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Finite time exergoeconomic performance optimization of a thermoacoustic heat engine

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Abstract
Finite time exergoeconomic performance optimization of a generalized irreversible thermoacoustic heat engine with heat resistance, heat leakage, thermal relaxation, and internal dissipation is investigated in this paper. Both the real part and the imaginary part of the complex heat transfer exponent change the optimal profit rate versus efficiency relationship quantitatively. The operation of the generalized irreversible thermoacoustic engine is viewed as a production process with exergy as its output. The finite time exergoeconomic performance optimization of the generalized irreversible thermoacoustic engine is performed by taking profit rate as the objective. The analytical formulas about the profit rate and thermal efficiency of the thermoacoustic engine are derived. Furthermore, the comparative analysis of the influences of various factors on the relationship between optimal profit rate and the thermal efficiency of the generalized irreversible thermoacoustic engine is carried out by detailed numerical examples. The optimal zone on the performance of the thermoacoustic heat engine is obtained by numerical analysis. The results obtained herein may be useful for the selection of the operation parameters for real thermoacoustic heat engines.

Keywords: Thermoacoustic heat engine, Complex heat transfer exponent, Exergoeconomic performance, Optimization zone.

1. Introduction

Compared with the conventional heat engines, thermoacoustic engines (including prime mover and refrigerator) [1-4] have many advantages, such as simple structure, no or least moving parts, high reliability, working with environmental friendly fluid and materials, and etc. With this great potential, more and more scholars have been investigating the performance of thermoacoustic engine.

Recently, Wu *et al.* [5-7] have studied the performance of generalized irreversible thermoacoustic heat engine (or cooler) cycle by using the finite-time thermodynamics [8-15]. A relatively new method that combines exergy with conventional concepts from long-run engineering economic optimization to evaluate and optimize the design and performance of energy systems is exergoeconomic (or thermoeconomic) analysis [16, 17]. Salamon and Nitzan's work [18] combined the endoreversible model with exergoeconomic analysis. It was termed as finite time exergoeconomic analysis [19-28] to distinguish it from the endoreversible analysis with pure thermodynamic objectives and the exergoeconomic analysis with long-run economic optimization. Similarly, the performance bound at maximum profit was termed as finite time exergoeconomic performance bound to distinguish it from the

finite time thermodynamic performance bound at maximum thermodynamic output.
Some authors have assessed the influence of the heat transfer law on the finite time exergoeconomic performance optimization of heat engines and refrigerators [20, 23, 26]. In these researches, the heat transfer exponent is assumed to be a real. But for thermoacoustic heat engines, whose principle parts are the stack and two adjacent heat exchangers, the acoustic wave carries the working gas back and forth within these components, a longitudinal pressure oscillating in the sound channel induces a temperature oscillation in time with angular frequency ω. In this circumstance the gas temperature can be taken as complex. It results in a time-averaged heat exchange with complex exponent between the gas and the environment by hot and cold heat exchangers. Wu *et al.* [6] studied the optimization of a thermoacoustic engine with a complex heat transfer exponent. In this paper, a further investigation for finite time exergoeconomic performance optimization of the generalized thermoacoustic engine based on a generalized heat transfer law $\dot{Q} \propto \Delta(T^n)$, where *n* is a complex, is performed. Numerical examples are provided to show the effects of complex heat transfer exponent, heat leakage and internal irreversibility on the optimal performance of the generalized irreversible thermoacoustic engine. The result obtained herein may be useful for the selection of the operation parameters for real thermoacoustic engines.

2. The model of thermoacoustic heat engine

The energy flow in a thermoacoustic heat engine is schematically illustrated in Figure 1, where $\dot{W}_{in}$ and $\dot{W}_{out}$ are the flows of power inside the acoustic channel. To simulate the performance of a real thermoacoustic engine more realistically, the following assumptions are made for this model.
(1) External irreversibilities are caused by heat-transfer in the high- and low-temperature side heat-exchangers between the engine and its surrounding heat reservoirs. Because of the heat-transfer, the time average temperatures (T_{H0} and T_{L0}) of the working fluid are different from the heat-reservoir temperatures (T_H and T_L). The second law of thermodynamics requires $T_H > T_{H0} > T_{L0} > T_L$. As a result of thermoacoustic oscillation, the temperatures (T_{HC} and T_{LC}) of the working fluid can be expressed as complexes:

$$T_{HC} = T_{H0} + T_1 e^{i\omega t} \tag{1}$$

$$T_{LC} = T_{L0} + T_2 e^{i\omega t} \tag{2}$$

where T_1 and T_2 are the first-order acoustic quantities, and $i = \sqrt{-1}$. Here the reservoir temperatures (T_H and T_L) are assumed as real constants.
(2) Consider that the heat transfer between the engine and its surroundings follows a generalized radiative law $\dot{Q} \propto \Delta(T^n)$, then

$$\dot{Q}'_{HC} = k_1 F_1 (T_H^n - T_{HC}^n)\,\mathrm{sgn}(n_1) \tag{3}$$

$$\dot{Q}'_{LC} = k_2 F_2 (T_{LC}^n - T_L^n)\,\mathrm{sgn}(n_1) \tag{4}$$

with sign function

$$\mathrm{sgn}(n_1) = \begin{cases} 1 & n_1 > 0 \\ -1 & n_1 < 0 \end{cases} \tag{5}$$

where $n = n_1 + n_2 i$ is a complex heat transfer exponent, k_1 is the overall heat transfer coefficient and F_1 is the total heat transfer surface area of the hot-side heat exchanger, k_2 is the overall heat transfer coefficient and F_2 is the total heat transfer surface area of the cold-side heat exchanger. Here the imaginary part n_2 of *n* indicates the relaxation of a heat transfer process. Defining $\dot{Q}_{HC} = < \dot{Q}'_{HC} >_t$ and

