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Finite difference approach on magnetohydrodynamic flow and heat transfer in a viscous incompressible fluid between two parallel porous plates

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Abstract
This paper considers the magnetohydrodynamic flow and heat transfer in a viscous incompressible fluid between two parallel porous plates experiencing a discontinuous change in wall temperature. An explicit finite difference scheme has been employed to solve the coupled non-linear equations governing the flow. The flow phenomenon has been characterized by Hartmann number M, suction Reynolds number R, channel Reynolds number R^* and Prandtl number P_r. The effects of these parameters on the velocity and temperature distribution have been analyzed and the results are presented with the aid of figures. It is observed that a growing suction parameter R retards the velocity of the flow field both in MHD ($M \neq 0$) as well as non-MHD ($M = 0$) flow. The effect of increasing Hartmann number M is to decrease the transverse component of velocity for both suction ($R > 0$) and injection ($R < 0$) and in absence of suction and injection ($R = 0$), while it decreases the axial component of velocity up to the middle of the channel and beyond this the effect reverses. There is a sharp fluctuation in temperature near the walls and at the middle of the channel which may be attributed to the discontinuous change in wall temperature. For fluids having low Prandtl number such as air ($P_r = 0.71$), the temperature assumes negative values.
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Keywords: Magnetohydrodynamic flow; Viscous fluid; Heat transfer; Finite difference.

1. Introduction

The phenomenon of fluid flow between two porous boundaries is of great theoretical as well as practical interest. Some of the practical interests include problems dealing with gaseous diffusion, transpiration-cooling, lubrication of porous bearings; methods of decreasing rates of heat transfer in combustion chambers exhaust nozzles and porous walled flow reactors etc. In view of the above interests, several researchers have solved the heat transfer problems between two impermeable parallel walls under different physical situations.
Berman [1] analyzed the laminar flow in channels with porous walls. Verma and Bansal [2] discussed the flow of an incompressible fluid between two parallel plates, one in uniform motion and the other at rest with uniform suction at the stationary plate. Gupta and Goyal [3] investigated plane couette flow between two parallel plates with uniform suction at the stationary plate. Soundalgekar and Wavre [4]

explained the unsteady free convection flow past an infinite vertical plate with constant suction and mass transfer. Soundalgekar and Ramamurty [5] developed heat transfer in MHD flow with pressure gradient, suction and injection. Newal Kishore *et al*. [7] studied unsteady MHD flow through two parallel porous flat plates. Kaviany [8] analyzed laminar flow through a porous channel bounded by isothermal parallel plates. Sharma and Singh [9] reported the steady flow and heat transfer of an ordinary viscous fluid between two parallel plates. Goyal and Bansal [10] developed the unsteady magnetohydrodynamic boundary layer flow over a flat plate.

MHD flow between two parallel plates with heat transfer was studied by Attia and Kotb [11]. Raptis and Soundalgekar [12] analyzed the steady laminar free convection flow of an electrically conducting fluid along a porous hot vertical plate in presence of heat source/sink. Vajravelu and Hadjinicolaou [13] discussed the convective heat transfer in an electrically conducting fluid at a stretching surface with uniform free stream. Chowdhury and Islam [14] approached analytically MHD free convection flow of visco elastic fluid past an infinite vertical porous plate. Hossain *et al*. [15] explained natural convection flow past a vertical permeable flat plate with variable surface temperature and species concentration. Raptis and Kafousias [16] investigated the heat transfer in flow through a porous medium bounded by an infinite vertical plate under the action of a magnetic field. Sharma *et al*. [17] analyzed the steady laminar flow and heat transfer of a non-Newtonian fluid through a straight horizontal porous channel in the presence of heat source. Das and his associates [18] discussed the three dimensional couette flow and heat transfer in presence of a transverse magnetic field. Recently, Das [19] estimated the effect of suction and injection on MHD three dimensional couette flow and heat transfer through a porous medium.

