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Optimal placement of horizontal - and vertical - axis wind turbines in a wind farm for maximum power generation using a genetic algorithm

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Abstract

In this paper, we consider the Wind Farm layout optimization problem using a genetic algorithm. Both the Horizontal –Axis Wind Turbines (HAWT) and Vertical-Axis Wind Turbines (VAWT) are considered. The goal of the optimization problem is to optimally position the turbines within the wind farm such that the wake effects are minimized and the power production is maximized. The reasonably accurate modeling of the turbine wake is critical in determination of the optimal layout of the turbines and the power generated. For HAWT, two wake models are considered; both are found to give similar answers. For VAWT, a very simple wake model is employed.

Keywords: Wind farm layout; Genetic algorithm; Horizontal-axis wind turbines (HAWT); Vertical-axis wind turbines (VAWT).

1. Introduction

With increased emphasis on wind power generation worldwide, the optimal placement of large number of wind-turbines in a wind farm is currently a problem of great interest. Several studies have addressed the problem of optimal placement of horizontal axis wind turbines (HAWT) for maximum power generation capacity [1-3]. These studies employ a genetic algorithm for determining the optimal placement of turbines to maximize the generated wind power while limiting the number of turbines installed and the acreage of land occupied. The optimal spacing between the turbines in general depends upon the terrain, the wind direction and the speed, and turbine size. The optimization strategy requires the models for the wake and investment cost for the turbines (which depends on the number of turbines and their size). Most of the studies have employed a very simple wake model of Jensen [4] and a simple cost model of Mosetti et al. [1]. For a HAWT, this study also employs a more accurate wake model due to Werle [5] and a more realistic cost model for large turbines (80-120m diameter) with more realistic constraints on turbine placement than random distribution of Mosetti et al. [1] or uniformly distributed square grid arrangement of Grady et al. [2]. This study finds that the uniformly distributed square grid arrangement of Grady et al. [2] is indeed optimal for large HAWT even with improved wake and cost models. We also study the wind farms with vertical-axis wind turbines (VAWT) such as Darrieus rotor. A simple wake model following the work of Jensen [4] is developed for the VAWT. We have also developed a more complex wake model for VAWT using the double stream-tube model of Paraschivoiu

[6]. However this model is not employed in the results reported in this paper. It is found that a uniform grid arrangement is also best in the case of a VAWT for optimal power generation.

2. Wake, power and cost modeling of a HAWT

2.1 Jensen's wake modeling of a HAWT

All the results reported to date in the literature on optimal layout of wind turbines in a wind farm employ the simple wake model of Jensen [4] and use a genetic algorithm for optimization of an objective function based on power output or a combination of power output and cost [1-3]. In Jensen's model, the near field effects of the turbine wake are neglected and the near wake is simplified as an axisymmetric wake with a velocity defect which linearly spreads with distance downstream into the far – field where it encounters another turbine as shown in Figure 1. Let U be the mean wind speed, then employing the inviscid actuator disc theory of Betz, it can be shown that

$$V_0 = (1-2a)U \quad \text{and} \quad r_0 = r_r\sqrt{\frac{1-a}{1-2a}} \tag{1}$$

where r_0 is the radius of the axisymmetric wake immediately behind the turbine rotor, a is the axial induction factor, and r_r is the rotor radius of the turbine.

The wind velocity in the wake at a distance x downstream can then be determined using the principle of conservation of momentum as:

$$u = U[1-\frac{2a}{(1+\alpha(x/r_0))^2}] \tag{2}$$

It can be shown by the Betz's theory that the turbine thrust coefficient C_T is related to the axial induction factor a by the following relation:

$$a = \frac{1-\sqrt{1-C_T}}{2} \tag{3}$$

The entrainment constant α is empirically given as [2]:

$$\alpha = \frac{0.5}{\ln(z/z_0)} \tag{4}$$

where z is the turbine hub height and z_0 is the surface roughness.

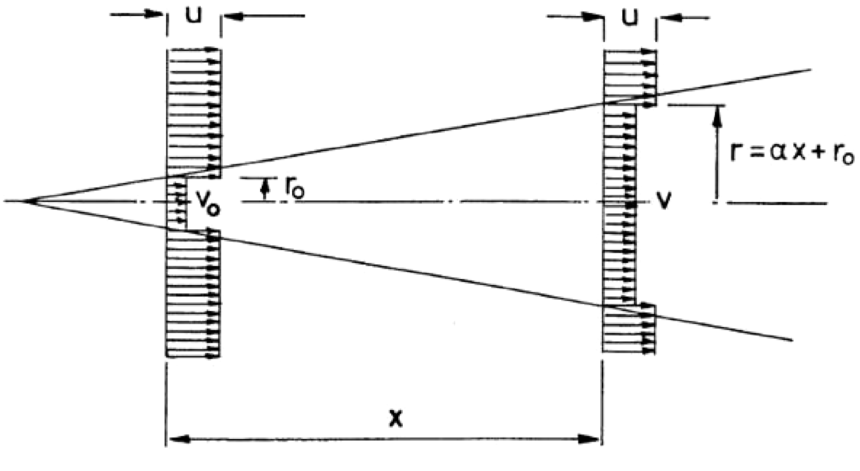


Figure 1. Schematic of the Jensen's wake model [4]

