1})^{1/(m+1)}]^{m+1}} \left( \frac{1}{T_L x} - \frac{1}{T_H} \right) + q \left( \frac{1}{T_L} - \frac{1}{T_H} \right) \quad (17)$$

$$E = \frac{\alpha F (T_H^n - T_L^n x^n)^m}{[1 + (r x^{nm-1})^{1/(m+1)}]^{m+1}} \left[ \left(1 + \frac{T_0}{T_H}\right) - \frac{1}{x} \left(1 + \frac{T_0}{T_L}\right) \right] - q T_0 \left( \frac{1}{T_L} - \frac{1}{T_H} \right) \quad (18)$$

The power output ( $P$ ) and ecological function ( $E$ ) versus efficiency ( $\eta$ ) curves are loop-shaped ones, and the entropy generation rate ( $\sigma$ ) versus efficiency ( $\eta$ ) curve is a parabolic-like one.

(3). If the engine is an endoreversible one (i.e.,  $\Phi = 1, q = 0$ ), Equations (7)-(10) become

$$P = \frac{\alpha F (1 - 1/x) (T_H^n - T_L^n x^n)^m}{[1 + r^{1/(m+1)} x^{(nm-1)/(1+m)}]^{m+1}} \quad (19)$$

$$\eta = 1 - 1/x \quad (20)$$

$$\sigma = \frac{\alpha F (T_H^n - T_L^n x^n)^m}{[1 + (r x^{nm-1})^{1/(m+1)}]^{m+1}} \left( \frac{1}{T_L x} - \frac{1}{T_H} \right) \quad (21)$$

The power output ( $P$ ) and ecological function ( $E$ ) versus efficiency ( $\eta$ ) curves are parabolic-like ones, and the entropy generation rate ( $\sigma$ ) is a monotonically decreasing function of efficiency ( $\eta$ ).

#### 4.2 Effects of heat transfer law on the optimal characteristics

(1) Equations (7)-(10) can be written as follows when  $m = 1$

$$P = \frac{\alpha F (1 - \Phi/x) (T_H^n - T_L^n x^n)}{[1 + (\Phi r)^{1/2} x^{(n-1)/2}]^2} \quad (22)$$

$$\eta = \frac{\alpha F (1 - \Phi/x) (T_H^n - x^n T_L^n)}{\alpha F (T_H^n - T_L^n x^n) + q [1 + (\Phi r)^{1/2} x^{(n-1)/2}]^2} \quad (23)$$

$$\sigma = \frac{\alpha F(T_H^n - T_L^n x^n)}{[1 + (\Phi r x^{n-1})^{1/2}]^2} \left( \frac{\Phi}{T_L x} - \frac{1}{T_H} \right) + q \left( \frac{1}{T_L} - \frac{1}{T_H} \right) \quad (24)$$

$$E = \frac{\alpha F(T_H^n - T_L^n x^n)}{[1 + (\Phi r x^{n-1})^{1/2}]^2} \left[ \left(1 + \frac{T_0}{T_H}\right) - \frac{\Phi}{x} \left(1 + \frac{T_0}{T_L}\right) \right] - q T_0 \left( \frac{1}{T_L} - \frac{1}{T_H} \right) \quad (25)$$

They are the same results as those obtained in Ref. [57]. If  $n=1$ , they are the results of irreversible Carnot heat engine with Newtownian heat transfer law [22, 33, 56, 57]. If  $n=-1$ , they are the results of irreversible Carnot heat engine with linear phenomenological heat transfer law [54, 57]. If  $n=4$ , they are the results of irreversible Carnot heat engine with radiative heat transfer law [55, 57].

