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Finite time exergoeconomic performance optimization of a thermoacoustic cooler with a complex heat transfer exponent

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Abstract
The finite time exergoeconomic performance optimization of a generalized irreversible thermoacoustic cooler with heat resistance, heat leakage, relaxation effect, and internal dissipation, in which heat transfer between the working fluid and heat reservoirs obeys a complex generalized heat transfer law $\dot{Q} \propto \Delta(T^n)$, where n is a complex heat transfer exponent, is investigated in this paper. Both the real part and the imaginary part of the complex heat transfer exponent change the optimal profit rate versus the coefficient of performance (COP) relationship quantitatively. The operation of the generalized irreversible thermoacoustic cooler is viewed as a production process with exergy as its output. The finite time exergoeconomic performance optimization of the generalized irreversible thermoacoustic cooler is performed by taking profit rate as the objective. The analytical formulae about the profit rate and the COP of the thermoacoustic cooler are derived. Furthermore, the comparative analysis of the influences of various factors on the relationship between optimal profit rate and the COP of the generalized irreversible thermoacoustic cooler is carried out by detailed numerical examples. The optimal zone on the performance of the thermoacoustic cooler is obtained.

Keywords: Thermoacoustic cooler; Complex heat transfer exponent; Exergoeconomic performance; Optimization zone.

1. Introduction

Thermoacoustic engine (prime mover and refrigerator) [1-4] is a new type of engine, which is based on the thermoacoustic effect. It has many advantages of high reliability, non-pollution, simple construction, low noise, non-parts of motion, ability to self-start, etc, with the comparison to the traditional engines.
Recently, Wu *et al.* [5-7] have studied a generalized irreversible thermoacoustic engine (or refrigerator) cycle model and the performance using the finite-time thermodynamics [8-15]. A relatively new method that combines exergy with conventional concepts from long-run engineering economic optimization to evaluate and optimize the performance of energy systems is exergoeconomic (or thermoeconomic) analysis [16, 17]. Salamon and Nitzan's work [18] combined the endoreversible model with exergoeconomic analysis. It was termed as finite time exergoeconomic analysis [19-34] to distinguish it from the endoreversible analysis with pure thermodynamic objectives and the exergoeconomic analysis

with long-run economic optimization. Similarly, the performance bound at maximum profit was termed as finite time exergoeconomic performance bound to distinguish it from the finite time thermodynamic performance bound at maximum thermodynamic output. A similar idea was provided by Ibrahim et al. [19], Bejan [20] and De Vos [21, 22]. de Vos [21, 22] made the use of the basic idea of finite time thermodynamics into the thermoeconomics for heat engine in which the heat transfer between the working fluid and the heat reservoirs obeys Newtonian (linear) heat transfer law, and derived the relation between the optimal efficiency and economic returns. Chen *et al.* [29] investigated the endoreversible thermoeconomic performance of heat engine with the linear phenomenological heat transfer law based on Ref. [21]. Sahin *et al.* [24, 35-42] provided a new thermoeconomic optimization criterion, thermodynamic output rates (power, cooling load or heating load for heat engine, refrigerator or heat pump) per unit total cost, and investigated the performances of endoreversible and irreversible simple-cycle heat engines, refrigerators and heat pumps, combined-cycle refrigerators and heat pumps, as well as three-heat-reservoir absorption refrigerators and heat pumps. This method was also applied to the optimization of an endoreversible four-heat-reservoir absorption-refrigerator by Qin *et al.* [43].

From these previous works, the heat transfer exponent is assumed to be real. But for thermoacoustic heat engines (or refrigerators), the interactions between the entropy wave and the oscillating flow produce a rich variety of thermoacoustic phenomenon such as self-excited gas oscillation. A longitudinal pressure oscillating in the sound channel induces a temperature oscillation in time with angular frequency ω. In this circumstance the gas temperature can be taken as complex [6, 7, 44]. It results in a time-averaged heat exchange with complex exponent between the gas and the environment by hot and cold heat exchangers.

In this paper, finite time exergoeconomic performance optimization of a generalized irreversible thermoacoustic cooler with the losses of heat resistance, heat leakage and internal irreversibility, in which heat transfer between the working fluid and heat reservoirs obeys a complex generalized heat transfer law $\dot{Q} \propto \Delta\left(T^n\right)$, where n is a complex heat transfer exponent, is derived by taking the finite time exergoeconomic performance optimization criterion as the objective. Numerical examples are provided to show the effects of complex heat transfer exponent, heat leakage and internal irreversibility on the optimal performance of the generalized irreversible thermoacoustic cooler.

2. Model of a thermoacoustic cooler

The energy flow in a thermoacoustic cooler is schematically illustrated in Figure 1, where $\dot{W}_{in}$ and $\dot{W}_{out}$ are the flows of power inside the acoustic channel. To simulate the performance of a real thermoacoustic cooler more realistically, the following assumptions are made for this model.