$\dot{Q}_{LC} = <\dot{Q}'_{LC}>_t$ as the time average of $\dot{Q}'_{HC}$ and $\dot{Q}'_{LC}$, respectively, equations (3) and (4) can be rewritten as

$$\dot{Q}_{HC} = \frac{k_1 F_T}{1+f}(T_H^n - T_{H0}^n)\mathrm{sgn}(n_1) \tag{6}$$

$$\dot{Q}_{LC} = \frac{k_2 F_T f}{1+f}(T_{L0}^n - T_L^n)\mathrm{sgn}(n_1) \tag{7}$$

where $f = F_2 / F_1$ and $F_T = F_1 + F_2$. Here, the total heat transfer surface area F_T of the two heat exchangers is assumed to be a constant.

(3) There is a constant rate of heat leakage (q) from the heat source at the temperature T_H to heat sink at T_L such that

$$\dot{Q}_H = \dot{Q}_{HC} + q \tag{8}$$

$$\dot{Q}_L = \dot{Q}_{LC} + q \tag{9}$$

where $\dot{Q}_H$ and $\dot{Q}_L$ are the rates of total heat-transfer absorbed from the heat source and released to the heat sink.

(4) Other than irreversibilies due to heat resistance between the working substance and the heat reservoirs, as well as the heat leakage between the heat reservoirs, there are more irreversibilities such as friction, turbulence, and non-equilibrium activities inside the engine. Thus the power output produced by the irreversible thermoacoustic heat engine is less than that of the endoreversible thermoacoustic heat engine with the same heat input. In other words, the rate of heat transfer ($\dot{Q}_{LC}$) from the cold working fluid to the heat sink for the irreversible thermoacoustic engine is larger than that ($\dot{Q}'_{LC}$) of the endoreversible thermoacoustic heat engine with the same heat input. A constant coefficient (φ) is introduced in the following expression to characterize the additional miscellaneous irreversible effects:

$$\varphi = \dot{Q}_{LC} \big/ \dot{Q}'_{LC} \geq 1 \tag{10}$$

The thermoacoustic heat engine being satisfied with above assumptions is called the generalized irreversible thermoacoustic heat engine with a complex heat transfer exponent. It is similar to a generalized irreversible Carnot heat engine model with heat resistance, heat leakage and internal irreversibility in some aspects [27, 29-32].

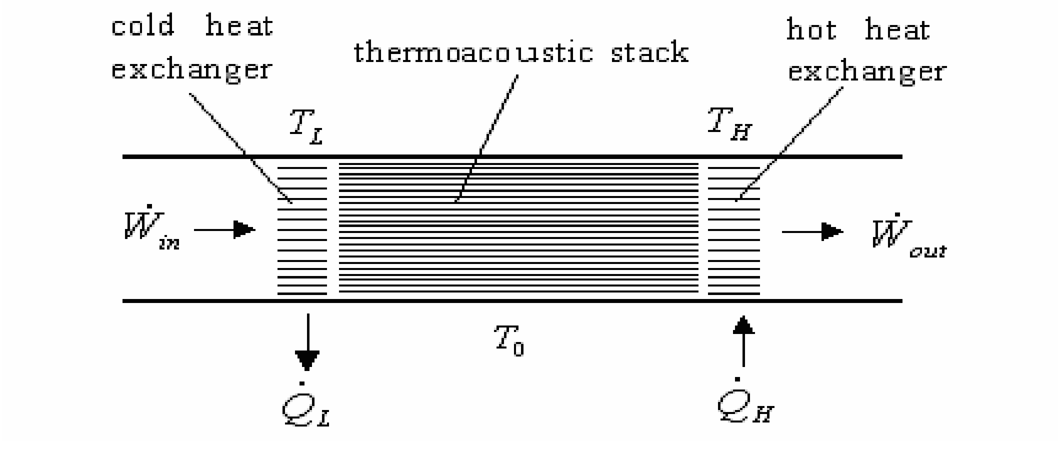

Figure 1. Energy flows in a thermoacoustic heat engine

3. Optimal characteristics

For an endoreversible thermoacoustic heat engine, the second law of thermodynamics requires

$$\dot{Q}'_{LC}/T_{L0} = \dot{Q}_{HC}/T_{H0} \tag{11}$$

Combining Eqs. (10) and (11) gives

$$\dot{Q}_{LC} = \varphi x \dot{Q}_{HC} \tag{12}$$

where $x = T_{L0}/T_{H0}$ ($T_L/T_H \le x \le 1$) is the temperature ratio of the working fluid.
Combining Eqs. (6)- (12) yields

$$T_{H0}{}^n = \frac{k_1 f \varphi x T_H{}^n + k_2 T_L{}^n}{k_2 x^n + k_1 x f \varphi} \tag{13}$$

$$\dot{Q}_{HC} = \frac{k_1 f F_T (x^n T_H{}^n - T_L{}^n)}{(1+f)(x^n + \varphi x f k_1/k_2)} \mathrm{sgn}(n_1) \tag{14}$$

$$\dot{Q}_{LC} = \varphi x \frac{k_1 f F_T (x^n T_H{}^n - T_L{}^n)}{(1+f)(x^n + \varphi x f k_1/k_2)} \mathrm{sgn}(n_1) \tag{15}$$