(2) Equations (7)-(10) can be written as follows when  $n=1$

$$P = \frac{\alpha F(1 - \Phi/x)(T_H - T_L x)^m}{[1 + (\Phi r)^{1/(m+1)} x^{(m-1)/(1+m)}]^{m+1}} \quad (26)$$

$$\eta = \frac{\alpha F(1 - \Phi/x)(T_H - x T_L)^m}{\alpha F(T_H - T_L x)^m + q[1 + (\Phi r)^{1/(m+1)} x^{(m-1)/(1+m)}]^{m+1}} \quad (27)$$

$$\sigma = \frac{\alpha F(T_H - T_L x)^m}{[1 + (\Phi r x^{m-1})^{1/(m+1)}]^{m+1}} \left( \frac{\Phi}{T_L x} - \frac{1}{T_H} \right) + q \left( \frac{1}{T_L} - \frac{1}{T_H} \right) \quad (28)$$

$$E = \frac{\alpha F(T_H - T_L x)^m}{[1 + (\Phi r x^{m-1})^{1/(m+1)}]^{m+1}} \left[ \left(1 + \frac{T_0}{T_H}\right) - \frac{\Phi}{x} \left(1 + \frac{T_0}{T_L}\right) \right] - q T_0 \left( \frac{1}{T_L} - \frac{1}{T_H} \right) \quad (29)$$

They are the same results as those obtained in Ref.[56]. If  $m=1$ , they are the results of irreversible Carnot heat engine with Newtownian heat transfer law [22, 33, 56, 57]. If  $m=1.25$ , they are the results of irreversible Carnot heat engine [56] with Dulong-Petit heat transfer law [61].

## 5. Numerical example

To show the ecological function, power output and the entropy generation rate versus the efficiency characteristics of the irreversible Carnot heat engine with the complex heat transfer law, one numerical example is provided. In the numerical calculations,  $T_H = 1000K$ ,  $T_L = 400K$ ,  $T_0 = 300K$ ,  $\alpha F = 4W/K^m$ ,  $\Phi = 1.0$  and  $1.2$ ,  $\alpha = \beta$  ( $r=1$ ),  $q = C_i(T_H^n - T_L^n)^m$  and  $C_i = 0.00W/K^m$  and  $0.02W/K^m$  are set, where  $C_i$  is the heat conductance of the heat leakage.

Figure 2 shows the relations between ecological function, power output, entropy generation rate and the efficiency of the irreversible Carnot heat engine with  $n=4$  and  $m=1.25$ . This case means the heat transfer obeys inner radiative and outer Dulong and Petit laws. The dimensionless ecological function and power output are defined as ratios of the ecological function and power output of the heat engine to the maximum ecological function and the maximum power output, respectively. The dimensionless entropy generation rate is defined as a ratio of the entropy generation rate of the heat engine to the minimum entropy generation rate when  $\eta=0$ . It can be seen that the characteristic curve of the power output versus the efficiency is similar to that of the ecological function versus the efficiency. But the efficiency ( $\eta_p$ ) at the maximum power output is smaller than that ( $\eta_E$ ) at the maximum  $E$  objective, and the entropy generation rate versus efficiency curve is the parabolic shaped one. The entropy generation rate ( $\sigma_E$ ) at maximum ecological function is lower greatly than that ( $\sigma_p$ ) at maximum power output of the upper point. The ecological function ( $E_p$ ) at maximum power output does not exist. The results of this case show that  $\eta_E/\eta_p = 1.5151$ , the upper points  $P_E/P_{\max} = 0.6543$ ,  $\sigma_E/\sigma_p = 0.3229$  and the lower points  $P_E/P_{\max} = 0.4154$ ,  $\sigma_E/\sigma_p = 1.5131$ . It can be seen that the engine should operate at the upper point and the optimization of the ecological function makes the entropy generation rate of the cycle

decrease greatly and the thermal efficiency increase significantly with some decrease of the power output.

