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Combined natural convection and mass transfer effects on unsteady flow past an infinite vertical porous plate embedded in a porous medium with heat source

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Abstract
This paper theoretically investigates the combined natural convection and mass transfer effects on unsteady flow of a viscous incompressible fluid past an infinite vertical porous plate embedded in a porous medium with heat source. The governing equations of the flow field are solved analytically for velocity, temperature, concentration distribution, skin friction and the rate of heat transfer using multi parameter perturbation technique and the effects of the flow parameters such as permeability parameter K_p, Grashof number for heat and mass transfer G_r, G_c; heat source parameter S, Schmidt number S_c, Prandtl number P_r etc. on the flow field are analyzed and discussed with the help of figures and tables. The permeability parameter K_p is reported to accelerate the transient velocity of the flow field at all points for small values of K_p (≤ 1) and for higher values the effect reverses. The effect of increasing Grashof numbers for heat and mass transfer or heat source parameter is to enhance the transient velocity of the flow field at all points while a growing Schmidt number retards its effect at all points. A growing permeability parameter or heat source parameter increases the transient temperature of the flow field at all points, while a growing Prandtl number shows reverse effect. The effect of increasing Schmidt number is to decrease the concentration boundary layer thickness of the flow field at all points. Further, a growing permeability parameter enhances the skin friction at the wall and a growing Prandtl number shows reverse effect. The effect of increasing Prandtl number or permeability parameter leads to increase the magnitude of the rate of heat transfer at the wall.

Keywords: Mass transfer; Natural convection; Porous medium; Heat source.

1. Introduction
The phenomenon of natural convection unsteady flow with mass transfer in a viscous incompressible fluid past a porous plate embedded in a porous medium has attracted the attention of a good number of researchers because of its varied applications in many engineering problems such as plasma studies, nuclear reactors, oil exploration, geothermal energy extractions and in the boundary layer control in the

field of aerodynamics. Heat transfer in laminar flow is important in problems dealing with chemical reactions and in dissociating fluids.

Several investigators reported such flows under various physical situations. Gersten and Gross [1] analyzed the flow and heat transfer along a plane wall with periodic suction. Yamamoto and Iwamura [2] discussed the flow with convective acceleration through a porous medium. Raptis and Kafousias [3] investigated the problem of heat transfer in flow through a porous medium bounded by an infinite vertical plate under the action of a magnetic field. Raptis and Singh [4] studied the MHD free convection flow past an accelerated

vertical plate. Raptis [5] explained the flow through a porous medium in the presence of a magnetic field. Mansutti *et al*. [6] analyzed the steady flows of non-Newtonian fluids past a porous plate with suction or injection. Sattar [7] reported the free convection and mass transfer flow through a porous medium past an infinite vertical porous plate with time dependant temperature and concentration. Takhar and Ram [8] discussed the magnetohydrodynamic free convection flow of water at 4^0C through a porous medium. Acharya *et al*. [9] estimated the effect of heat and mass transfer over an accelerating surface with heat source in presence of suction and blowing.

Kim [10] analyzed the unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction. Sharma and Pareek [11] discussed the steady free convection MHD flow past a vertical porous moving surface. Gokhale and Alsamman [12] estimated the effects of mass transfer on the transient free convection flow of a dissipative fluid along a semi-infinite vertical plate with constant heat flux. Makinde and his team [13] discussed the unsteady free convection flow with suction on an accelerating porous plate. Singh *et al*. [14] explained the MHD free convection transient flow through a porous medium in a vertical channel. Das and his associates [15] numerically solved the mass transfer effects on unsteady flow past an accelerated vertical porous plate with suction. Ogulu and Prakash [16] investigated the problem of heat transfer to unsteady magneto-hydrodynamic flow past an infinite vertical moving plate with variable suction. Mbeledogu *et al*. [17] reported the unsteady MHD free convective flow of a compressible fluid past a moving vertical plate in the presence of radiative heat transfer. Recently, Das and his co-workers [18] estimated the effect of mass transfer on MHD flow and heat transfer past a vertical porous plate through a porous medium under oscillatory suction and heat source.

The work reported herein estimates the effect of natural convection and mass transfer on unsteady flow of a viscous incompressible fluid past an infinite vertical porous plate embedded in a porous medium with heat source. Approximate solutions are obtained analytically for velocity, temperature, concentration distribution, skin friction and the rate of heat transfer using multi parameter perturbation technique and the effects of the flow parameters on the flow field are analyzed with the help of figures and tables.

2. Formulation of the problem

We consider the unsteady natural convection mass transfer flow of a viscous incompressible fluid past an infinite vertical porous plate in presence of constant suction and heat source. The x'-axis is taken in vertically upward direction along the plate and y'-axis is chosen normal to it. Neglecting the Joulean heat dissipation and applying Boussinesq’s approximation the governing equations of the flow field are given by:

Continuity equation:

$$\frac{\partial v'}{\partial y'} = 0 \Rightarrow v' = -v_0' \text{ (Constant)}, \tag{1}$$

Momentum equation:

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta\left(T' - T'_\infty\right) + g\beta^*\left(C' - C'_\infty\right) - \frac{\nu}{K'}u', \tag{2}$$

Energy equation:

$$\frac{\partial T'}{\partial t'}+v'\frac{\partial T'}{\partial y'}=k\frac{\partial^2 T'}{\partial y'^2}+\frac{\nu}{C_p}\left(\frac{\partial u'}{\partial y'}\right)^2+S'(T'-T'_\infty), \tag{3}$$