The study reported herein considers the magnetohydrodynamic flow and heat transfer in a viscous incompressible fluid between two parallel porous plates experiencing a discontinuous change in wall temperature by explicit finite difference scheme. The effects of the flow parameters on the velocity and temperature distribution have been analyzed and presented with the aid of graphs.

2. Formulation of the problem

Consider the flow of a viscous incompressible electrically conducting fluid with electrical conductivity σ between two horizontal parallel non-conducting porous plates in presence of a uniform transverse magnetic field B_0. The physical sketch and geometry of the problem is shown in Figure 1. A Cartesian co-ordinate system is chosen where the axes x- and y- are parallel and perpendicular to the channel walls respectively. Let u and v be the velocity components in x- and y- directions, respectively. Let the temperature of the walls and the fluid be $T = T_0$ for $x < x_0$ and let $T = T_1$ be the constant temperature of the walls for $x > x_0$. Assuming that the magnetic Reynolds number for the flow is very small, the induced magnetic field is neglected. The external electric field is zero and the electric field due to polarization of the charges is negligible. With The above assumptions and neglecting Joules heating effect the equations governing the steady flow of a viscous fluid are:

Equation of continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

Equations of motion:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \frac{\sigma B_0^2 u}{\rho} \tag{2}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \tag{3}$$

Equation of energy:

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$$\rho c\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=K\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

where c is the specific heat, K is the thermal conductivity. ν is the kinematic viscosity, ρ is the density of the fluid and p is the pressure.
The boundary conditions are

$$u(x,0)=0 \quad \text{and} \quad v(x,0)=0,$$
$$u(x,h)=0 \quad \text{and} \quad v(x,h)=V \tag{5}$$

where h is the channel width and V is the constant suction velocity at the wall.
Introducing a non-dimensional distance variable

$$\eta=\frac{y}{h} \tag{6}$$

The boundary conditions (5) now reduce to the form

$$u(x,0)=0 \quad \text{and} \quad v(x,0)=0,$$
$$u(x,1)=0 \quad \text{and} \quad v(x,1)=V \tag{7}$$

Following Berman's procedure, it is assumed that for a two dimensional incompressible flow there exists a 'stream function' of the form

$$\psi(x,\eta)=g(x)s(\eta) \tag{8}$$

The velocity components are given by

$$u=\frac{\partial\psi}{\partial y}=\frac{1}{h}\frac{\partial\psi}{\partial\eta}=\frac{g(x)}{h}\frac{ds}{d\eta} \tag{9}$$

And

$$v=-\frac{\partial\psi}{\partial x}=-s\frac{dg}{dx} \tag{10}$$

To obtain an ordinary non-linear differential equation for $s(\eta)$, it is necessary to have

$$\frac{dg}{dx}=\text{Constant} \tag{11}$$

Which on integration gives

$$g(x)=A+Bx \tag{12}$$

where A and B are constants. Now from equation (8), we have

$$\psi(x,\eta)=(A+Bx)s(\eta) \tag{13}$$

The constants A and B are determined by defining the entrance velocity i.e. the velocity of the fluid in the x-direction at x=0 to be

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$$U = \int_0^1 u(0,\eta)d\eta \tag{14}$$

And by using the law of conservation of matter. Now using equations (14), (9), (10) and (7) we get

$$A = \frac{hU}{s(1)-s(0)}, B = -\frac{V}{s(1)} \text{ and } \frac{s(0)}{s(1)} = 0 \tag{15}$$

The stream function from equations (13) and (15) can then be expressed as

$$\psi(x,\eta) = (hU - Vx)f(\eta) \tag{16}$$

where $f(\eta)$ is a new function defined by

$$f(\eta) = \frac{s(\eta)}{s(1)} \tag{17}$$

The velocity components (9) and (10) with the help of equations (12), (15) and (17) assume the form

$$u(x,\eta) = \frac{1}{h}(hU - Vx)f'(\eta) \tag{18}$$

$$v(\eta) = Vf(\eta) \tag{19}$$

where the prime denotes the differentiation with respect to η. Substituting the partial derivatives of u and v with respect to x and η using equation (18), (19) into equations (2) - (4), we get