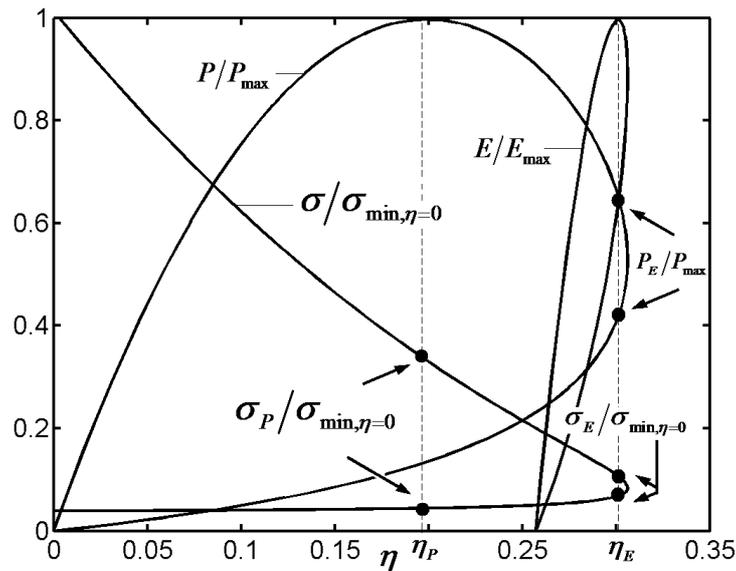


Figure 2. Ecological function, power output and the entropy generation rate versus efficiency relationships for  $m = 1.25$  and  $n = 4$

The effects of heat-leakage and internal irreversibility on the relations between power output, ecological function, entropy generation rate and efficiency are shown in Figures 3-5, respectively. In Figures 3-5,  $n = 4$  and  $m = 1.25$  are set. From Figures 3-5, it can be seen that the bypass heat-leakage change the power output, ecological function and entropy generation rate versus efficiency relations qualitatively. The characteristics of power output and ecological function versus efficiency are become the loop-shaped curves from the parabolic shaped ones if the engine suffers a heat leakage loss. The characteristic of entropy generation rate versus efficiency is become the parabolic shaped curve from the decreasing shaped one if the engine suffers a heat leakage loss. The internal irreversibility change the power output, ecological function and entropy generation rate versus efficiency relationships quantitatively. The maximum-power output, maximum-ecological function value, the minimum-entropy generation rate and the corresponding efficiencies with internal irreversibility are smaller than those without internal irreversibility.

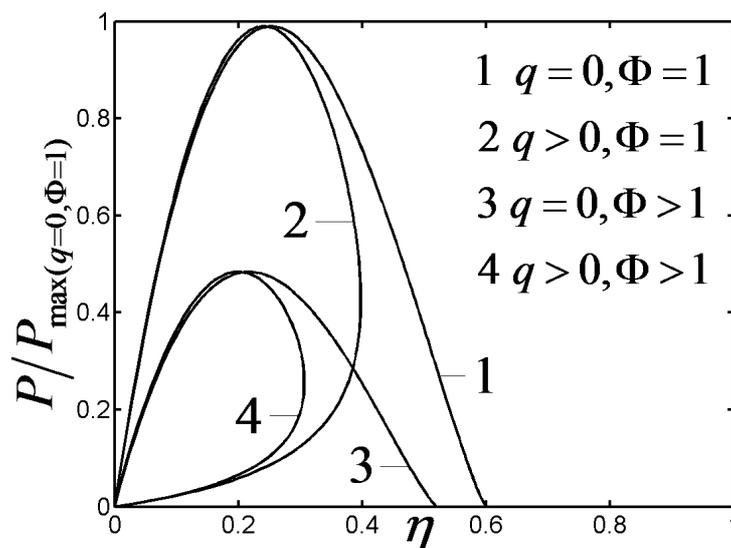


Figure 3. The effects of heat-leakage and internal irreversibility on relation between power output and efficiency