The first law of thermodynamics gives that the power output and the efficiency of the thermoacoustic heat engine are

$$P' = \dot{Q}_H - \dot{Q}_L = \dot{Q}_{HC} - \dot{Q}_{LC} \tag{16}$$

$$\eta' = P'/\dot{Q}_H = \left(\dot{Q}_{HC} - \dot{Q}_{LC}\right)/(\dot{Q}_{HC} + q) \tag{17}$$

From equations (14)-(17), one can obtain the complex power output (P') and the complex efficiency (η') of the thermoacoustic heat engine

$$P' = \dot{Q}_{HC} - \dot{Q}_{LC} = \frac{k_1 f F_T (1-\varphi x)[T_H{}^n - (T_L/x)^n]}{(1+f)(1+\varphi\delta f x^{1-n})} \mathrm{sgn}(n_1) \tag{18}$$

$$\eta' = \frac{k_1 f F_T (1-\varphi x)[T_H{}^n - (T_L/x)^n]}{q(1+f)(1+\varphi\delta f x^{1-n}) + k_1 f F_T [T_H{}^n - (T_L/x)^n]} \mathrm{sgn}(n_1) \tag{19}$$

where $\delta = k_1/k_2$.
Assuming the environmental temperature is T_0, the rate of exergy input of the thermoacoustic heat engine is:

$$A = Q_H(1 - T_0/T_H) - Q_L(1 - T_0/T_L) = Q_H \varepsilon_1 - Q_L \varepsilon_2 \tag{20}$$

where $\varepsilon_1 = 1 - T_0/T_H$ and $\varepsilon_2 = 1 - T_0/T_L$ are the Carnot coefficients of the reservoirs.
Substituting Eqs. (8), (9), (14) and (15) into Eq. (20) yields the complex rate of exergy input

$$A' = \frac{k_1 f F_T (1-\varphi x)\left(\varepsilon_1 - \varphi x \varepsilon_2\right)[T_H{}^n - (T_L/x)^n]}{(1+f)(1+\varphi\delta f x^{1-n})} \mathrm{sgn}(n_1) + q\left(\varepsilon_1 - \varepsilon_2\right) \tag{21}$$

Assuming that the prices of power output and the exergy input rate be ψ_1 and ψ_2, the profit of the thermoacoustic heat engine is:

$$\pi = \psi_1 P - \psi_2 A \tag{22}$$

Combining Eqs. (18), (21) and (22) gives the complex profit rate of the thermoacoustic heat engine

$$\pi' = \left[\psi_1(1-\varphi x) - \psi_2(\varepsilon_1 - \varphi x \varepsilon_2)\right] \frac{k_1 f F_T (1-\varphi x)[T_H{}^n - (T_L/x)^n]}{(1+f)(1+\varphi\delta f x^{1-n})} \mathrm{sgn}(n_1) - q(\varepsilon_1 - \varepsilon_2)\psi_2 \tag{23}$$

From equations (19) and (23), one can obtain the real parts of efficiency and the profit rate are, respectively,

$$\eta = \mathrm{R}_e(\eta') = \frac{(1-\varphi x)\left[A_1(A_1+B_1) + A_2(A_2+B_2)\right]}{(A_1+B_1)^2 + (A_2+B_2)^2} \tag{24}$$

$$\pi = \mathrm{R}_e(\pi') = \frac{A_1\left[1 + f\varphi\delta x^{1-n_1}\cos(n_2 \ln x)\right] - A_2 f\varphi\delta x^{1-n_1}\sin(n_2 \ln x)]}{1 + 2\delta\varphi f x^{1-n_1}\cos(n_2 \ln x) + f^2\delta^2\varphi^2 x^{2(1-n_1)}}$$
$$\frac{k_1 f F_T\left[\psi_1(1-\varphi x) - \psi_2(\varepsilon_1 - \varphi x\varepsilon_2)\right]}{1+f} - q(\varepsilon_1 - \varepsilon_2)\psi_2 \tag{25}$$

where $B_1 = \frac{q(1+f)}{k_1 f F_T}\left[1 + \varphi\delta f x^{1-n_1}\cos(n_2 \ln x)\right]$, $B_2 = -\frac{q(1+f)\varphi\delta f x^{1-n_1}\sin(n_2 \ln x)}{k_1 f F_T}$,
$A_1 = R_e\left[T_H^n - (T_L/x)^n\right]\mathrm{sgn}(n_1)$, and $A_2 = I_m\left[T_H^n - (T_L/x)^n\right]\mathrm{sgn}(n_1)$, where $R_e(\ )$ and $I_m(\ )$ indicate the real part and imaginary part of complex number.
Maximizing η and π with respect to f by setting $d\eta/df = 0$ or $d\pi/df = 0$ in Eqs. (24) and (25) yields the same optimal ratio of heat-exchanger area (f_{opt})

$$f = f_{opt} = \frac{1}{4}(b - \sqrt{8y + b^2 - 4c}) + \frac{1}{2}\left[\frac{1}{4}(b - \sqrt{8y + b^2 - 4c})^2 - 4\left(y - \frac{by - d}{\sqrt{8y + b^2 - 4c}}\right)\right]^{0.5} \tag{26}$$