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} = \frac{1}{h^2}(hU - Vx)\left[V\left(ff'' - f'^2\right) - \frac{\nu}{h}f'''\right] + \frac{\sigma}{\rho}B_0^2(hU - Vx)\frac{f'}{h} \tag{20}$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} = V^2 ff' - \frac{\nu V}{h}f'' \tag{21}$$

$$\frac{\rho c}{h}\left[(hU - Vx)f'\frac{\partial T}{\partial x} + Vf\frac{\partial T}{\partial \eta}\right] = K\left(\frac{\partial^2 T}{\partial x^2} + \frac{1}{h^2}\frac{\partial T^2}{\partial \eta^2}\right) \tag{22}$$

Since the left hand side of equation (21) is a function of x only, its differentiation with respect to η gives

$$\frac{\partial^2 p}{\partial x \partial \eta} = 0 \tag{23a}$$

Differentiating (20) with respect to η and using equation (23a), we get

$$\frac{\mathrm{d}}{\mathrm{d}\eta}\left[V\left(ff'' - f'^2\right) - \frac{\nu}{h}f'''\right] + \frac{\sigma B_0^2}{\rho}hf' = 0 \tag{23b}$$

which on integration gives

$$f''' + R(f'^2 - ff'') - M^2 f' = C^* \tag{24}$$

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The boundary conditions on $f(\eta)$ are

$$f(0) = 0, \qquad f'(0) = 0,$$
$$f(1) = 1, \qquad f'(1) = 0 \tag{25}$$

where C^* is an arbitrary constant to be determined, $R = \frac{Vh}{\nu}$, the suction Reynolds number, $M = B_0 h\sqrt{\frac{\sigma}{\rho\nu}}$, the Hartmann number.

We assume $R > 0$ for suction and $R < 0$ for injection at both the walls.
Neglecting the longitudinal heat conduction, equation (22) can be rewritten as

$$\left(1-\frac{R}{R^*}\xi\right)f'(\eta)\frac{\partial\theta}{\partial\xi}+\frac{R}{R^*}f(\eta)\frac{\partial\theta}{\partial\eta}=\frac{1}{P_r R^*}\frac{\partial^2\theta}{\partial\eta^2} \tag{26}$$

where $R^* = \frac{Uh}{\nu}$, the channel Reynolds number, $P_r = \frac{c\eta_0}{K}$, the Prandtl number, $\theta = \frac{(T-T_1)}{(T_0-T_1)}$, non-dimensional temperature, $\xi = \frac{(x-x_0)}{h}$, dimensionless distance along the channel.

The boundary conditions on θ are.

$$\theta = 0 \quad \text{at} \quad \eta = \pm 1, \quad \xi > 0,$$
$$\theta = 1 \quad \text{at} \quad \xi = 0, \quad -1 \le \eta \le 1 \tag{27}$$