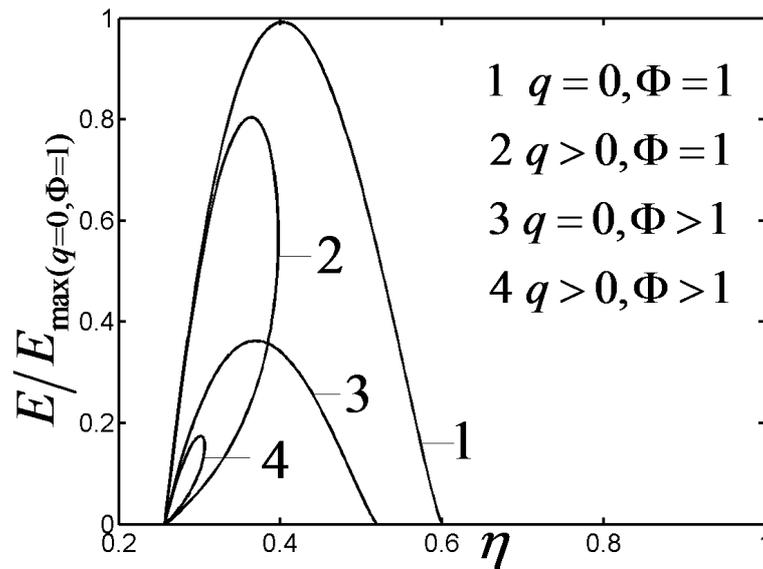


Figure 4. The effects of heat-leakage and internal irreversibility on relation between ecological function and efficiency

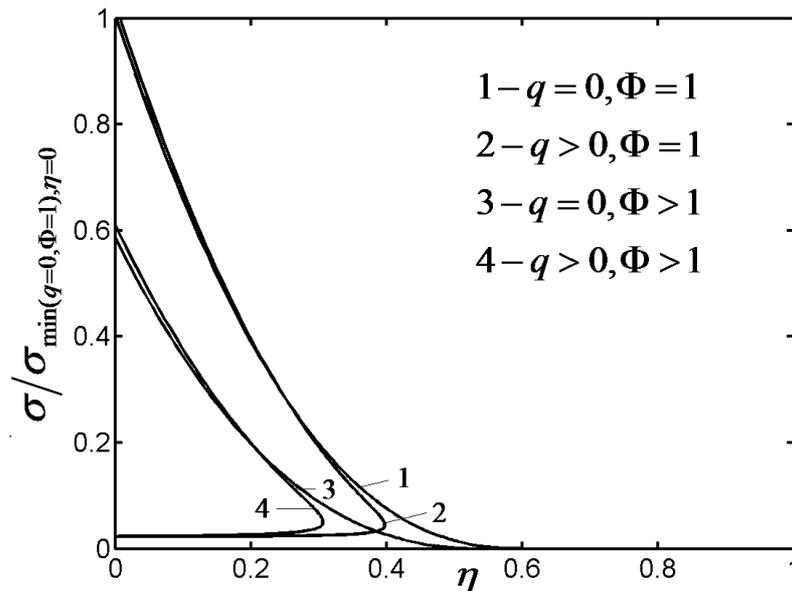


Figure 5. The effects of heat-leakage and internal irreversibility on relation between entropy generation rate and efficiency

The effects of heat transfer laws on relations between power output, ecological function, entropy generation rate and efficiency are shown in Figures 6-8, respectively. In Figures 6-8,  $\Phi = 1.2$  and  $C_i = 0.02W/K^m$  are set. From Figures 6-8, it can be seen that heat transfer law changes the power output, ecological function and entropy generation rate versus efficiency relations quantitatively.