Concentration equation:

$$\frac{\partial C'}{\partial t'}+v'\frac{\partial C'}{\partial y'}=D\frac{\partial^2 C'}{\partial y'^2}. \tag{4}$$

The boundary conditions of the problem are:

$$u'=0, v'=-v'_0, T'=T'_w+\varepsilon(T'_w-T'_\infty)e^{i\omega' t'},\ C'=C'_w+\varepsilon(C'_w-C'_\infty)e^{i\omega' t'} \text{ at } y'=0,$$
$$u'\to 0,\ T'\to T'_\infty,\ C'\to C'_\infty \text{ as } y'\to\infty. \tag{5}$$

Introducing the following non-dimensional variables and parameters,

$$y=\frac{y'v'_0}{\nu}, t=\frac{t'v_0'^2}{4\nu}, \omega=\frac{4\nu\omega'}{v_0'^2}, u=\frac{u'}{v'_0}, \nu=\frac{\eta_0}{\rho},\ K_p=\frac{v_0^2K'}{\nu^2}, T=\frac{T'-T'_\infty}{T'_w-T'_\infty},\ C=\frac{C'-C'_\infty}{C'_w-C'_\infty}, P_r=\frac{\nu}{k},$$
$$G_r=\frac{\nu g\beta(T'_w-T'_\infty)}{v_0'^3}, G_c=\frac{\nu g\beta^*(C'_w-C'_\infty)}{v_0'^3}, S_c=\frac{\nu}{D}, S=\frac{4S'\nu}{v_0'^2}, E_c=\frac{v_0'^2}{C_p(T'_w-T'_\infty)}, \tag{6}$$

where g, ρ, ν, β, β^*, ω, η_0, k, T', T'_w, T'_∞, C', C'_w, C'_∞, C_p, D, P_r, S_c, G_r, G_c, S, K_p and E_c are respectively the acceleration due to gravity, density, coefficient of kinematic viscosity, volumetric coefficient of expansion for heat transfer, volumetric coefficient of expansion for mass transfer, angular frequency, coefficient of viscosity, thermal diffusivity, temperature, temperature at the plate, temperature at infinity, concentration, concentration at the plate, concentration at infinity, specific heat at constant pressure, molecular mass diffusivity, Prandtl number, Schmidt number, Grashof number for heat transfer, Grashof number for mass transfer, heat source parameter, permeability parameter and Eckert number.
Substituting equation (6) in equations (2), (3) and (4) under boundary conditions (5), we get

$$\frac{1}{4}\frac{\partial u}{\partial t}-\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}+G_rT+G_cC-\frac{1}{K_p}u, \tag{7}$$

$$\frac{1}{4}\frac{\partial T}{\partial t}-\frac{\partial T}{\partial y}=\frac{1}{P_r}\frac{\partial^2 T}{\partial y^2}+\frac{1}{4}ST+E_c\left(\frac{\partial u}{\partial y}\right)^2, \tag{8}$$

$$\frac{1}{4}\frac{\partial C}{\partial t}-\frac{\partial C}{\partial y}=\frac{1}{S_c}\frac{\partial^2 C}{\partial y^2}. \tag{9}$$

The corresponding boundary conditions are:

$$u=0, T=1+\varepsilon e^{i\omega t}, C=1+\varepsilon e^{i\omega t} \text{ at } y=0,$$
$$u\to 0, T\to 0, C\to 0 \text{ as } y\to\infty. \tag{10}$$

3. Method of solution

To solve equations (7), (8) and (9), we assume ε to be very small and the velocity, temperature and concentration distribution of the flow field in the neighbourhood of the plate as

$$u(y,t) = u_0(y) + \varepsilon e^{i\omega t} u_1(y), \tag{11}$$

$$T(y,t) = T_0(y) + \varepsilon e^{i\omega t} T_1(y), \tag{12}$$

$$C(y,t) = C_0(y) + \varepsilon e^{i\omega t} C_1(y). \tag{13}$$

Substituting equations (11)-(13) in equations (7)-(9) respectively, equating the harmonic and non-harmonic terms and neglecting the coefficients of ε^2, we get
Zeroth order:

$$u_0'' + u_0' - \frac{1}{K_p} u_0 = -G_r T_0 - G_c C_0, \tag{14}$$

$$T_0'' + P_r T_0' + \frac{P_r S}{4} T_0 = -P_r E_c \left(\frac{\partial u_0}{\partial y} \right)^2, \tag{15}$$

$$C_0'' + S_c C_0' = 0. \tag{16}$$

First order:

$$u_1'' + u_1' - \left(\frac{i\omega}{4} + \frac{1}{K_p} \right) u_1 = -G_r T_1 - G_c C_1, \tag{17}$$

$$T_1'' + P_r T_1' - \frac{P_r}{4} (i\omega - S) T_1 = -2 P_r E_c \left(\frac{\partial u_0}{\partial y} \right) \left(\frac{\partial u_1}{\partial y} \right), \tag{18}$$

$$C_1'' + S_c C_1' - \frac{i\omega S_c}{4} C_1 = 0 \quad . \tag{19}$$

The corresponding boundary conditions are

$$\begin{aligned} &y = 0 : u_0 = 0, T_0 = 1, C_0 = 1, u_1 = 0, T_1 = 1, C_1 = 1 \\ &y \to \infty : u_0 = 0, T_0 = 0, C_0 = 0, u_1 = 0, T_1 = 0, C_1 = 0 \end{aligned} \tag{20}$$