where

$$y = \left\{-\frac{e}{2} + \left[\left(\frac{e}{2}\right)^2 - \left(\frac{c^2}{36}\right)^3\right]^{0.5}\right\}^{1/3} + \left\{-\frac{e}{2} - \left[\left(\frac{e}{2}\right)^2 - \left(\frac{c^2}{36}\right)^3\right]^{0.5}\right\}^{1/3} + \frac{c}{6} \tag{27}$$

$$b = \frac{2A_1 x^{n_1-1}}{A_1\cos(n_2 \ln x) - A_2\sin(n_2 \ln x)} \tag{28}$$

$$c = \frac{2A_1 x^{2n_1-2}\cos(n_2 \ln x) + A_1\varphi\delta x^{n_1-1}}{\varphi\delta\left[A_1\cos(n_2 \ln x) - A_2\sin(n_2 \ln x)\right]} - \frac{x^{2n_1-1}}{(\varphi\delta)^2} - \frac{2x^{n_1-1}\cos(n_2 \ln x)}{\varphi\delta} \tag{29}$$

$$d = -2x^{2n_1-2}/(\varphi\delta)^2 \tag{30}$$

$$e = \frac{e_1 c}{2} - \frac{c^3}{108} - \frac{A_1{}^2 e_1 x^{2n_1-2}}{2\left[A_1\cos(n_2 \ln x) - A_2\sin(n_2 \ln x)\right]^2} - \frac{x^{4n_1-4}}{2(\varphi\delta)^4} \tag{31}$$

$$e_1 = \frac{A_1 x^{3n_1-3}}{(\varphi\delta)^2\left[A_1\cos(n_2\ln x) - A_2\sin(n_2\ln x)\right]} \tag{32}$$

Substituting Eq.(26) into Eqs. (24) and (25), respectively, yields the optimal efficiency and the profit rate in the following forms:

$$\eta = \left\{\frac{(1-\varphi x)\left[A_1\left(A_1+B_1\right)+A_2(A_2+B_2)\right]}{\left(A_1+B_1\right)^2+(A_2+B_2)^2}\right\}_{f=f_{opt}} \tag{33}$$

$$\pi = \frac{A_1\left[1+f_{opt}\varphi\delta x^{1-n_1}\cos(n_2\ln x)\right]-A_2 f_{opt}\varphi\delta x^{1-n_1}\sin(n_2\ln x)]}{1+2\delta\varphi f_{opt}x^{1-n_1}\cos(n_2\ln x)+f_{opt}{}^2\delta^2\varphi^2x^{2(1-n_1)}} \\ \frac{k_1 f_{opt}F_T\left[\psi_1(1-\varphi x)-\psi_2\left(\varepsilon_1-\varphi x\varepsilon_2\right)\right]}{1+f_{opt}} - q\left(\varepsilon_1-\varepsilon_2\right)\psi_2 \tag{34}$$

The parameter equation defined by equations (33) and (34) gives the fundamental relationship between the optimal profit rate and efficiency consisting of the interim variable.
Maximizing π with respect to x by setting $d\pi/dx = 0$ in Eq. (34) can yield the optimal temperature ratio x_{opt} and the maximum profit rate π_{max} of the thermoacoustic heat engine. The corresponding efficiency η_π, which is the finite-time exergoeconomic bound of the generalized irreversible thermoacoustic heat engine can be obtained by substituting the optimal temperature ratio into Eq. (33).

4. Discussions

If $\varphi = 1$ and $q \neq 0$, equations (33) and (34) become:

$$\eta = \left\{\frac{(1-x)\left[A_1\left(A_1+B_1\right)+A_2(A_2+B_2)\right]}{\left(A_1+B_1\right)^2+(A_2+B_2)^2}\right\}_{f=f_{opt}} \tag{35}$$

$$\pi = \frac{A_1\left[1+f_{opt}\delta x^{1-n_1}\cos(n_2\ln x)\right]-A_2 f_{opt}\delta x^{1-n_1}\sin(n_2\ln x)]}{1+2\delta f_{opt}x^{1-n_1}\cos(n_2\ln x)+f_{opt}{}^2\delta^2x^{2(1-n_1)}} \\ \frac{k_1 f_{opt}F_T\left[\psi_1(1-x)-\psi_2\left(\varepsilon_1-x\varepsilon_2\right)\right]}{1+f_{opt}} - q\left(\varepsilon_1-\varepsilon_2\right)\psi_2 \tag{36}$$

Equations (35) and (36) are the relationship between the efficiency and the profit rate of the irreversible thermoacoustic heat engine with heat resistances and heat leakage losses.
If $\varphi > 1$ and $q = 0$, equations (33) and (34) become:

$$\eta = \left\{\frac{(1-\varphi x)\left[A_1\left(A_1+B_1\right)+A_2(A_2+B_2)\right]}{\left(A_1+B_1\right)^2+(A_2+B_2)^2}\right\}_{f=f_{opt},q=0} \tag{37}$$

$$\pi = \frac{A_1\left[1+f_{opt}\varphi\delta x^{1-n_1}\cos(n_2\ln x)\right]-A_2 f_{opt}\varphi\delta x^{1-n_1}\sin(n_2\ln x)]}{1+2\delta\varphi f_{opt}x^{1-n_1}\cos(n_2\ln x)+f_{opt}{}^2\delta^2\varphi^2x^{2(1-n_1)}} \\ \frac{k_1 f_{opt}F_T\left[\psi_1(1-\varphi x)-\psi_2\left(\varepsilon_1-\varphi x\varepsilon_2\right)\right]}{1+f_{opt}} \tag{38}$$

Equations (37) and (38) are the relationship between the efficiency and the profit rate of the irreversible thermoacoustic heat engine with heat resistance and internal irreversibility losses.