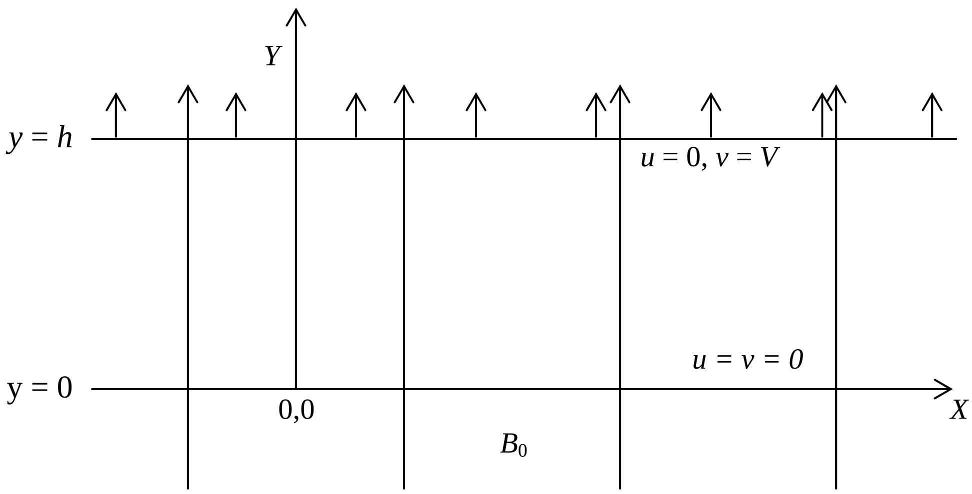

Figure 1. Physical configuration and geometry of the problem

3. Solution for velocity distribution

The non-linear equation (24) subject to boundary conditions (25) is solved using finite difference method. Taking a uniform mesh of step size h, the difference scheme is obtained by replacing the derivatives of $f(\eta)$ by the following difference approximations at the nodes ih, i = 0, 1, 2, ..., n.

$$\frac{df}{d\eta} \approx \frac{f_{i+1} - f_{i-1}}{2h} \tag{28}$$

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$$\frac{d^2 f}{d\eta^2} \approx \frac{1}{h^2}\left(f_{i+1} - 2f_i + f_{i-1} \right) \tag{29}$$

$$\frac{d^3 f}{d\eta^3} \approx \frac{1}{h^3}\left(f_{i+2} - 3f_{i+1} + 3f_i - f_{i-1} \right) \tag{30}$$

With the introduction of above approximations (28) - (30) into equation (24), the discretised form of the equation can be written as

$$f_{i+2} - 3f_{i+1} + 3f_i - f_{i-1} + Rh\left[\frac{1}{4}\left(f_{i+1} - f_{i-1} \right)^2 - f_i\left(f_{i+1} - 2f_i + f_{i-1} \right) \right] - \frac{M^2 h^2}{2}\left(f_{i+1} - f_{i-1} \right) - h^3 C^* = 0 \tag{31}$$

And the boundary conditions (25) on f take the form

$$f_0 = 0, \qquad f_n = 1,$$
$$f_{n+1} = f_{n-1}, \qquad f_{-1} = -f_1 \tag{32}$$

Eliminating f_{-1}, f_0, f_n, f_{n+1} from the system (31) using equation (32), we obtain the following non-linear system to be solved for the unknowns f_1 , f_2 ,..........., f_{n-1}

$$R_0 \equiv f_2 - 2f_1 + Rhf_1^2 - M^2 h^2 f_1 - C^* h^3 = 0,$$
$$\Rightarrow C^* h^3 = f_2 - 2f_1 + Rhf_1^2 - M^2 h^2 f_1 \tag{33}$$

$$R_1 \equiv f_3 - 4f_2 + 5f_1 + \frac{Rh}{4} f_2^2 + Rhf_1^2 - Rhf_1 f_2 - \frac{M^2 h^2}{2} f_2 + M^2 h^2 f_1 = 0,$$

$$R_i \equiv f_{i+2} - 3f_{i+1} + 3f_i - f_{i-1} + \frac{Rh}{4}\left(f_{i+1} - f_{i-1} \right)^2 - Rhf_i\left(f_{i+1} - 2f_i + f_{i-1} \right) - \frac{M^2 h^2}{2}\left(f_{i+1} - f_{i-1} \right)$$
$$- f_2 + 2f_1 - Rhf_1^2 + M^2 h^2 f_1 = 0, \quad for\ i = 2,3,4,5,6,\ldots\ldots\ldots\ldots, n-3 \tag{34a}$$

$$R_{n-1} \equiv 4f_{n-1} - 3 - f_{n-2} + \frac{Rh}{4}\left(1 - 2f_{n-2} \right)^2 - Rhf_{n-1}\left(1 - 2f_{n-1} + f_{n-2} \right) - \frac{M^2 h^2}{2}\left(1 - f_{n-2} \right)$$
$$- f_2 + 2f_1 - Rhf_1^2 + M^2 h^2 f_1 = 0 \tag{34b}$$