Solving equations (16) and (19) under boundary condition (20), we get

$$C_0 = e^{-S_c y}, \tag{21}$$

$$C_1 = e^{-m_1 y}, \tag{22}$$

Using multi parameter perturbation technique and assuming E_c<<1, we take

$$u_0 = u_{00} + E_c u_{01}, \tag{23}$$

$$T_0 = T_{00} + E_c T_{01}, \tag{24}$$

$$u_1 = u_{10} + E_c u_{11}, \tag{25}$$

$$T_1 = T_{10} + E_c T_{11}. \tag{26}$$

Now using equations (23)-(26) in equations (14), (15), (17) and (18) and equating the coefficients of like powers of E_c neglecting those of E_c^2, we get the following set of differential equations:
Zeroth order:

$$u''_{00} + u'_{00} - \frac{1}{K_p} u_{00} = -G_r T_{00} - G_c C_0, \tag{27}$$

$$u''_{10} + u'_{10} - \left(\frac{1}{K_p} + \frac{i\omega}{4} \right) u_{10} = -G_r T_{10} - G_c C_1, \tag{28}$$

$$T''_{00} + P_r T'_{00} + \frac{P_r S}{4} T_{00} = 0, \tag{29}$$

$$T''_{10} + P_r T'_{10} - \frac{P_r}{4} (i\omega - S) T_{10} = 0. \tag{30}$$

The corresponding boundary conditions now reduce to,

$$\begin{aligned} &y = 0 : u_{00} = 0, T_{00} = 1, u_{10} = 0, T_{10} = 1; \\ &y \to \infty : u_{00} = 0, T_{00} = 0, u_{10} = 0, T_{10} = 0. \end{aligned} \tag{31}$$

First order:

$$u''_{01} + u'_{01} - \frac{1}{K_p} u_{01} = -G_r T_{01}, \tag{32}$$

$$u''_{11} + u'_{11} - \left(\frac{1}{K_p} + \frac{i\omega}{4} \right) u_{11} = -G_r T_{11}, \tag{33}$$

$$T''_{01} + P_r T'_{01} + \frac{P_r S}{4} T_{01} = -P_r \left(u'_{00} \right)^2, \tag{34}$$

$$T''_{11} + P_r T'_{11} - \frac{P_r}{4} (i\omega - S) T_{11} = -2P_r \left(\frac{\partial u_{00}}{\partial y} \right) \left(\frac{\partial u_{10}}{\partial y} \right). \tag{35}$$

The corresponding boundary conditions become,

$$\begin{aligned} &y = 0 : u_{01} = 0, T_{01} = 0, u_{11} = 0, T_{11} = 0; \\ &y \to \infty : u_{01} = 0, T_{01} = 0, u_{11} = 0, T_{11} = 0. \end{aligned} \tag{36}$$

Solving equations (27)-(30) subject to boundary condition (31), we get

$$u_{00} = A_1 e^{-m_3 y} + A_2 e^{-S_c y} - A_3 e^{-m_7 y} \tag{37}$$

$$T_{00} = e^{-m_3 y}, \tag{38}$$

$$u_{10} = A_4 e^{-m_5 y} + A_5 e^{-m_1 y} - A_6 e^{-m_9 y}, \tag{39}$$

$$T_{10} = e^{-m_5 y}. \tag{40}$$

Solving equations (32)-(35) subject to boundary condition (36), we get

$$T_{01} = A_7 e^{-2S_c y} + A_8 e^{-2m_3 y} + A_9 e^{-2m_5 y} + A_{10} e^{-(m_3+S_c)y} + A_{11} e^{-(m_7+S_c)y} + A_{12} e^{-(m_3+m_7)y} - A_{13} e^{-m_3 y}, \tag{41}$$

$$\begin{aligned} T_{11} = {} & A_{14} e^{-(m_3+m_5)y} + A_{15} e^{-(m_1+m_3)y} + A_{16} e^{-(m_3+m_9)y} + A_{17} e^{-(m_5+S_c)y} + A_{18} e^{-(m_1+S_c)y} \\ & + A_{19} e^{-(m_9+S_c)y} + A_{20} e^{-(m_5+m_7)y} + A_{21} e^{-(m_1+m_7)y} + A_{22} e^{-(m_7+m_9)y} - A_{23} e^{-m_5 y}, \end{aligned} \tag{42}$$

$$\begin{aligned} u_{01} = {} & B_1 e^{-2S_c y} + B_2 e^{-2m_3 y} + B_3 e^{-2m_7 y} + B_4 e^{-(m_3+S_c)y} + B_5 e^{-(m_7+S_c)y} \\ & + B_6 e^{-(m_3+m_7)y} + B_7 e^{-m_3 y} - B_8 e^{-m_7 y}, \end{aligned} \tag{43}$$

$$\begin{aligned} u_{11} = {} & B_9 e^{-(m_3+m_5)y} + B_{10} e^{-(m_1+m_3)y} + B_{11} e^{-(m_3+m_9)y} + B_{12} e^{-(m_5+S_c)y} + B_{13} e^{-(m_1+S_c)y} + B_{14} e^{-(m_9+S_c)y} \\ & + B_{15} e^{-(m_5+m_7)y} + B_{16} e^{-(m_1+m_7)y} + B_{17} e^{-(m_7+m_9)y} + B_{18} e^{-m_5 y} - B_{19} e^{-m_9 y}. \end{aligned} \tag{44}$$