If $\varphi = 1$ and $q = 0$, equations (33) and (34) become:

$$\eta = \left\{ \frac{(1-x)\left[A_1(A_1+B_1)+A_2(A_2+B_2)\right]}{(A_1+B_1)^2+(A_2+B_2)^2} \right\}_{f=f_{opt},q=0} \tag{39}$$

$$\pi = \frac{A_1\left[1+f_{opt}\delta x^{1-n_1}\cos(n_2\ln x)\right]-A_2 f_{opt}\delta x^{1-n_1}\sin(n_2\ln x)]}{1+2\delta f_{opt}x^{1-n_1}\cos(n_2\ln x)+f_{opt}{}^2\delta^2x^{2(1-n_1)}} \\ \frac{k_1 f_{opt}F_T\left[\psi_1(1-x)-\psi_2(\varepsilon_1-x\varepsilon_2)\right]}{1+f_{opt}} \tag{40}$$

Equations (39) and (40) are the relationship between the efficiency and the profit rate of the endoreversible thermoacoustic heat engine.

The finite-time exergoeconomic performance bound at the maximum profit rate is different from the classical reversible bound and the finite-time thermodynamic bound at the maximum power output. It is dependent on T_H , T_L , T_0 and ψ_2/ψ_1. Note that for the process to be potential profitable, the following relationship must exist: $0<\psi_2/\psi_1<1$, because one unit of exergy input rate must give rise to at least one unit of power output. As the price of power output becomes very large compared with that of the exergy input rate, i.e. $\psi_2/\psi_1 \to 0$, equation (34) becomes

$$\pi = \psi_1\frac{A_1\left[1+f_{opt}\varphi\delta x^{1-n_1}\cos(n_2\ln x)\right]-A_2 f_{opt}\varphi\delta x^{1-n_1}\sin(n_2\ln x)]}{1+2\delta\varphi f_{opt}x^{1-n_1}\cos(n_2\ln x)+f_{opt}{}^2\delta^2\varphi^2x^{2(1-n_1)}} \\ \frac{k_1 f_{opt}F_T(1-\varphi x)}{1+f_{opt}} = \psi_1 P \tag{41}$$

That is the profit maximization approaches the power output maximization,

On the other hand, as the price of exergy input rate approaches the price of the power output, i.e. $\psi_2/\psi_1 \to 1$, equation (34) becomes

$$\pi = -\psi_1 T_0[(Q_{LC}+q)/T_L-(Q_{HC}+q)/T_H] = -\psi_1 T_0\sigma \tag{42}$$

where σ is the rate of entropy production of the thermoacoustic heat engine. That is the profit maximization approaches the entropy production rate minimization, in other word, the minimum waste of exergy. Eq. (42) indicates that the thermoacoustic heat engine is not profitable regardless of the efficiency is at which the thermoacoustic heat engine is operating. Only the thermoacoustic heat engine is operating reversibly ($\eta = \eta_C$) will the revenue equal to the cost, and then the maximum profit rate will be equal zero. The corresponding rate of entropy production is also zero.

Therefore, for any intermediate values of ψ_2/ψ_1, the finite-time exergoeconomic performance bound (η_π) lies between the finite-time thermodynamic performance bound and the reversible performance bound. η_π is related to the latter two through the price ratio, and the associated efficiency bounds are the upper and lower limits of η_π.

5. Numerical examples

To illustrate the preceding analysis, numerical examples are provided. In the calculations, it is set that $T_H = 1200K, T_L = 400K, T_0 = 298.15K$; $k_1 = k_2$; $\varphi = 1.0$, 1.1 , 1.2 ; $\psi_1 = 1000 yuan/kW$, $\psi_1/\psi_2 = 4$; $q = C_i(T_H{}^n - T_L{}^n)$ (same as Ref. [33]) and $C_i = 0.0, 0.02 kW/K$; C_i is the thermal conductance inside the thermoacoustic heat engine.

Figures (2-7) show the effects of the heat leakage, the internal irreversibility losses and the heat transfer exponent on the relationship between the profit rate and efficiency. One can see that for all heat transfer law, the influences of the internal irreversibility losses and the heat leakage on the relationship between the profit rate and efficiency are different obviously: the profit rate π decreases along with increasing of the internal irreversibility φ, but the curves of $\pi - \eta$ are not changeable; the heat leakage affects strongly the relationship between the profit rate and efficiency, the curves of $\pi - \eta$ are parabolic-like ones in the case of $q = 0$, while the curves are loop-shaped ones in the case of $q \neq 0$.