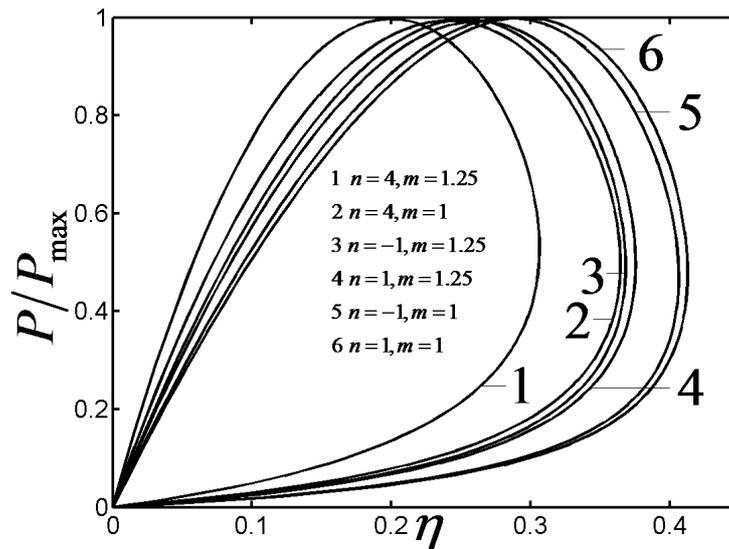


Figure 6. The effects of heat transfer laws on relation between power output and efficiency

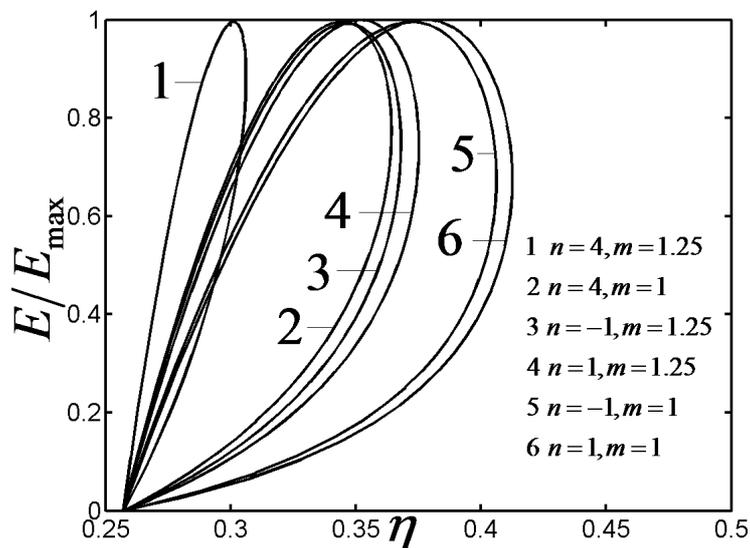


Figure 7. The effects of heat transfer laws on relation between ecological function and efficiency

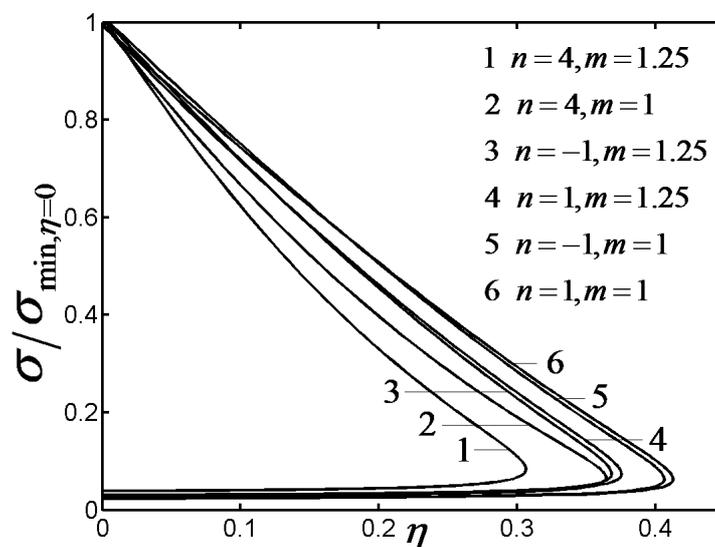


Figure 8. The effects of heat transfer laws on relation between entropy generation rate and efficiency