Substituting the values of C_0 and C_1 from equations (21) and (22) in equation (13) the solution for concentration distribution of the flow field is given by

$$C = e^{-S_c y} + \varepsilon e^{i\omega t - m_1 y}. \tag{45}$$

3.1 Skin friction

The wall shear stress i.e. the skin friction at the wall τ_w is given by

$$\begin{aligned} \tau_w = {} & \left(\frac{\partial u}{\partial y}\right)_{y=0} \\ = {} & -m_3 A_1 - S_c A_2 + m_7 A_3 - E_c \left[2S_c B_1 + 2m_3 B_2 + 2m_7 B_3 + (m_3 + S_c) B_4 + (m_7 + S_c) B_5 \right. \\ & \left. + (m_3 + m_7) B_6 + m_3 B_7 - m_7 B_8\right] - \varepsilon e^{i\omega t} \left\{m_5 A_4 + m_1 A_5 - m_9 A_6 + E_c \left[(m_3 + m_5) B_9 \right.\right. \\ & + (m_1 + m_3) B_{10} + (m_3 + m_9) B_{11} + (m_5 + S_c) B_{12} + (m_1 + S_c) B_{13} + (m_9 + S_c) B_{14} \\ & \left.\left. + (m_5 + m_7) B_{15} + (m_1 + m_7) B_{16} + (m_7 + m_9) B_{17} + m_5 B_{18} - m_9 B_{19}\right]\right\}. \end{aligned} \tag{46}$$

3.2 Heat flux

The rate of heat transfer i.e. heat flux at the wall in terms of Nusselt number N_u is given by

$$\begin{aligned} N_u = {} & \left(\frac{\partial T}{\partial y}\right)_{y=0} \\ = {} & -m_3 - E_c \left[2S_c A_7 + 2m_3 A_8 + 2m_5 A_9 - (m_3 + S_c) A_{10} + (m_7 + S_c) A_{11} + (m_3 + m_7) A_{12} - m_3 A_{13}\right] \\ & + \varepsilon e^{i\omega t} \left\{-m_5 - E_c \left[(m_3 + m_5) A_{14} + (m_1 + m_3) A_{15} + (m_3 + m_9) A_{16} + (m_5 + S_c) A_{17} + (m_1 + S_c) A_{18} \right.\right. \\ & \left.\left. + (m_9 + S_c) A_{19} + (m_5 + m_7) A_{20} + (m_1 + m_7) A_{21} + (m_7 + m_9) A_{22} - m_5 A_{23}\right]\right\}, \end{aligned} \tag{47}$$

where

$$m_1 = \frac{1}{2}\left[S_c + \sqrt{S_c^2 + i\omega S_c}\right],\ m_2 = \frac{1}{2}\left[-S_c + \sqrt{S_c^2 + i\omega S_c}\right],\ m_3 = \frac{1}{2}\left[P_r + \sqrt{P_r^2 - SP_r}\right],$$

$$m_4 = \frac{1}{2}\left[-P_r + \sqrt{P_r^2 - SP_r}\right],\ m_5 = \frac{1}{2}\left[P_r + \sqrt{P_r^2 - P_r(S - i\omega)}\right],\ m_6 = \frac{1}{2}\left[-P_r + \sqrt{P_r^2 - P_r(S - i\omega)}\right],$$

$$m_7 = \frac{1}{2}\left[1 + \sqrt{1 + \frac{4}{K_p}}\right],\ m_8 = \frac{1}{2}\left[-1 + \sqrt{1 + \frac{4}{K_p}}\right], m_9 = \frac{1}{2}\left[1 + \sqrt{1 + 4\left(\frac{1}{K_p} + \frac{i\omega}{4}\right)}\right],\ m_{10} = \frac{1}{2}\left[-1 + \sqrt{1 + 4\left(\frac{1}{K_p} + \frac{i\omega}{4}\right)}\right],$$

$$A_1 = \frac{G_r}{(m_7 - m_3)(m_8 + m_3)}, A_2 = \frac{G_c}{(m_7 - S_c)(m_8 + S_c)}, A_3 = A_1 + A_2,\quad A_4 = \frac{G_r}{(m_9 - m_5)(m_{10} + m_5)},$$

$$A_5 = \frac{G_c}{(m_9 - m_1)(m_{10} + m_1)}, A_6 = A_4 + A_5, A_7 = \frac{P_r S_c^2 A_2^2}{(m_3 - 2S_c)(m_4 + 2S_c)}, A_8 = \frac{-P_r m_3 A_1^2}{(m_4 + 2m_3)},$$

$$A_9 = \frac{P_r m_7^2 A_3^2}{(m_3 - 2m_7)(m_4 + 2m_7)}, A_{10} = -\frac{2A_1 A_2 m_3 P_r}{(m_3 + m_4 + S_c)}, A_{11} = -\frac{2P_r S_c A_2 A_3 m_7}{(m_3 - m_7 - S_c)(m_4 + m_7 + S_c)},$$

$$A_{12} = \frac{2P_r A_1 A_2 m_3}{(m_3 + m_4 + m_7)}, A_{13} = A_7 + A_8 + A_9 + A_{10} + A_{11} + A_{12},\ A_{14} = -\frac{2P_r A_1 A_6}{(m_3 + m_5 + m_6)},$$

$$A_{15} = \frac{2P_r A_1 A_5 m_1 m_3}{(m_5 - m_3 - m_1)(m_6 + m_3 + m_1)}, A_{16} = \frac{2P_r A_1 A_6 m_3 m_9}{(m_9 - m_5 + m_3)(m_9 + m_6 + m_3)}, A_{17} = -\frac{2P_r A_2 A_4 m_5}{(m_6 + m_5 + S_c)},$$