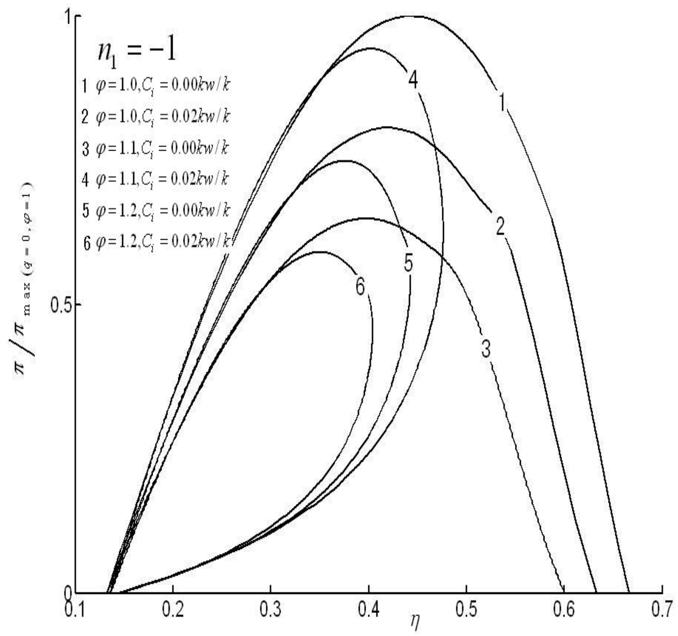

Figure 2. Influences of internal irreversibility and heat leakage on $\pi - \eta$ characteristic with $n_1 = -1$ and $n_2 = 0.1$

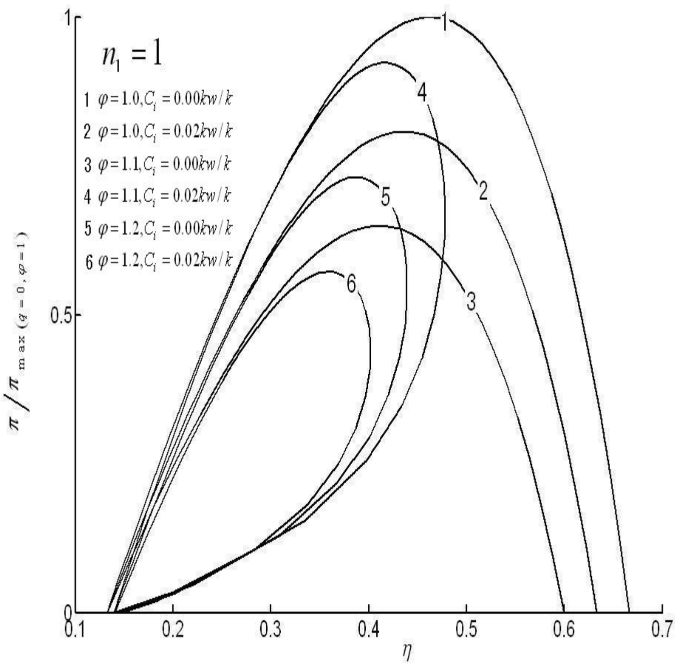

Figure 3. Influences of internal irreversibility and heat leakage on $\pi - \eta$ characteristic with $n_1 = 1$ and $n_2 = 0.1$

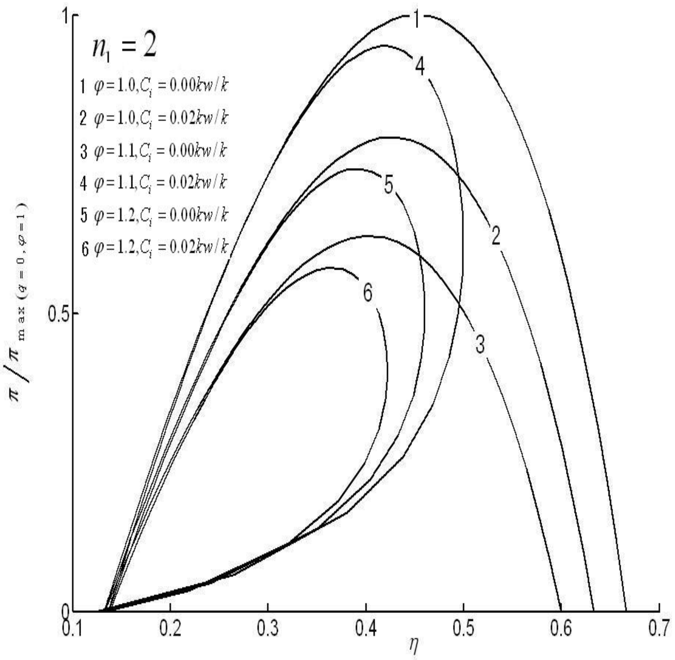

Figure 4. Influences of internal irreversibility and heat leakage on $\pi-\eta$ characteristic with $n_1=2$ and $n_2=0.1$