(1). External irreversibilities are caused by heat-transfers in the hot and cold heat exchangers between the cooler and its surrounding heat reservoirs. Because of the heat-transfers, the time average temperature (T_{H0} and T_{L0}) of the working fluid are different from the heat-reservoir temperatures (T_H and T_L). The second law of thermodynamics requires $T_{H0} > T_H > T_L > T_{L0}$. As a result of thermoacoustic oscillation, the temperatures (T_{HC} and T_{LC}) of the working fluid can be expressed as complexes [1-4, 6, 7, 44]:

$$T_{HC} = T_{H0} + T_1 e^{i\omega t} \tag{1}$$

$$T_{LC} = T_{L0} + T_2 e^{i\omega t} \tag{2}$$

where $i = \sqrt{-1}$ and ω is the oscillating angular frequency. In this paper the oscillating quantities are expanded by Taylor series, the first-order item of Taylor series is used, and the high-order items of Taylor series are neglected. So T_1 and T_2 are the first-order acoustic quantities. Here the reservoir temperatures (T_H and T_L) are assumed as real constants, so they have no imaginary part.

(2). Consider that the heat transfer between the cooler and its surroundings follow a generalized law $\dot{Q} \propto \Delta(T^n)$, then;

$$\dot{Q}'_{HC} = k_1 F_1 (T_{HC}^n - T_H^n)\,\mathrm{sgn}(n_1) \tag{3}$$

$$\dot{Q}'_{LC} = k_2 F_2 (T_L^n - T_{LC}^n)\,\mathrm{sgn}(n_1) \tag{4}$$

with sign function

$$\mathrm{sgn}(n_1) = \begin{cases} 1 & n_1 > 0 \\ -1 & n_1 < 0 \end{cases} \tag{5}$$

where $n = n_1 + n_2 i$ is a complex heat transfer exponent, k_1 is the overall heat transfer coefficient and F_1 is the total heat transfer surface area of the hot heat exchanger, k_2 is the overall heat transfer coefficient and F_2 is the total heat transfer surface area of the cold heat exchanger. Here the imaginary part n_2 of n indicates the relaxation of a heat transfer process, and k_1, k_2, F_1 and F_2 are all real constants. Defining $\dot{Q}_{HC} =< \dot{Q}'_{HC} >_t$ and $\dot{Q}_{LC} =< \dot{Q}'_{LC} >_t$ as the time average of $\dot{Q}'_{HC}$ and $\dot{Q}'_{LC}$, respectively, equations (8) and (9) can be rewritten as

$$\dot{Q}_{HC} = \frac{k_1 F_T}{1+f}(T_{H0}^n - T_H^n)\,\mathrm{sgn}(n_1) \tag{6}$$

$$\dot{Q}_{LC} = \frac{k_2 f F_T}{1+f}(T_L^n - T_{L0}^n)\,\mathrm{sgn}(n_1) \tag{7}$$

where $f = F_2 / F_1$ and $F_T = F_1 + F_2$. Here, the total heat transfer surface area F_T of the two heat exchangers is assumed to be a constant.

(3). There is a constant rate of heat leakage (q) [45] from the heat source at the temperature T_H to heat sink at T_L such that

$$\dot{Q}_H = \dot{Q}_{HC} - q \tag{8}$$

$$\dot{Q}_L = \dot{Q}_{LC} - q \tag{9}$$

where $\dot{Q}_H$ and $\dot{Q}_L$ are the rates of total heat-transfer released to the heat sink and absorbed from the heat source, and the cooling load of the cooler is $\dot{Q}_L$.

(4). Other than irreversibilies due to heat resistance between the working fluid and the heat reservoirs, as well as the heat leakage between the heat reservoirs, there are more irreversibilities such as friction, turbulence, and non-equilibrium activities inside the cooler. Thus the power input to the irreversible thermoacoustic cooler is larger than that of the endoreversible thermoacoustic cooler with the same cooling load. In other words, the rate of heat flow ($\dot{Q}_{HC}$) from the warm working fluid to the heat sink for the real thermoacoustic cooler is larger than that ($\dot{Q}'_{HC}$) of the endoreversible thermoacoustic cooler with the same cooling load. A constant coefficient (φ) is introduced in the following expression to characterize the additional miscellaneous irreversible effects:

$$\varphi = \dot{Q}_{HC} / \dot{Q}'_{HC} \geq 1 \tag{10}$$

The cooler being satisfied with above assumptions is called the generalized irreversible thermoacoustic cooler with a complex heat transfer exponent. In fact, the parameters F_1, F_2 and φ should not be constants in a realistic optimization. In order to simply the problem, they are assumed as two constants, as did for irreversible Carnot heat engine [30] and refrigerator [32, 45, 46].