$$A_{18} = \frac{2P_r S_c A_2 A_5 m_1}{(m_5 - m_1 - S_c)(m_6 + m_1 + S_c)}, A_{19} = \frac{2P_r S_c A_2 A_6 m_9}{(m_9 - m_5 + S_c)(m_9 + m_6 + S_c)}, A_{20} = \frac{2P_r A_3 A_4 m_5}{(m_7 + m_6 + m_5)},$$

$$A_{21} = \frac{2P_r A_3 A_5 m_1 m_7}{(m_7 - m_5 + m_1)(m_7 + m_6 + m_1)}, A_{22} = \frac{2P_r A_3 A_6 m_7 m_9}{(m_9 + m_7 + m_5)(m_9 + m_7 + m_6)},$$

$$A_{23} = A_{14} + A_{15} + A_{16} + A_{17} + A_{18} + A_{19} + A_{20} + A_{21} + A_{22},\ B_1 = \frac{G_r A_7}{(m_7 - 2S_c)(m_8 + 2S_c)},$$

$$B_2 = \frac{G_r A_8}{(m_7 - 2m_3)(m_8 + 2m_3)}, B_3 = \frac{-G_r A_9}{m_7(m_8 + 2m_7)}, B_4 = \frac{G_r A_{10}}{(m_7 - m_3 - S_c)(m_8 + m_3 + S_c)},$$

$$B_5 = \frac{-G_r A_{11}}{S_c(m_8 + m_7 + S_c)}, B_6 = \frac{-G_r A_{12}}{m_3(m_8 + m_7 + m_3)}, B_7 = \frac{G_r A_{13}}{(m_3 - m_7)(m_8 + m_3)},$$

$$B_8 = B_1 + B_2 + B_3 + B_4 + B_5 + B_6 + B_7, B_9 = \frac{G_r A_{14}}{(m_9 - m_5 - m_3)(m_{10} + m_5 + m_3)},$$

$$B_{10} = \frac{G_r A_{15}}{(m_9 - m_3 - m_1)(m_{10} + m_3 + m_1)}, B_{11} = \frac{-G_r A_{16}}{m_3(m_{10} + m_9 + m_3)}, B_{12} = \frac{G_r A_{17}}{(m_9 - m_5 - S_c)(m_{10} + m_5 + S_c)},$$

$$B_{13} = \frac{G_r A_{18}}{(m_9 - m_1 - S_c)(m_{10} + m_1 + S_c)}, B_{14} = \frac{-G_r A_{19}}{S_c(m_{10} + m_9 + S_c)}, B_{15} = \frac{G_r A_{20}}{(m_9 - m_7 - m_5)(m_{10} + m_7 + m_5)},$$

$$B_{16} = \frac{G_r A_{21}}{(m_9 - m_7 - m_1)(m_{10} + m_7 + m_1)}, B_{17} = \frac{-G_r A_{22}}{m_7(m_{10} + m_9 + m_7)}, B_{18} = \frac{G_r A_{23}}{(m_9 - m_5)(m_{10} + m_5)},$$

$$B_{19} = B_9 + B_{10} + B_{11} + B_{12} + B_{13} + B_{14} + B_{15} + B_{16} + B_{17} + B_{18}. \tag{48}$$

4. Results and discussions

The effect of combined natural convection and mass transfer on unsteady flow of a viscous incompressible fluid past an infinite vertical porous plate embedded in a porous medium with heat source has been studied. The governing equations of the flow field are solved for velocity, temperature, concentration distribution, skin friction and the rate of heat transfer employing multi parameter perturbation technique. The effects of the pertinent parameters on the flow field are analyzed and discussed with the help of velocity profiles shown in Figures 1-5, temperature profiles shown in Figures 6-7, concentration distribution shown in Figure 8 and Tables 1-2 respectively. During numerical calculations we choose P_r =0.71 representing air at 20°C, S_c =0.66 representing O_2, G_r >0 corresponding to cooling of the plate and S >0 representing heat source in order to have a realistic approach.

4.1 Velocity field

The flow parameters play an important role in determining the magnitude of velocity of the flow field. The flow parameters affecting the velocity flow field are permeability parameter K_p, Grashof number for heat and mass transfer G_r, G_c; Schmidt number S_c and heat source parameter S. Figures 1-5 depict the effects of these parameters on the velocity of the flow field.

In Figure 1, we present the effect of permeability parameter on the transient velocity of the flow field. The permeability parameter K_p is reported to accelerate the transient velocity at all points for small values of K_p (≤1) and for higher values the effect reverses. The effects of Grashof numbers for heat and mass transfer G_r, G_c; on the transient velocity of the flow field have been shown in Figures 2 and 3 respectively. Both the parameters are found to enhance the transient velocity of the flow field at all points due to the action of free convection current and the presence of foreign mass respectively in the flow field. Figure 4 depicts the effect of heat source parameter on the velocity of the flow field. A growing heat source parameter is found to enhance the velocity of the flow field at all points. In Figure 5, we discuss the effect of Schmidt number S_c on the transient velocity of the flow field. A growing S_c is found to decelerate the transient velocity of the flow field at all points due to the injection of heavier diffusive species into the flow field.