The system (34a) is solved by Damped Newton method using the algorithm described in Conte and Boor [6]. For the implementation of the method, the non-zero elements of the Jacobian matrix $\frac{\partial R_i}{\partial f_i}$ are computed as follows:

$$\frac{\partial R_1}{\partial f_1} = 5 + 2Rh\left(f_1 - f_2 \right) + M^2 h^2,$$

$$\frac{\partial R_1}{\partial f_2} = -4 + Rh\frac{f_2}{2} - Rhf_1 - \frac{M^2 h^2}{2},$$

$$\frac{\partial R_2}{\partial f_1} = 1 - 2Rhf_1 - \frac{Rh}{2}\left(f_3 - f_1 \right) - 2Rhf_2 + \frac{3M^2 h^2}{2},$$

$$\frac{\partial R_2}{\partial f_2} = 2 - Rhf_3 - 4Rhf_2 + Rhf_1 ,$$

$$\frac{\partial R_2}{\partial f_3} = -3 + \frac{Rh}{2}\left(f_3 - f_1 - 2f_2\right) - \frac{M^2h^2}{2} ,$$

$$\frac{\partial R_2}{\partial f_4} = 1 ,$$

$$\frac{\partial R_3}{\partial f_1} = 2 - 2Rhf_1 + M^2h^2 ,$$

$$\frac{\partial R_3}{\partial f_2} = -2 + Rh\left[\frac{1}{2}\left(f_2 - f_4\right) - f_3\right] + \frac{M^2h^3}{2} ,$$

$$\frac{\partial R_3}{\partial f_3} = 3 - Rh\left[f_4 - 4f_3 + f_2\right],$$

$$\frac{\partial R_3}{\partial f_4} = -3 + Rh\left[\frac{1}{2}\left(f_4 - f_2\right) - f_3\right] - \frac{M^2h^2}{2} ,$$

$$\frac{\partial R_3}{\partial f_5} = 1 ,$$

$$\frac{\partial R_i}{\partial f_1} = 2 - 2Rhf_1 + M^2h^2 ,$$

$$\frac{\partial R_i}{\partial f_2} = -1 ,$$

$$\frac{\partial R_i}{\partial f_{i-1}} = -1 - \frac{Rh}{2}\left(f_{i+1} + 2f_i - f_{i-1}\right) + \frac{M^2h^2}{2} ,$$

$$\frac{\partial R_i}{\partial f_i} = 3 - Rh\left(f_{i+1} - 4f_i + f_{i-1}\right),$$

$$\frac{\partial R_i}{\partial f_{i+1}} = -3 + \frac{Rh}{2}\left(f_{i+1} - 2f_i - f_{i-1}\right) - \frac{M^2h^2}{2} ,$$

$\frac{\partial R_i}{\partial f_{i+2}} = 1$, where $i = 4, 5, \ldots\ldots, n\text{-}3$.

$$\frac{\partial R_{n-2}}{\partial f_1} = 2 - 2Rhf_1 + M^2h^2 ,$$

$$\frac{\partial R_{n-2}}{\partial f_2} = -1 ,$$

$$\frac{\partial R_{n-2}}{\partial f_{n-2}} = 3 - Rh\left(f_{n-1} + f_{n-3}\right) + 4Rhf_{n-2} ,$$

$$\frac{\partial R_{n-2}}{\partial f_{n-1}} = -3 + \frac{Rh}{2}\left(f_{n-1} - f_{n-3} - 2f_{n-2}\right) - \frac{M^2h^2}{2} ,$$

$$\frac{\partial R_{n-1}}{\partial f_1} = 2 - 2Rhf_1 + M^2h^2 ,$$

$$\frac{\partial R_{n-1}}{\partial f_2} = -1 ,$$

$$\frac{\partial R_{n-1}}{\partial f_{n-2}} = -1 + \frac{Rh}{2}\left(f_{n-2} - 2f_{n-1} - 1\right) + \frac{M^2h^2}{2} ,$$