## 6. Conclusion

The optimal ecological performance of a generalized irreversible Carnot heat engine with the losses of heat-resistance, heat leakage and internal irreversibility, in which the heat transfer between the working fluid and the heat reservoirs obeys a complex heat transfer law  $Q \propto (\Delta T^n)^m$ , is derived by taking into account an ecological optimization criterion as the objective, which consists of maximizing a function representing the best compromise between the power output and entropy production rate of the heat engine. The effects of heat-leakage, internal irreversibility and heat transfer law on relations between power output, ecological function, entropy generation rate and efficiency are obtained. The results include those obtained in many literatures, such as the optimal ecological performance of endoreversible Carnot heat engine with different heat transfer laws ( $m \neq 0, n \neq 0, q = 0, \Phi = 1$ ), the optimal ecological performance of the Carnot heat engine with heat resistance and internal irreversibility ( $m \neq 0, n \neq 0, q = 0, \Phi > 1$ ), the optimal ecological performance of the Carnot heat engine with heat resistance and heat leakage ( $m \neq 0, n \neq 0, q > 0, \Phi = 1$ ), and optimal ecological performance of the irreversible Carnot heat engine ( $q > 0, \Phi > 1$ ) with generalized heat transfer laws  $Q \propto (\Delta T^n)$  ( $m = 1, n \neq 0$ ) and  $Q \propto (\Delta T)^m$  ( $n = 1, m \neq 0$ ). They can provide some theoretical guidelines for the design of practical heat engines.

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## References

- [1] Bejan A. Entropy generation through heat and fluid flow. New York: Wiley, 1982.
- [2] Andresen B. Finite-time thermodynamics. Physics Laboratory II, University of Copenhagen, 1983.
- [3] Sieniutycz S, Salamon P.(eds.). Advances in thermodynamics. Volume 4: Finite time thermodynamics and thermoeconomics. New York: Taylor&Francis, 1990.
- [4] De Vos A. Endoreversible thermodynamics of solar energy conversion. Oxford: Oxford University Press, 1992.
- [5] Feidt M. Thermodynamique et optimisation energetique des systems et procedes (2nd Ed.). Paris: Technique et Documentation, Lavoisier, 1996 (in French).
- [6] Bejan A. Entropy generation minimization. Boca Raton: CRC Press, 1996.
- [7] Berry R S, Kazakov V A, Sieniutycz S, Szwasz Z, Tsirlin A M. Thermodynamic optimization of finite time processes. Chichester: Wiley, 1999.
- [8] Chen L, Wu C, Sun F. Finite time thermodynamic optimization or entropy generation minimization of energy systems. J. Non-Equilib. Thermodyn., 1999,24(4):327-359.
- [9] Wu C, Chen L, Chen J. (eds). Recent advances in finite time thermodynamics. New York: Nova Science Publishes, 1999.
- [10] Mironova V A, Amelkin S A, Tsirlin A M. Mathematical methods of finite time thermodynamics. Moscow: Khimia, 2000(in Russian).
- [11] Sieniutycz S, De Vos A. Thermodynamics of energy conversion and transport. New York: Springer-Verlag, 2000.
- [12] Chen L, Wu C, Sun F. The recent advances in finite time thermodynamics and its future application. Int. J. Energy, Environment and Economics, 2001, 11(1): 69-81.
- [13] Bejan A. Fundamentals of exergy analysis, entropy generation minimization, and the generation of flow architecture. Int. J. Energy Research, 2002, 26(7): 545-565.
- [14] Hoffman K H, Burzler J, Fischer A, Schaller M, Schubert S. Optimal process paths for endoreversible systems. J. Non-Equilib. Thermodyn., 2003, 28(3): 233-268.
- [15] Chen L, Sun F. (eds) Advances in finite time thermodynamics: Analysis and optimization. New York: Nova Science Publishers, 2004.
- [16] Chen L. Finite-time thermodynamic analysis of irreversible processes and cycles. Beijing: Higher Education Press, 2005 (in Chinese).