4.2 Temperature field

Figures 6-8 elucidate the temperature profiles of the flow field with the variation of the flow parameters in the flow field such as permeability parameter K_p, Prandtl number P_r and heat source parameter S. In Figure 6, we discuss the effect of permeability parameter on the temperature field. A growing permeability parameter is found to increase the temperature of the flow field at all points. The effect of Prandtl number on the temperature field is shown in Figure 7. Comparing the curves of the said figure one may note that the transient temperature decreases at all points of the flow field with an increase in the values of Prandtl number P_r. Figure 8 presents the effect of heat source parameter on the temperature field. A growing heat source parameter is observed to enhance the temperature of the flow field at all points.

4.3 Concentration field

The Schmidt number S_c plays a dominant role in determining the concentration boundary layer thickness of the flow field. These variations are shown in Figure 9. Analyzing the curves of the figure, it is seen that a growing Schmidt number decreases the concentration boundary layer thickness of the flow field at all points.

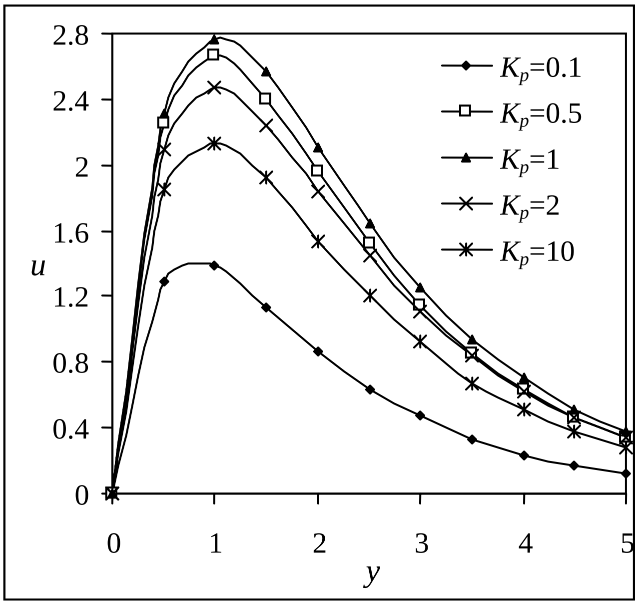

Figure 1. Transient velocity against y for different values of K_p with G_r=2, G_c=2, E_c=0.002, S=0.1, S_c=0.66, P_r=0.71, ω=5.0, ε=0.2, ωt=π/2

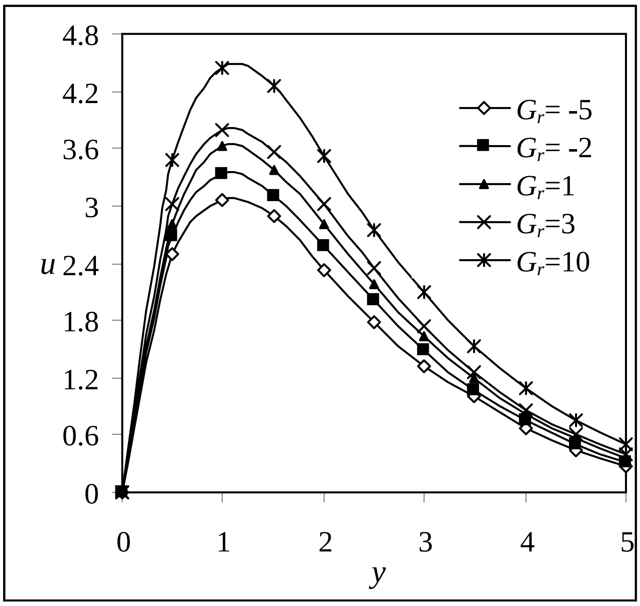

Figure 2. Transient velocity against y for different values of G_r with G_c=2, K_p=1, S=0.1, S_c=0.66, P_r=0.71, E_c=0.002, ω=5.0, ε=0.2, ωt=π/2

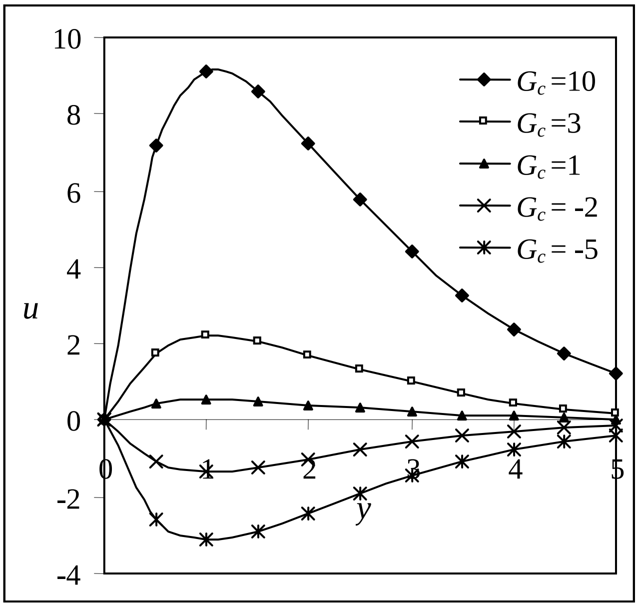

Figure 3. Transient velocity against y for different values of G_c with G_r=2, K_p=1, S=0.1, S_c=0.66, P_r=0.71, E_c=0.002, ω=5.0, ε=0.2, ωt=π/2

4.4 Skin friction

The variations in the values of skin friction at the wall against the permeability parameter K_p for different values of heat source parameter S are entered in Table 1. It is observed that a growing permeability parameter enhances the skin friction at the wall, while a growing Prandtl number reverses the effect.