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$$\frac{\partial R_{n-1}}{\partial f_{n-1}} = 4 - Rh + 4Rhf_{n-1} - Rhf_{n-2} \tag{35}$$

4. Solution for temperature distribution

In order to solve equation (26), we use the following difference approximations for the derivatives of θ.

$$\frac{\partial \theta}{\partial \xi} \approx \frac{\left(\theta_i^{j+1} - \theta_i^j\right)}{k} \tag{36}$$

$$\frac{\partial \theta}{\partial \eta} \approx \frac{\left(\theta_{i+1}^j - \theta_{i-1}^j\right)}{2h} \tag{37}$$

$$\frac{\partial^2 \theta}{\partial \eta^2} \approx \frac{\left(\theta_{i+1}^j - \theta_i^j + \theta_{i-1}^j\right)}{h^2} \tag{38}$$

where h and k are the uniform mesh-sizes in the directions of η and ξ respectively (Figure 2).

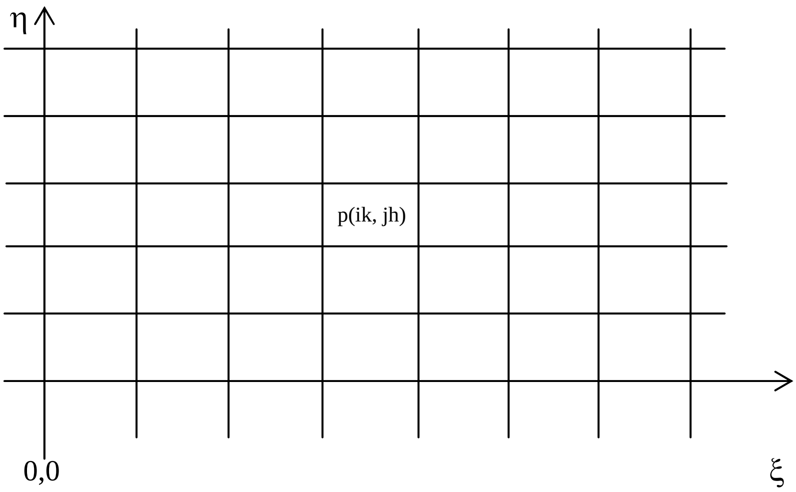

Figure 2. Geometry of a discretised domain showing mesh points

Introducing the difference approximations (36) - (38) in equation (26), it produces the discretised form of the equation

$$\theta_i^{j+1} = \frac{\left[\frac{2}{P_r R^*}\left(\frac{k}{h}\right) - \frac{R}{R^*} kf_i\right]}{\left(1 - \frac{R}{R^*}\xi_j\right)\left(f_{i+1} - f_{i-1}\right)}\theta_{i+1}^j + \frac{\left[\frac{2}{P_r R^*}\left(\frac{k}{h}\right) + \frac{R}{R^*} kf_i\right]}{\left(1 - \frac{R}{R^*}\xi_j\right)\left(f_{i+1} - f_{i-1}\right)}\theta_{i-1}^j + \left[1 - \frac{4(k/h)}{P_r R^*\left(1 - \frac{R}{R^*}\xi j\right)\left(f_{i+1} - f_{i-1}\right)}\right]\theta_i^j \tag{39}$$

$i = -n, -n+1, \ldots\ldots\ldots, -1, 0, 1, \ldots\ldots\ldots, n.$

The corresponding boundary conditions on θ become.

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$$\begin{aligned} &\theta_{-n}^{j} = 0, \qquad \theta_{n}^{j} = 0 \quad \text{for } \xi_j, \qquad j = 1, 2, \ldots\ldots \\ &\theta_i^j = 1 \qquad \text{for i = -n, - n + 1, ..., -1, 0, 1, ..., n} \\ &\qquad\qquad j = 0 \end{aligned} \tag{40}$$

Using equations (40) and the nodal values of *f*, the values of θ at all interior nodes for $j = 1$, are computed from the equation (39). The procedure is repeated for $j = 2, 3.....$ until the required value of ξ is reached.