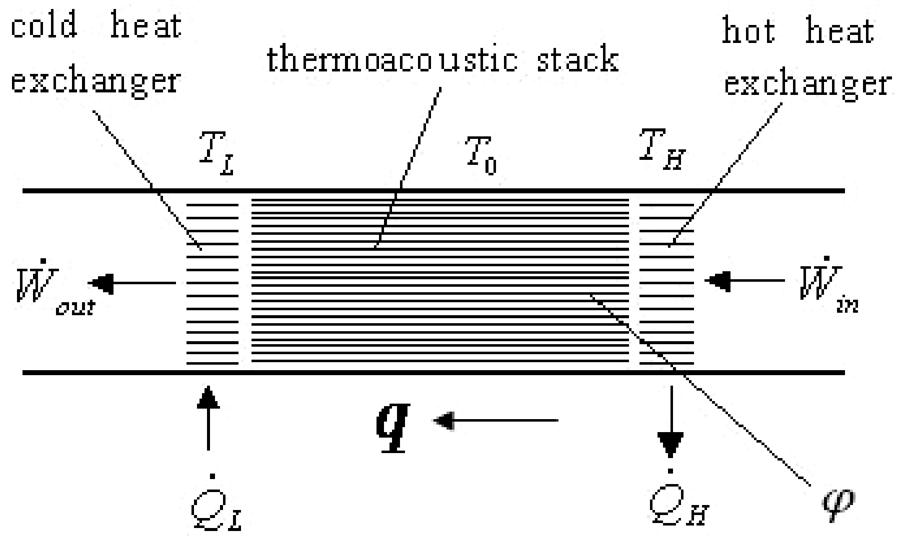

Figure 1. Energy flows in a thermoacoustic cooler

3. Optimal characteristics

For an endoreversible thermoacoustic cooler, the second law of thermodynamics requires

$$\dot{Q}_{HC}/T_{H0} = \dot{Q}_{LC}/T_{L0} \quad (11)$$

Combining Eqs. (10) and (11) gives

$$\dot{Q}_{HC} = \varphi x \dot{Q}_{LC} \quad (12)$$

where $x = T_{H0}/T_{L0}$ ($x \ge T_H/T_L$) is the temperature ratio of the working fluid.

Combining Eqs. (6)- (12) yields

$$T_{L0}{}^{n} = \frac{k_2 f \varphi x T_L{}^{n} + k_1 T_H{}^{n}}{k_1 x^n + k_2 x f \varphi} \quad (13)$$

$$\dot{Q}_{LC} = \frac{k_2 f F_T (x^n T_L{}^{n} - T_H{}^{n})}{(1+f)(x^n + \varphi x f k_2/k_1)} \mathrm{sgn}(n_1) \quad (14)$$

$$\dot{Q}_{HC} = \varphi x \frac{k_2 f F_T (x^n T_L{}^{n} - T_H{}^{n})}{(1+f)(x^n + \varphi x f k_2/k_1)} \mathrm{sgn}(n_1) \quad (15)$$

The first law of thermodynamics gives the cooling load, COP and power input of the thermoacoustic cooler are, respectively

$$R' = \dot{Q}_L = \dot{Q}_{LC} - q \quad (16)$$

$$\varepsilon' = \dot{Q}_L \Big/ \left(\dot{Q}_H - \dot{Q}_L\right) = \left(\dot{Q}_{LC} - q\right) \Big/ (\dot{Q}_{HC} - \dot{Q}_{LC}) \quad (17)$$

$$P' = \dot{Q}_H - \dot{Q}_L = \dot{Q}_{HC} - \dot{Q}_{LC} \tag{18}$$

Assuming the environmental temperature is T_0, the rate of exergy output of the thermoacoustic cooler is:

$$A' = \dot{Q}_L (T_0/T_L - 1) - \dot{Q}_H (T_0/T_H - 1) = \dot{Q}_L \eta_2 - \dot{Q}_H \eta_1 \tag{19}$$

where η_i is the Carnot coefficient of the reservoir i, and is defined as $\eta_1 = T_0/T_H - 1$ and $\eta_2 = T_0/T_L - 1$.
Assuming that the prices of exergy output rate and the power input be ψ_1 and ψ_2, respectively, the profit rate of the cooler is:

$$\pi' = \psi_1 A' - \psi_2 P' \tag{20}$$

Combining Eqs. (8)-(20) gives the complex cooling load (R'), the complex COP (ε') and the complex profit rate (π') of the thermoacoustic cooler

$$R' = \frac{k_2 f F_T \left[T_L{}^n - (T_H/x)^n\right]}{(1+f)(1+\varphi\delta x^{1-n} f)} \mathrm{sgn}(n_1) - q \tag{21}$$

$$\varepsilon' = (\varphi x - 1)^{-1} \left\{1 - \frac{q(1+f)(1+\varphi\delta f x^{1-n})}{k_2 f F_T [T_L{}^n - (T_H/x)^n]} \mathrm{sgn}(n_1)\right\} \tag{22}$$

$$\pi' = \left[(\psi_1\eta_2 + \psi_2) - \varphi x(\psi_1\eta_1 + \psi_2)\right] \frac{k_2 f F_T \left[T_L{}^n - (T_H/x)^n\right]}{(1+f)(1+\varphi\delta x^{1-n} f)} \mathrm{sgn}(n_1) + q\psi_1(\eta_1 - \eta_2) \tag{23}$$

where $\delta = k_2/k_1$.
From equations (22) and (23), one can obtain the real parts of COP and the profit rate, respectively