4.5 Rate of heat transfer

The variations in the heat flux i.e. the rate of heat transfer at the wall for different values of Prandtl number P_r and permeability parameter K_p are entered in Table 2. From the table we observe that a growing Prandtl number or permeability parameter enhances the magnitude of the rate of heat transfer at the wall.

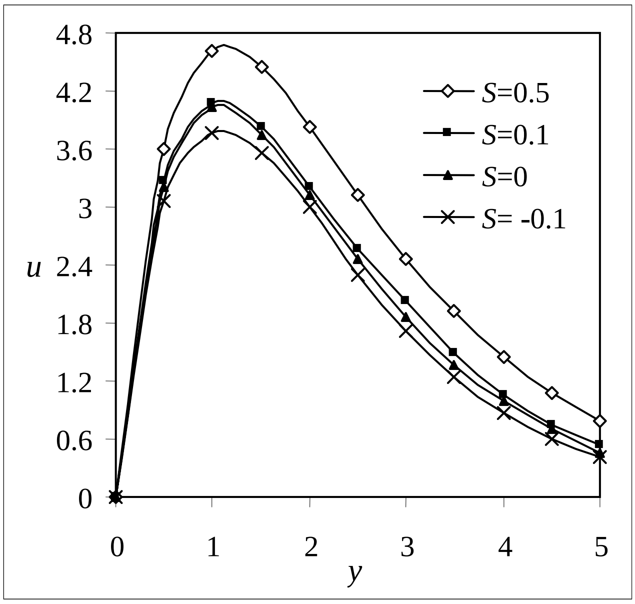

Figure 4. Transient velocity against y for different values of S with G_r=2, G_c=2, K_p=1, S_c=0.66, P_r=0.71, E_c=0.002, ω=5.0, ε=0.2, ωt=π/2

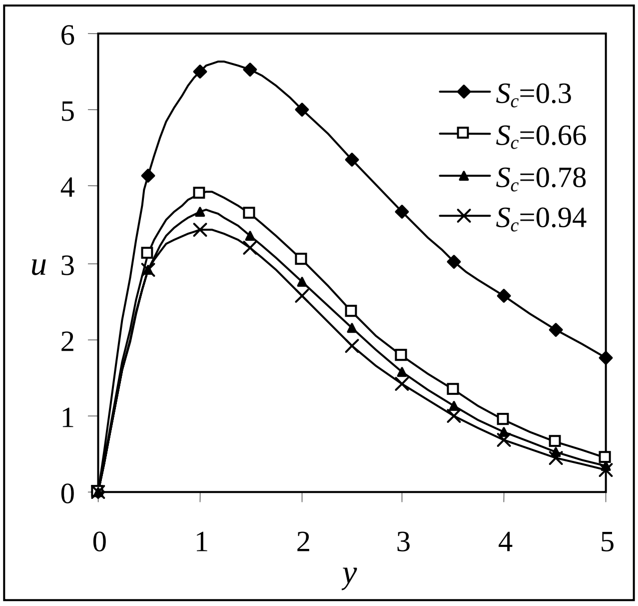

Figure 5. Transient velocity against y for different values of S_c with G_r=2, G_c=2, K_p=1, S=0.1, P_r=0.71, E_c=0.002, ω=5.0, ε=0.2, ωt=π/2

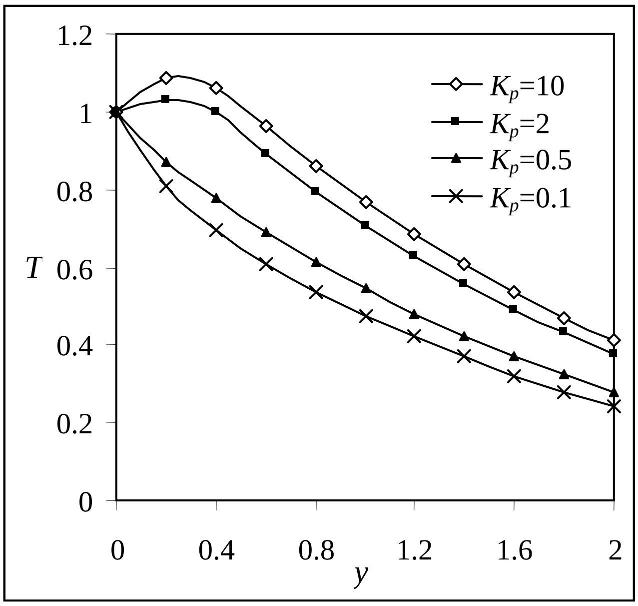

Figure 6. Transient temperature against y for different values of K_p with G_r=2, S=0.1, P_r=0.71, G_c=2, S_c= 0.66, E_c=0.002, ω=5.0, ε=0.2, ωt=π/2

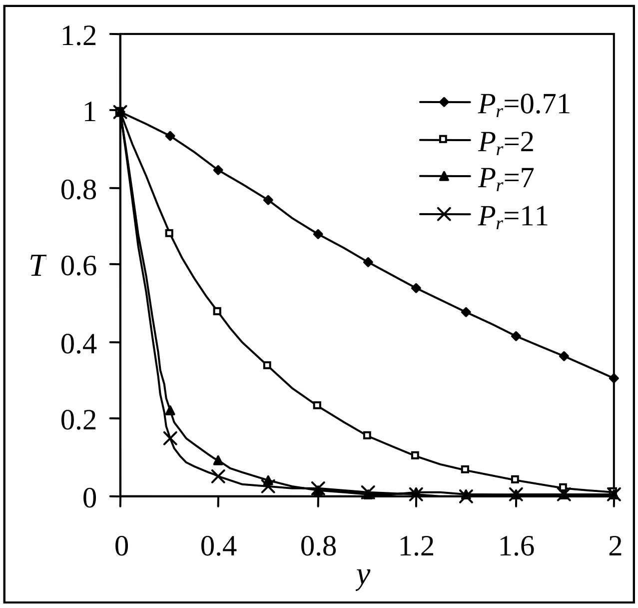

Figure 7. Transient temperature against y for different values of P_r with G_r=2, G_c=2, K_p=1, S=0.1, S_c= 0.66, E_c=0.002, ω=5.0, ε=0.2, ωt=π/2