- [17] Tsirlin A M. Optimal processes in open controllable macrosystems. *Automation and Remote Control*, 2006, 67(1): 132–147.
- [18] Huleihil M, Andresen B. Convective heat transfer law for an endoreversible engine. *J. Appl. Phys.*, 2006, 100(1): 014911.
- [19] Hoffman K H. An introduction to endoreversible thermodynamics. *Atti dell'Accademia Peloritana dei Pericolanti Classe di Scienze Fisiche, Matematiche e Naturali*, 2008, LXXXVI (C1S0801011): 1-18.
- [20] De Vos A. *Thermodynamics of solar energy conversion*. WILEY--VCH Verlag, 2008.
- [21] Angulo-Brown F. An ecological optimization criterion for finite-time heat engines. *J. Appl. Phys.*, 1991, 69(11): 7465-7469.
- [22] Yan Z. Comment on “ecological optimization criterion for finite-time heat engines”. *J. Appl. Phys.*, 1993, 73(7):3583.
- [23] Arias-Hernandez L A, Angulo-Brown F. A general property of endoreversible thermal engines. *J. Appl. Phys.*, 1997, 81(7): 2973-2979.
- [24] Chen L, Sun F, Chen W. The ecological optimization performance of finite time heat engines with heat transfer  $q \propto (\Delta T^{-1})$ . *Gas Turbine Tech.*, 1995, 8(1):16-18 (in Chinese).
- [25] Cheng C, Chen C. The ecological optimization of an irreversible Carnot heat engine. *J. Phys. D: Appl. Phys.*, 1997, 30(11): 1602-1609.
- [26] Cheng C, Chen C. Ecological optimization of an endoreversible Brayton-cycle. *Energy Convers. Manage.*, 1998, 39(1/2):33-44.
- [27] Khaliq A, Kumar R. Finite-time heat-transfer analysis and ecological optimization of an endoreversible and regenerative gas-turbine power-cycle. *Appl. Energy*, 2005, 81(1):73-84.
- [28] Ust Y, Safa A, Sahin B. Ecological performance analysis of an endoreversible regenerative Brayton heat-engine. *Appl. Energy*, 2005, 80(3): 247-260.
- [29] Xia D, Chen L, Sun F, Wu C. Universal ecological performance for endoreversible heat engine cycles. *Int. J. Ambient Energy*, 2006, 27(1): 15-20.
- [30] Zhang W, Chen L, Sun F, Wu C. Exergy-based ecological optimal performance for a universal endoreversible thermodynamic cycle. *Int. J. Ambient Energy*, 2007, 28(1): 51-56.
- [31] Chen L, Sun F. The optimal performance of an irreversible Carnot engine. *Sci. Tech. Bull.* 1995, 11(2):128. (in Chinese).
- [32] Chen L, Sun F, Wu C. A generalized model of real heat engines and its performance. *J. Inst. Energy*, 1996, 69(481): 214-222.
- [33] Chen L, Zhou J, Sun F, Wu C. Ecological optimization for generalized irreversible Carnot engines. *Appl. Energy*, 2004, 77(3): 327-338.
- [34] Tyagi S, Kaushik S, Salhotra R. Ecological optimization and performance study of irreversible Stirling and Ericsson heat-engines. *J. Phys. D: Appl. Phys.*, 2002, 35(20):2668-2675.
- [35] Tyagi S, Kaushik S. Ecological optimization of an irreversible regenerative intercooled Brayton heat engine with direct heat loss. *Int. J. Ambient Energy*, 2005, 26(2): 81-92.
- [36] Wu C. Output power and efficiency upper bounds of real solar energy heat engine. *Int. J. Ambient Energy*, 1988, 9(1): 17-21.
- [37] Bejan A. *Advanced engineering thermodynamics*. New York: Wiley, 1988.
- [38] Wu C. Power optimization of a finite-time solar radiant heat engine. *Int. J. Ambient Energy*, 1989, 10(3): 145-150.
- [39] Gutowicz-Krusin D, Procaccia J, Ross J. On the efficiency of rate processes: power and efficiency of heat engines. *J. Chem. Phys.*, 1978, 69(9):3898-3906.
- [40] Chen L, Sun F, Wu C. Influence of heat transfer law on the performance of a Carnot engine. *Appl. Thermal Engineering*, 1997, 17(3): 277-282.
- [41] Angulo-Brown F, Paez-Hernandez R. Endoreversible thermal cycle with a nonlinear heat transfer law. *J. Appl. Phys.*, 1993, 74(4): 2216-2219.
- [42] De Vos A. Efficiency of some heat engines at maximum power conditions. *Am. J. Phys.*, 1985, 53(6): 570-573.
- [43] De Vos A. Reflections on the power delivered by endoreversible engines. *J. Phys. D: Appl. Phys.*, 1987, 20(2): 232-236.
- [44] Chen L, Yan Z. The effect of heat transfer law on the performance of a two-heat-source endoreversible cycle. *J. Chem. Phys.*, 1989, 90(7): 3740-3743.