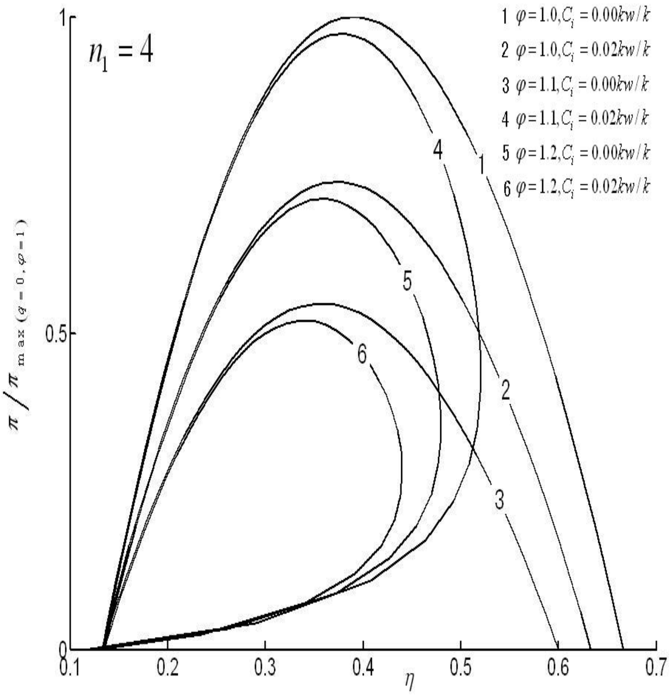

Figure 5. Influences of internal irreversibility and heat leakage on $\pi-\eta$ characteristic with $n_1=4$ and $n_2=0.1$

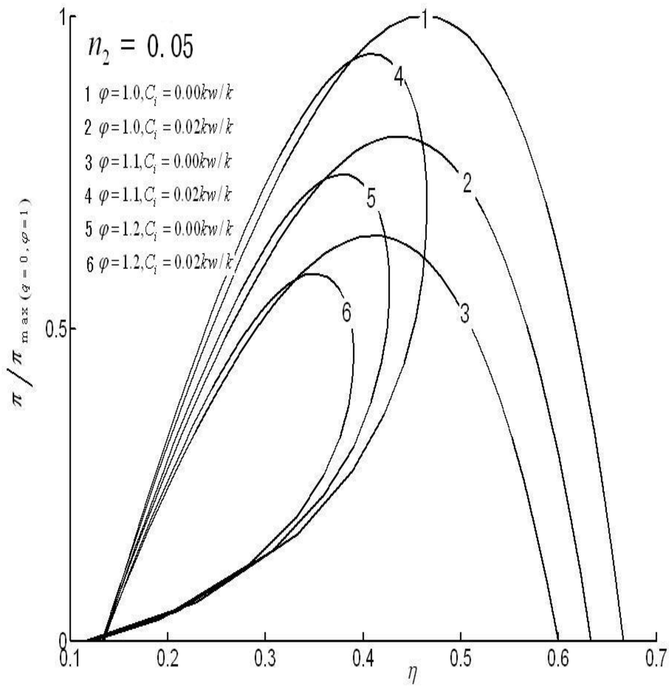

Figure 6. Influences of internal irreversibility and heat leakage on $\pi-\eta$ characteristic with $n_1=1$ and $n_2=0.05$

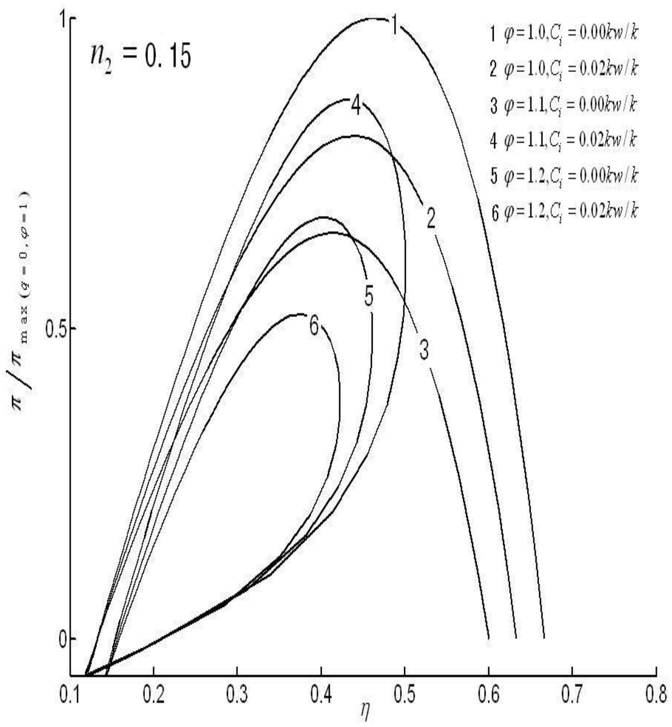

Figure 7. Influences of internal irreversibility and heat leakage on $\pi-\eta$ characteristic with $n_1=1$ and $n_2=0.15$

From Figures (2-7), one can also see that both the real part n_1 and the imaginary part n_2 of the complex heat transfer exponent n don't change the shape of the curves of $\pi-\eta$. Figures (2-5) illustrate that when the imaginary part $n_2 = 0.1$ is fixed, the corresponding efficiency η_π at the maximum profit rate decreases with the increase of absolute value of the real part n_1, the reason is that the power output is sensitive to the temperature, when the absolute value of the real part n_1 increases, it sacrifices a little part of the temperature ratio, decreases the thermal efficiency to some extent, but increase the power output to a great extent induced by the increases of the temperature differences between the heat exchangers and the working fluid. Figures (3, 6, 7) show that when the real part $n_1 = 1$ is fixed, the maximum profit rate decreases with the increase of the imaginary part n_2 of the complex heat transfer exponent n, it illustrates that the imaginary part n_2 of the complex heat transfer exponent n indicates energy dissipation.