$$\varepsilon = \mathrm{R}_e(\varepsilon') = \frac{1}{(1-\varphi x)} \left\{1 - \frac{q(1+f)}{k_2 f F_T} \frac{A_1\left[1 + f\varphi\delta x^{1-n_1}\cos(n_2 \ln x)\right] + A_2 f\varphi\delta x^{1-n_1}\sin(n_2 \ln x)]}{A_1{}^2 + A_2{}^2}\right\} \tag{24}$$

$$\pi = \mathrm{R}_e(\pi') = \frac{k_2 f F_T}{(1+f)} \frac{A_1\left[1 + f\varphi\delta x^{1-n_1}\cos(n_2 \ln x)\right] - A_2 f\varphi\delta x^{1-n_1}\sin(n_2 \ln x)}{1 + 2\delta\varphi f x^{1-n_1}\cos(n_2 \ln x) + f^2\delta^2\varphi^2 x^{2(1-n_1)}}$$
$$\left[(\psi_1\eta_2 + \varphi_2) - \varphi x(\psi_1\eta_1 + \psi_2)\right] + q\psi_1(\eta_1 - \eta_2) \tag{25}$$

where $A_1 = R_e\left[T_L^n - (T_H/x)^n\right]\mathrm{sgn}(n_1)$ and $A_2 = I_m\left[T_L^n - (T_H/x)^n\right]\mathrm{sgn}(n_1)$, here $R_e(\ )$ and $I_m(\ )$ indicate the real part and imaginary part of a complex number.
Maximizing ε (or π) with respect to f by setting $d\varepsilon/df = 0$ (or $d\pi/df = 0$) using Eq. (22) (or (23)) yields the same optimal ratio of heat-exchanger area (f_{opt})

$$f = f_{opt} = \frac{1}{4}(b - \sqrt{8y + b^2 - 4c}) + \frac{1}{2}[\frac{1}{4}(b - \sqrt{8y + b^2 - 4c})^2 - 4(y - \frac{by - d}{\sqrt{8y + b^2 - 4c}})]^{0.5} \tag{26}$$

where

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$$y=\left\{-\frac{e}{2}+[(\frac{e}{2})^2-(\frac{c^2}{36})^3]^{0.5}\right\}^{1/3}+\left\{-\frac{e}{2}-[(\frac{e}{2})^2-(\frac{c^2}{36})^3]^{0.5}\right\}^{1/3}+\frac{c}{6} \tag{27}$$

$$b=\frac{2A_1x^{n_1-1}}{A_1\cos(n_2\ln x)-A_2\sin(n_2\ln x)} \tag{28}$$

$$c=\frac{2A_1x^{2n_1-2}\cos(n_2\ln x)+A_1\varphi\delta x^{n_1-1}}{\varphi\delta\left[A_1\cos(n_2\ln x)-A_2\sin(n_2\ln x)\right]}-\frac{x^{2n_1-1}}{(\varphi\delta)^2}-\frac{2x^{n_1-1}\cos(n_2\ln x)}{\varphi\delta} \tag{29}$$

$$d=-2x^{2n_1-2}/(\varphi\delta)^2 \tag{30}$$

$$e_1=\frac{A_1x^{3n_1-3}}{(\varphi\delta)^2\left[A_1\cos(n_2\ln x)-A_2\sin(n_2\ln x)\right]} \tag{31}$$

$$e=\frac{e_1c}{2}-\frac{c^3}{108}-\frac{A_1^{\,2}e_1x^{2n_1-2}}{2\left[A_1\cos(n_2\ln x)-A_2\sin(n_2\ln x)\right]^2}-\frac{x^{4n_1-4}}{2(\varphi\delta)^4} \tag{32}$$

Substituting Eq.(26) into Eqs. (24) and (25), respectively, yields the optimal COP and the profit rate in the following:

$$\varepsilon=\frac{1}{(1-\varphi x)}\left\{1-\frac{q(1+f_{opt})}{k_2f_{opt}F_T}\frac{A_1\left[1+f_{opt}\varphi\delta x^{1-n_1}\cos(n_2\ln x)\right]+A_2f_{opt}\varphi\delta x^{1-n_1}\sin(n_2\ln x)]}{A_1^{\,2}+A_2^{\,2}}\right\} \tag{33}$$

$$\pi=\frac{k_2f_{opt}F_T}{(1+f_{opt})}\frac{A_1\left[1+f_{opt}\varphi\delta x^{1-n_1}\cos(n_2\ln x)\right]-A_2f_{opt}\varphi\delta x^{1-n_1}\sin(n_2\ln x)}{1+2\delta\varphi f_{opt}x^{1-n_1}\cos(n_2\ln x)+f_{opt}^{\,2}\delta^2\varphi^2x^{2(1-n_1)}}$$
$$\left[(\psi_1\eta_2+\psi_2)-\varphi x(\psi_1\eta_1+\psi_2)\right]+q\psi_1(\eta_1-\eta_2) \tag{34}$$

The parameter equations defined by equations (33) and (34) give the fundamental relationship between the optimal profit rate and the COP involving the interim variable x.
Maximizing π with respect to x by setting $d\pi/dx=0$ using Eq. (34) yields the optimal temperature ratio x_{opt} and the maximum profit $\pi_{\max}$ of the thermoacoustic cooler. The corresponding COP (ε_π), which is the finite-time exergoeconomic bound of the generalized irreversible thermoacoustic cooler, will be obtained by substituting the optimal temperature ratio into Eq. (33).