5. Discussions and result

The problem of hydromagnetic flow and heat transfer between two parallel porous plates experiencing a discontinuous change in wall temperature is investigated. The effects of the magnetic parameter *M*, suction/injection parameter *R* and Prandtl number P_r on the velocity and temperature of the flow field are discussed with the help of Figures 3-5.

The variation of *y*-component of velocity $f(\eta)$ for different values of *M* and *R* is shown in Figure 3. Comparing the curves 1, 2 and 4, 5; it is observed that the magnetic parameter *M* decreases the velocity of the flow field in the channel for suction (R>0) and injection ($R < 0$). Analyzing the curves 1, 4 and 2, 3; it is noticed that a growing suction parameter leads to a decrease in velocity both for MHD ($M \neq 0$) and non-MHD ($M = 0$) flow. Figure 4 elucidates the effect of magnetic parameter *M* and suction/ injection parameter *R* on the axial component of the velocity of the flow field. Comparing curves 1, 2; 4, 5 and 1, 4; it is observed that the magnetic parameter *M* and suction/injection parameter *R* decrease the axial component of the velocity up to the middle of the channel ($0 \leq \eta \leq 0.5$) and there after the effect reverses.

Figure 5 elucidates the temperature profiles of the flow field for different values of the Prandtl number P_r keeping $R = 2.0$, $R^* = 5.0$ and $\xi = 0.01$. When there is a fluid suction at the wall ($R = 2.0$), the temperature rises sharply near the wall for different values of P_r and goes on increasing up to $\eta = -0.5$. It then decreases slowly with a sharp fall at $\eta = -0.1$ and suddenly assumes a sharp rise with a peak in the neighbourhood of $\eta = 0.7$ and decreases up to $\eta = 0.2$ and afterwards increases up to $\eta = 0.5$. The temperature then falls sharply to zero. This fluctuation in temperature may be attributed to the discontinuous change in wall temperature. For fluids having low Prandtl number such as air ($P_r = 0.71$), the temperature assumes negative values. In case of injection ($R = -2$), there is a slight increase in temperature and the temperature profiles closely agree with the case of suction ($R = 2$).

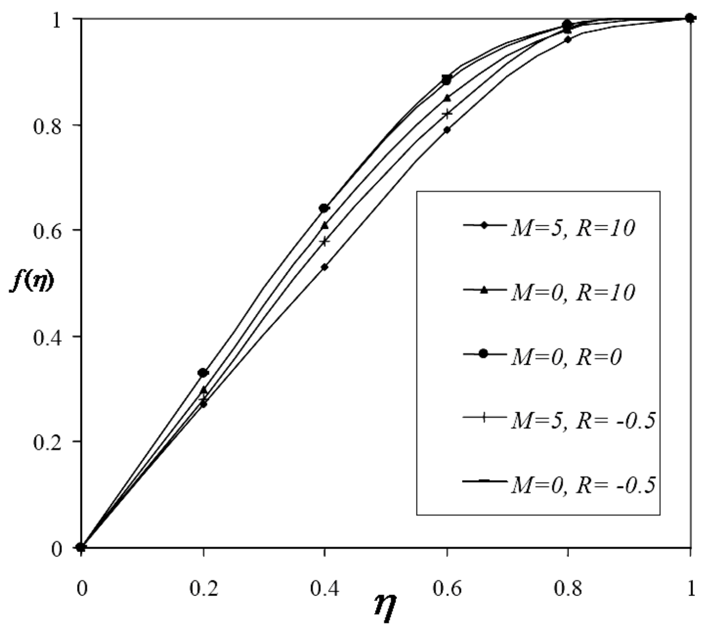

Figure 3. Variation of y-component of velocity $f(\eta)$ against η for different values of M and R