The effects of complex exponent $n = n_1 + in_2$ on the optimal profit rate versus efficiency characteristics with $T_H = 1200K$, $T_L = 400K$, $T_0 = 298.15K$, $\delta = 1$, $q = 16W$, and $\varphi = 1.05$ are shown in Figures (8, 9). They show that π versus η characteristics of a generalized irreversible thermoacoustic heat engine with a complex heat transfer exponent is a loop-shaped curve. For all n_1 and n_2, $\pi = \pi_{\max}$ when $\eta = \eta_0$ and $\eta = \eta_{\max}$ when $\pi = \pi_0$. For example, when $n_1 = 1$, the π bound ($\pi_{\max}$) corresponding to $n_2 = 0.05$, 0.10 and 0.15 are $10979(yuan)$, $8468.7(yuan)$ and $4659.5(yuan)$, respectively, and the maximum thermal efficiency $\eta_{\max}$ corresponding to $n_2 = 0.05, 0.10, 0.15$ are 0.4459, 0.1583 and 0.4808, respectively.

The optimization criteria of the thermoacoustic heat engine can been obtained from parameters $\pi_{\max}$, π_0, $\eta_{\max}$ and η_0 as follows:

$$\pi_0 \le \pi \le \pi_{\max} \text{ and } \eta_0 \le \eta \le \eta_{\max} \tag{43}$$

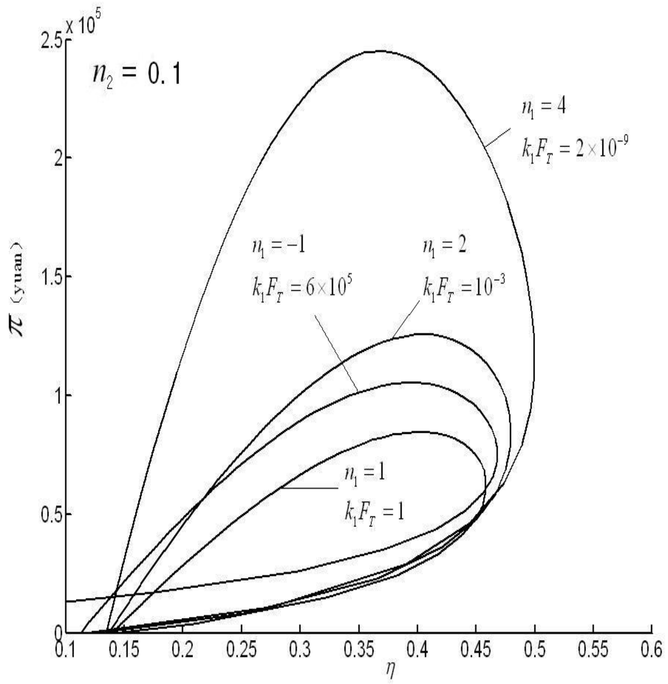

Figure 8. Optimal profit rate versus efficiency with $n_2 = 0.1, n_1 = -1, n_1 = 1, n_1 = 2$ and $n_1 = 4$

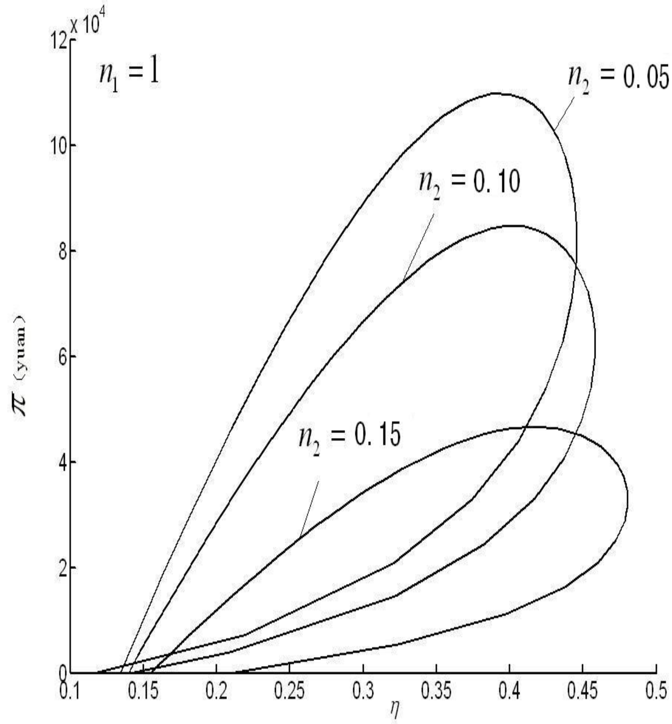

Figure 9. Optimal profit rate versus efficiency with $n_1 = 1, n_2 = 0.05, n_2 = 0.1$ and $n_2 = 0.15$

6. Conclusion

The generalized irreversible cycle model of a thermoacoustic heat engine with a complex heat transfer exponent established in this paper reveals the effects of heat resistance, heat leakage, thermal relaxation, internal irreversibility and complex heat transfer exponent on the relationship between the profit rate and efficiency. The heat transfer exponent for a thermoacoustic heat engine must be complex number due to the thermal relaxation induced by the thermoacoustic oscillation. The comparative analysis of the influences of various factors on the relationship between optimal profit rate and the thermal efficiency of the generalized irreversible thermoacoustic heat engine is carried out by detailed numerical examples, the optimal zone of the thermoacoustic engine with a complex heat transfer exponent is analyzed. The results obtained herein are helpful for the selection of the optimal mode of operation of the real thermoacoustic heat engines.

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