4. Discussions

If $\varphi=1$ and $q\neq 0$, equations (33) and (34) become:

$$\varepsilon=\frac{1}{(1-x)}\left\{1-\frac{q(1+f_{opt})}{k_2f_{opt}F_T}\frac{A_1\left[1+f_{opt}\delta x^{1-n_1}\cos(n_2\ln x)\right]+A_2f_{opt}\delta x^{1-n_1}\sin(n_2\ln x)]}{A_1^{\,2}+A_2^{\,2}}\right\} \tag{35}$$

$$\pi = \frac{k_2 f_{opt} F_T}{(1+f_{opt})} \frac{A_1\left[1+f_{opt}\delta x^{1-n_1}\cos(n_2 \ln x)\right] - A_2 f_{opt}\delta x^{1-n_1}\sin(n_2 \ln x)}{1+2\delta f_{opt} x^{1-n_1}\cos(n_2 \ln x) + f_{opt}{}^2\delta^2 x^{2(1-n_1)}} \left[(\psi_1\eta_2+\varphi_2) - x(\psi_1\eta_1+\psi_2)\right] + q\psi_1(\eta_1-\eta_2) \tag{36}$$

Equations (35) and (36) are the relationship between the COP and the profit rate of the irreversible thermoacoustic cooler with losses of heat resistances and heat leakage.
If $\varphi > 1$ and $q = 0$, equations (33) and (34) become:

$$\varepsilon = 1/(1-\varphi x) \tag{37}$$

$$\pi = \frac{k_2 f_{opt} F_T}{(1+f_{opt})} \frac{A_1\left[1+f_{opt}\varphi\delta x^{1-n_1}\cos(n_2 \ln x)\right] - A_2 f_{opt}\varphi\delta x^{1-n_1}\sin(n_2 \ln x)}{1+2\delta\varphi f_{opt} x^{1-n_1}\cos(n_2 \ln x) + f_{opt}{}^2\delta^2\varphi^2 x^{2(1-n_1)}} \left[(\psi_1\eta_2+\psi_2) - \varphi x(\psi_1\eta_1+\psi_2)\right] \tag{38}$$

Equations (37) and (38) are the relationship between the COP and the profit rate of the irreversible thermoacoustic cooler with losses of heat resistance and internal irreversibility.
If $\varphi = 1$ and $q = 0$, equations (33) and (34) become:

$$\varepsilon = 1/(1-x) \tag{39}$$

$$\pi = \frac{k_2 f_{opt} F_T}{(1+f_{opt})} \frac{A_1\left[1+f_{opt}\delta x^{1-n_1}\cos(n_2 \ln x)\right] - A_2 f_{opt}\delta x^{1-n_1}\sin(n_2 \ln x)}{1+2\delta f_{opt} x^{1-n_1}\cos(n_2 \ln x) + f_{opt}{}^2\delta^2 x^{2(1-n_1)}} \left[(\psi_1\eta_2+\psi_2) - x(\psi_1\eta_1+\psi_2)\right] \tag{40}$$

Equations (39) and (40) are the relationship between the COP and the profit rate of the endoreversible thermoacoustic cooler.
The finite-time exergoeconomic performance bound at the maximum profit is different from the classical reversible bound and the finite-time thermodynamic bound at the maximum cooling load. It is dependent on T_H, T_L, T_0 and ψ_2/ψ_1. Note that for the process to be potential profitable, the following relationship must exist: $0 < \psi_2/\psi_1 < 1$, because one unit of power input must give rise to at least one unit of exergy output rate. As the price of exergy output rate becomes very large compared with that of the power input, i.e., $\psi_2/\psi_1 \to 0$, equation (34) becomes

$$\pi = \psi_1\left\{(\eta_2 - \varphi x\eta_1)\frac{k_2 f_{opt} F_T}{(1+f_{opt})} \frac{A_1\left[1+f_{opt}\varphi\delta x^{1-n_1}\cos(n_2 \ln x)\right] - A_2 f_{opt}\varphi\delta x^{1-n_1}\sin(n_2 \ln x)}{1+2\delta\varphi f_{opt} x^{1-n_1}\cos(n_2 \ln x) + f_{opt}{}^2\delta^2\varphi^2 x^{2(1-n_1)}} + q(\eta_1-\eta_2)\right\} \tag{41}$$