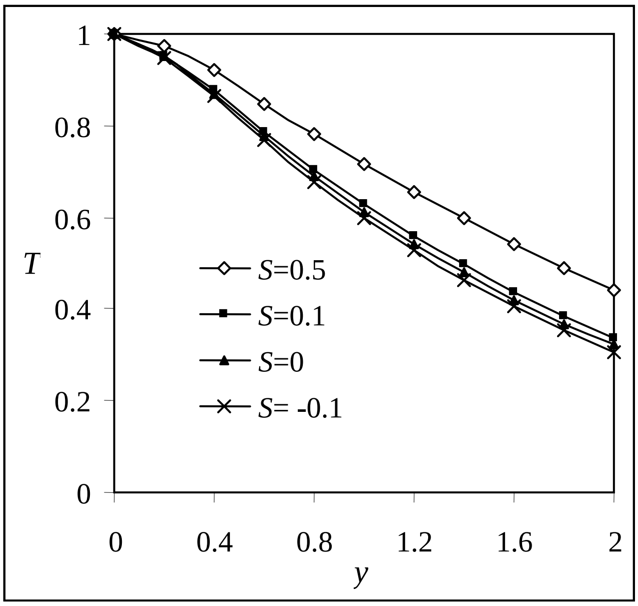

Figure 8. Transient temperature against y for different values of S with G_r=2, G_c=2, K_p =1, P_r=0.71, S_c= 0.66, E_c=0.002, ω=5.0, ε=0.2, ωt=π/2

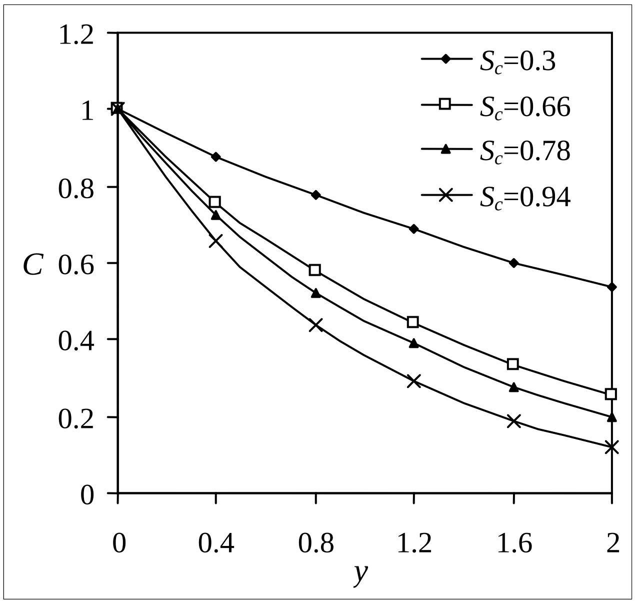

Figure 9. Concentration distribution against y for different values of S_c with ω=5.0, ε=0.2, ωt=π/2

Table 1. Variation in the value of skin friction τ at the wall against P_r for different values of K_p with G_r =2, G_c =2, S=0.1, S_c=0.66, E_c=0.002, ω=5.0, ε=0.2, ωt=π/2

K_p	τ			
	P_r=0.71	P_r=1	P_r=7	P_r=9
0.1	5.49361	5.16386	3.52134	3.20586
0.5	8.84566	8.09659	5.11587	4.87769
10	11.53014	10.32638	6.12064	5.74037
20	11.58724	10.46573	6.17686	5.7778

Table 2. Variation in the value of heat flux N_u at the wall against P_r for different values of K_p with G_r =2, G_c =2, S=0.1, S_c=0.66, E_c=0.002, ω=5.0, ε=0.2, ωt=π/2

K_p	N_u			
	P_r=0.71	P_r=1	P_r=7	P_r=9
0.1	-0.21273	-0.36874	-5.22845	-6.63074
0.5	0.24534	0.61752	-6.47356	-7.96538
10	1.61793	5.86762	-9.04647	-10.6853
20	1.64671	5.94199	-9.36456	-10.9487

5. Conclusion

We present here some of the results of physical interest on the velocity, temperature, concentration distribution and also on the wall shear stress and the rate of heat transfer at the wall.

1. A growing permeability parameter K_p accelerates the transient velocity at all points for small values of K_p (≤1) and for higher values the effect reverses.
2. The effect of increasing Grashof numbers for heat and mass transfer or heat source parameter is to enhance the transient velocity of the flow field at all points, while a growing Schmidt number retards its effect at all points.
3. A growing permeability parameter or heat source parameter increases transient temperature of the flow field at all points, while a growing Prandtl number P_r reverses the effect.
4. The effect of increasing Schmidt number is to decrease the concentration boundary layer thickness of the flow field at all points.
5. A growing permeability parameter enhances the skin friction at the wall, while a growing Prandtl number shows reverse effect.
6. The effect of increasing Prandtl number or permeability parameter leads to increase the magnitude of the rate of heat transfer at the wall.

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