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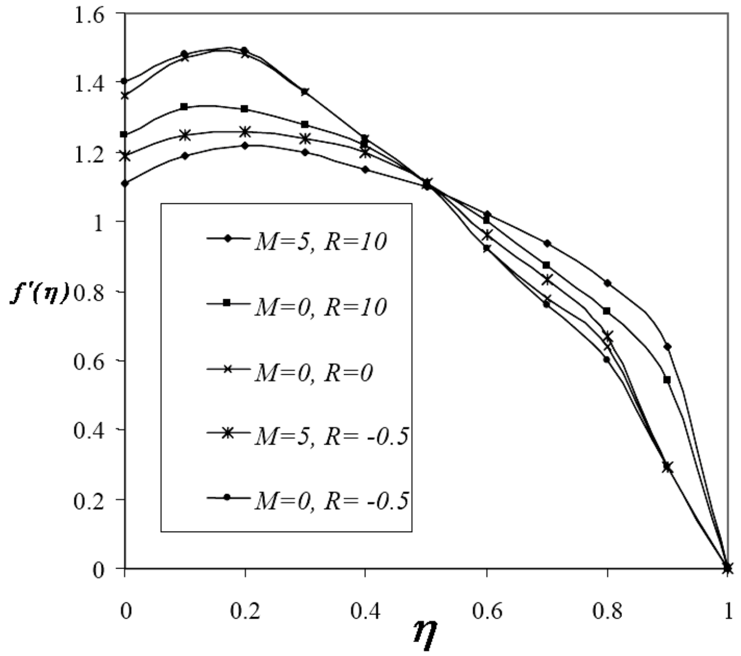

Figure 4. Variation of axial component of velocity $f'(\eta)$ against η for different values of R and M

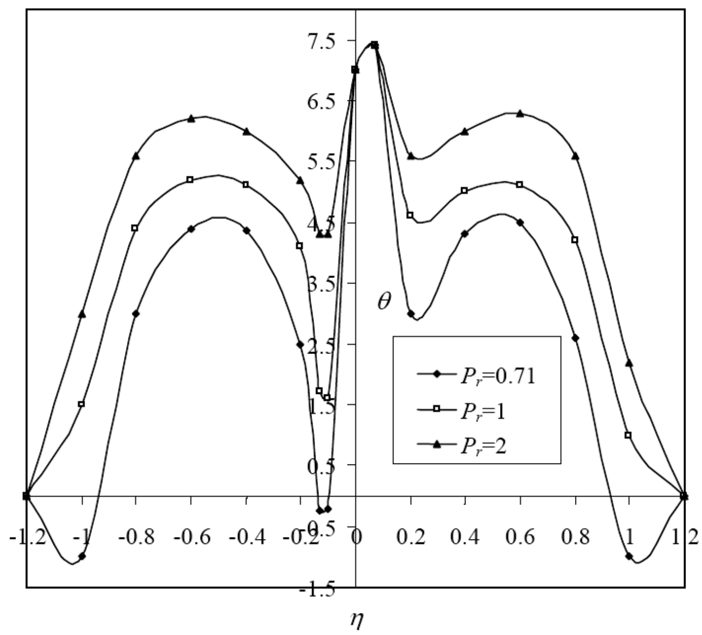

Figure 5. Temperature distribution against η for different values of P_r with R = 2.0, R^* = 5.0 and ξ = 0.01

6. Conclusion

We summarize below some of the essential features of physical interest from the above analysis.

1. A growing suction parameter R retards the velocity of the flow field both in MHD ($M \neq 0$) as well as non-MHD (M = 0) flow.
2. The effect of increasing Hartmann number M is to decrease the transverse component of velocity for both suction (R > 0) and injection (R <0), while it decreases the axial component of velocity up to the middle of the channel and beyond this the effect reverses.
3. There is a sharp fluctuation in temperature near the walls and at the middle of the channel which may be attributed to the discontinuous change in wall temperature.
4. For fluids having low Prandtl number such as air (P_r = 0.71), the temperature assumes negative values.

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