One can see that the cooling load and the profit are both monotonic decreasing functions of temperature ratio of working fluid. That is, the profit maximization approaches to the cooling load maximization. When $T_H \to T_0$, equation (41) becomes

$$\pi = \psi_1\eta_2 R \tag{42}$$

On the other hand, as the price of exergy output rate approaches to the price of the power input, i.e. $\psi_2/\psi_1 \to 1$, equation (34) becomes

$$\pi = -\psi_1 T_0[(\dot{Q}_{HC} - q)/T_H - (\dot{Q}_{LC} - q)/T_L] = -\psi_1 T_0 \sigma \quad (43)$$

where σ is entropy generation rate of the thermoacoustic cooler. That is, the profit maximization approaches to the entropy generation rate minimization, in other word, the minimum waste of exergy. Eq. (43) indicates that the thermoacoustic cooler is not profitable regardless of the COP is at which the cooler is operating. Only the thermoacoustic cooler is operating reversibly ($\varepsilon = \varepsilon_C$) will the revenue equal to the cost, and then the maximum profit will equal to zero. The corresponding entropy generation rate is also zero.

Therefore, for any intermediate values of ψ_2/ψ_1, the finite-time exergoeconomic performance bound (ε_π) lies between the finite-time thermodynamic performance bound and the reversible performance bound. ε_π is related to the latter two through the price ratio, and the associated COP bounds are the upper and lower limits of ε_π.

5. Numerical examples

To illustrate the preceding analyses, numerical examples are provided. In the calculations, it is set that $T_H = 300K, T_L = 260K, T_0 = 290K$; $k_1 = k_2$; $\varphi = 1.0, 1.05, 1.10$ $\psi_1 = 1000 yuan/kW$, $\psi_2/\psi_1 = 0.3$; $q = C_i(T_H{}^n - T_L{}^n)$ (same as Ref.[46]) and $C_i = 0.0, 0.005kW/K$. C_i is the thermal conductance inside the cooler.

Figures 2 and 3 show the effects of the heat leakage, the internal irreversibility and the heat transfer exponent on the relationship between the profit rate and the COP. One can see that for all heat transfer law, the influences of the internal irreversibility and the heat leakage on the relationship between the profit rate and COP are different obviously: the profit rate π decreases with the increase of the internal irreversibility φ, but the curves of $\pi - \varepsilon$ are not changeable; the heat leakage affects strongly the relationship between the profit rate and COP, the curves of $\pi - \varepsilon$ are parabolic-like ones in the case of $q = 0$, while the curves are loop-shaped ones in the case of $q \neq 0$. From Figures 2 and 3, one can also see that both the real part n_1 and the imaginary part n_2 of the complex heat transfer exponent n don't change the shape of the curves of $\pi - \varepsilon$.

The effects of complex exponent $n = n_1 + in_2$ on the optimal profit rate versus the COP characteristics with $T_H = 300K$, $T_L = 260K$, $T_0 = 290K$, $\delta = 1$, $q = 0.5kW$ and $\phi = 1.05$ are shown in Figures 4 and 5. They show that π versus ε characteristics of the generalized irreversible thermoacoustic cooler with a complex heat transfer exponent is a loop-shaped curve. When the real part $n_1 = 1$ is fixed, the maximum profit rate decreases with the increase of the imaginary part n_2 of the complex heat transfer exponent n. It shows that the imaginary part n_2 of the complex heat transfer exponent n indicates the energy dissipation. When the imaginary part $n_2 = 0.1$ is fixed, it is worth notice that the real part of n_1 affects strongly the optimal performance between the profit rate and COP. From Figures 4 and 5, one can also see that the influence of heat transfer exponent on the COP ε_π corresponding to the maximum profit rate is weakly, the reason is that the difference between the heat source and heat sink is small, the sensitivity of the heat transfer exponent to the temperature difference is not obvious. For all of n_1 and n_2, $\pi = \pi_{\max}$ when $\varepsilon = \varepsilon_0$, and $\varepsilon = \varepsilon_{\max}$ when $\pi = \pi_0$. For example, when $n_1 = 1$, the π bound ($\pi_{\max}$) corresponding to $n_2 = 0.05, 0.10, 0.15$ are $1316.5(yuan)$, $1089.3(yuan)$, $735.95(yuan)$, respectively, and the maximum COP ($\varepsilon_{\max}$) corresponding to $n_2 = 0.05, 0.10, 0.15$ are $3.8377, 3.9071, 4.0287$, respectively.

The optimization criteria of the thermoacoustic cooler can been obtained from parameters $\pi_{\max}$, π_0, $\varepsilon_{\max}$ and ε_0 as follows:

$$\pi_0 \le \pi \le \pi_{\max} \text{ and } \varepsilon_0 \le \varepsilon \le \varepsilon_{\max} \quad (44)$$