- [45] Gordon J M. Observations on efficiency of heat engines operating at maximum power. *Am. J. Phys.*, 1990, 58(4): 370-375.
- [46] Chen L, Sun F, Wu C. Effect of heat transfer law on the performance of a generalized irreversible Carnot engine. *J. Phys. D: Appl. Phys.*, 1999, 32(2): 99-105.
- [47] Zhou S, Chen L, Sun F. Optimal performance of a generalized irreversible Carnot engine. *Appl. Energy*, 2005, 81(4):376-387.
- [48] Feidt M, Costea M, Petre C, Petrescu S. Optimization of direct Carnot cycle. *Appl. Thermal Engng.*, 2007, 27(5-6): 829-839.
- [49] Chen L, Zhu X, Sun F, Wu C. Optimal configurations and performance for a generalized Carnot cycle assuming the heat transfer law  $Q \propto (\Delta T)^m$ . *Appl. Energy*, 2004, 78(3): 305-313.
- [50] Li J, Chen L, Sun F. Power vs. Efficiency characteristic of an endoreversible Carnot heat engine with heat transfer law  $q \propto (\Delta T)^m$ . *Int. J. Ambient Energy*, 2008, 29(3):149-152.
- [51] Chen L, Li J, Sun F. Generalized irreversible heat engine experiencing a complex heat transfer law. *Appl. Energy*, 2008, 85(1):52-60.
- [52] Li J, Chen L, Sun F. Ecological performance of an endoreversible Carnot heat engine with complex heat transfer law. *Int. J. Sustainable Energy*, in press.
- [53] Chen L, Sun F, Chen W. The ecological optimization performance of finite time heat engines with heat transfer  $q \propto \Delta(T^{-1})$ . *Gas Turbine Tech.*, 1995, 8(1): 16-18 (in Chinese).
- [54] Chen L, Zhu X, Sun F, Wu C. Exergy-based ecological optimization of linear phenomenological heat transfer law irreversible Carnot engines. *Appl. Energy*, 2006, 83(6): 573-582.
- [55] Sogut O, Durmayaz A. Ecological performance optimisation of a solar driven heat engine. *J. Energy Institute*, 2006, 79(4): 246-250.
- [56] Zhu X, Chen L, Sun F, Wu C. The ecological optimization of a generalized irreversible Carnot engine with a generalized heat transfer law. *Int. J. Ambient Energy*, 2003, 24(4): 189-194.
- [57] Zhu X, Chen L, Sun F, Wu C. Effect of heat transfer law on the ecological optimization of a generalized irreversible Carnot engine. *Open Systems & Information Dynamics*, 2005, 12(3): 249-260.
- [58] Wu C, Kiang R L. Finite time thermodynamic analysis of a Carnot engine with internal irreversibility. *Energy*, 1992, 17(12):1173-1178.
- [59] Bejan A. Theory of heat transfer irreversible power plants. *Int. J. Heat Mass Transfer*, 1988, 31(6):1211-1219.
- [60] Chen L, Wu C, Sun F. The influence of internal heat-leaks on the power versus efficiency characteristics of heat engines. *Energy Convers. Manage*, 1997, 38(14):1501-1507.
- [61] O'Sullivan C T. Newton's law of cooling-A critical assessment. *Am. J. Phys.*, 1990, 58(12): 956-960.



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