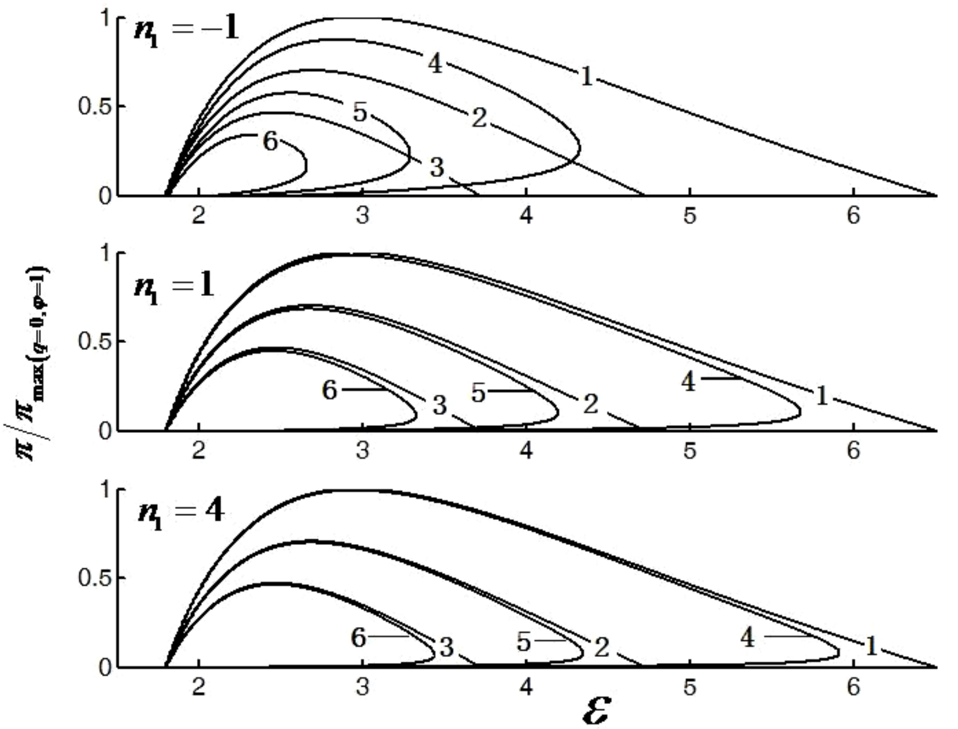

Figure 2. $\pi - \varepsilon$ performance characteristic for the thermoacoustic cooler with $n_2 = 0.1$

1. φ=1, $C_i = 0$; 2. φ=1.05, $C_i = 0$; 3. φ=1.10, $C_i = 0$;

4. φ=1, $C_i > 0$; 5. φ=1.05, $C_i > 0$; 6. φ=1.10, $C_i > 0$.

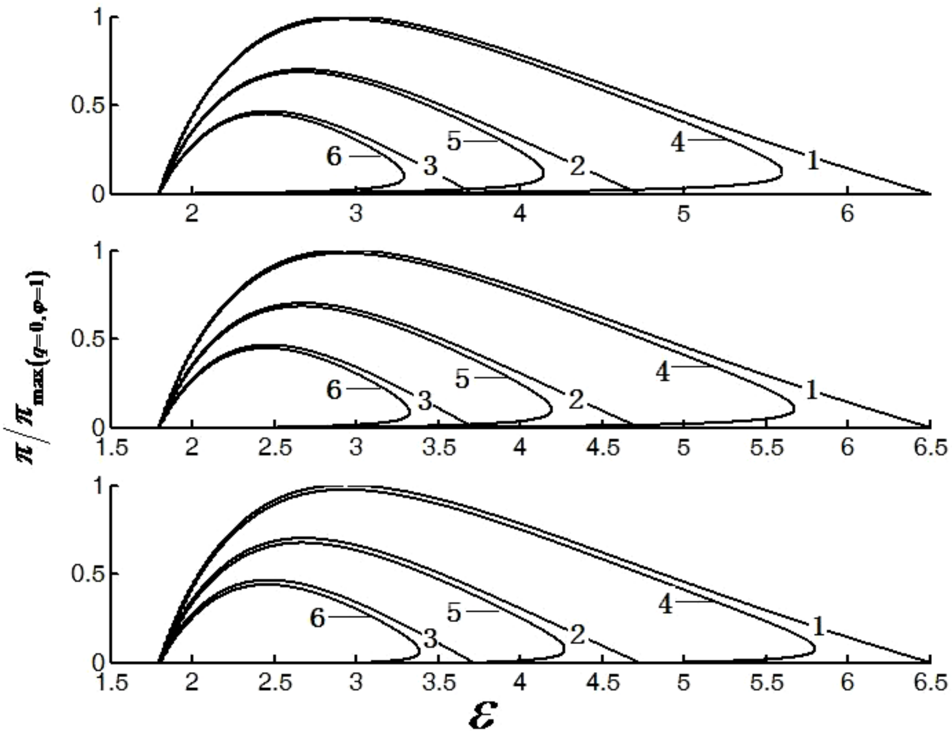

Figure 3. $\pi - \varepsilon$ performance characteristic for the thermoacoustic cooler with $n_1 = 1$

1. φ=1, $C_i = 0$; 2. φ=1.05, $C_i = 0$; 3. φ=1.10, $C_i = 0$;

4. φ=1, $C_i > 0$; 5. φ=1.05, $C_i > 0$; 6. φ=1.10, $C_i > 0$.

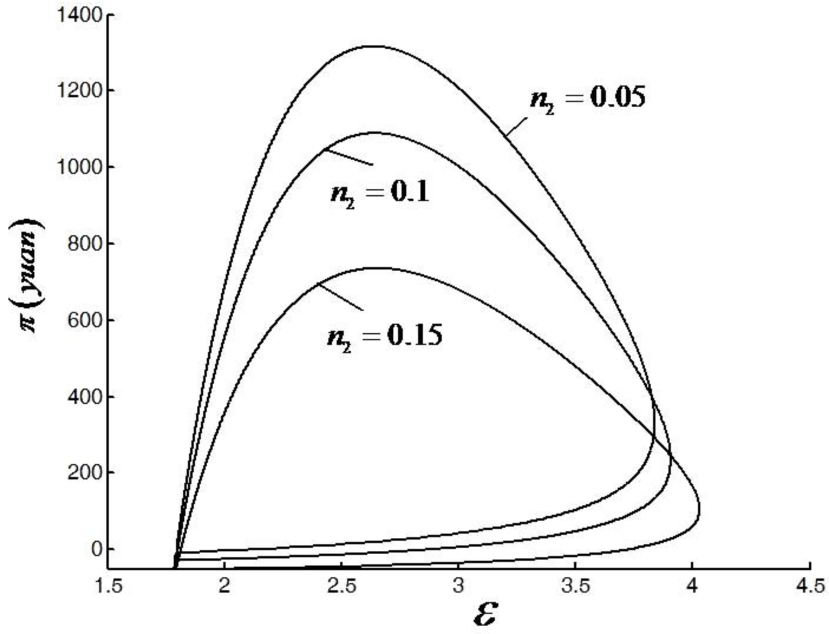

Figure 4. Optimal profit rate versus COP with $n_1 = 1, n_2 = 0.05, n_2 = 0.1, n_2 = 0.15$

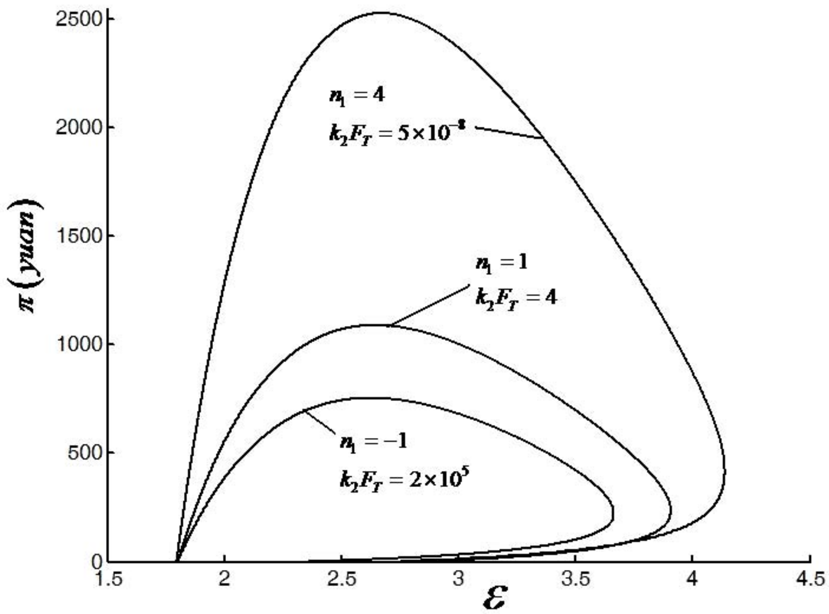

Figure 5. Optimal profit rate versus COP with $n_2 = 0.1, n_1 = -1, n_1 = 1, n_1 = 4$

6. Conclusion

The optimal profit rate performance of a generalized irreversible thermoacoustic cooler with the losses of heat-resistance, heat leakage and internal irreversibility, in which the transfer between the working fluid and the heat reservoirs obeys a generalized heat transfer law $Q \propto (\Delta T)^n$, where n is complex, is derived in this paper. The heat transfer exponent for the thermoacoustic cooler must be complex number due to the thermal relaxation induced by the thermoacoustic oscillation. The effects of the complex heat transfer exponent on the optimal performance for the thermoacoustic cooler are discussed by numerical examples, the optimal zone of the thermoacoustic cooler with a complex heat transfer exponent is obtained. The results obtained herein are helpful for the selection of the optimal mode of operation of the real thermoacoustic coolers. The theoretical results can provide some guidelines in the selection of heat transfer surface areas, working temperatures of working fluid, and region of COP of the cooler. The optimization work in future is possibly more practical and significant for the analysis and design of a thermoacoustic cooler if considering in a real sense more variables.

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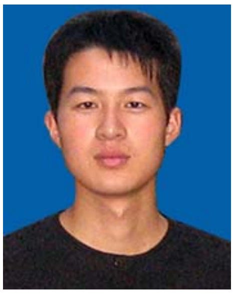

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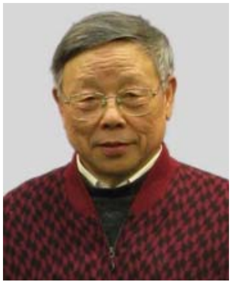

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