2.2 Werle's wake modeling for HAWT

Werle [5] attempted to improve on the simple wake model of Jensen [4]. He divided the wake into three parts: the near wake, the intermediate wake and the far wake. His model is supposedly better since it considers the near wake region, where the velocity is slightly higher compared to intermediate wake region as shown in Figure 2. Jenson's model does not consider the near wake region. Let X be the non-dimensional distance downstream of the turbine:

$$X \equiv x / D_p \tag{5}$$

where D_p is the turbine diameter $D_p = 2r_r$. Let D_w denote the diameter of the velocity defect in the wake at X. Then Werle's wake model can be described by the following expressions which give the wind speed and wake growth downstream of the turbine:

For $X < X_m$,

$$u = 1 - \frac{1-U}{2}[1 + \frac{2X}{\sqrt{1+4X^2}}] \tag{6}$$

$$D_w / D_p = \sqrt{\frac{1+U}{2u}} \tag{7}$$

For $X > X_m$,

$$u = 1 - \frac{1-u_m}{[(X-X_m)(2(1-u_m))^{3/2} / C_T^{1/2} + 1]^{2/3}} \tag{8}$$

$$D_w / D_p = D_m / D_p[C_T(X - X_m) / (D_m / D_p)^3 + 1]^{1/3} \tag{9}$$

In equations (6)-(9), X_m is the location where the far wake model is coupled to the near wake, D_m is the diameter of the wake at X_m and u_m is the velocity in the wake at X_m. X_m is given by:

$$X_m = 2 + K_m \frac{2r_o}{D_p} \frac{1+U}{1-U} \tag{10}$$

D_m / D_p is given by:

$$D_m / D_p = \sqrt{\frac{1+U}{2u_m}} \tag{11}$$

and u_m is given by:

$$u_m = 1 - \frac{1-U}{2}[1 + \frac{2X_m}{\sqrt{1+4X_m^2}}] \tag{12}$$

2.3 Multiple wake and cost modeling for HAWT

In general a HAWT downstream of an array of turbines may encounter multiple wakes due to several turbines upstream of it. Since various wakes of the array of turbines form a mixed wake, the kinetic energy of this mixed wake is assumed to be equal to the sum of the kinetic energy of various wake deficits. This results in the following expression for the velocity downstream of N turbines [1]:

$$(1-\frac{\bar{u}}{U})^2 = \sum_{i=1}^{N}(1-\frac{u_i}{U})^2 \tag{13}$$

In equation (13), $\bar{u}$ is the average velocity experienced by the turbine due to the wake deficit velocity of multiple turbines given by u_i, $i = 1\ldots\ldots N$. Assuming the non-dimensionalized cost/year of a single turbine to be 1, a maximum cost reduction of 1/3 can be obtained for each turbine if a large number of machines are installed. We then assume that the total cost/year of the whole wind farm can be expressed by the following relation [2]:

$$cost = N(\frac{2}{3}+\frac{1}{3}e^{-0.00174N^2}) \tag{14}$$

The power curve presented in Mosetti et al. [1] for the HAWT gives the following expression for power output of the whole wind farm:

$$P = \sum_{i=1}^{N} 0.3\bar{u}_i^3 \tag{15}$$

The optimization is based on the following objective function:

$$Objective\ function = \frac{cost}{P} \tag{16}$$

Equation (16) is the cost function for the optimization with genetic algorithm.

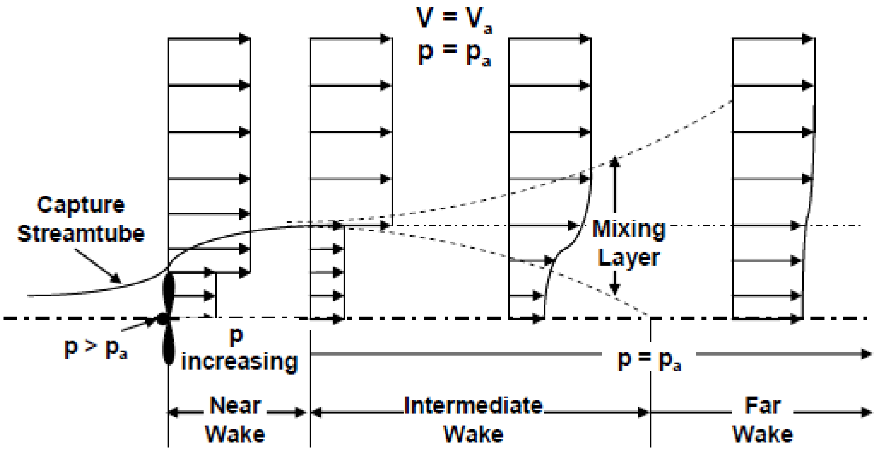


Figure 2. Schematic of Werle's wake model [5]

3. Wake, power and cost modeling of a VAWT

3.1 Single stream model of a VAWT

The book by Manwell, McGowan and Rogers [7] describes the analysis of the single stream tube model for a two-dimensional single straight-blade vertical axis wind turbine. The geometry of this simple model is shown in Figure 3; the blade is rotating in the counter-clockwise direction while the wind blows from left to the right. Some modifications are made to the model described in the book by Manwell et al. [7] so that it can be applied to the flow field with multiple wakes. Let $e = u_{local}/U$, where u_{local} represents the velocity u in the wake of a single turbine or $\bar{u}$, the velocity due to multiple wakes. Now, applying the blade element theory together with the principle of momentum conservation and assuming high tip speed ratios λ, the following expressions for induction factor a and power coefficient Cp for a single vertical axis wind turbine are obtained [7]:

$$a \approx \frac{1}{16}\frac{Bc}{R} e C_{l,\alpha} \lambda \tag{17}$$

$$C_p \approx 4ea(1-a)^2 - \frac{1}{2}\frac{Bc}{R} C_{d,0} \lambda^3 \tag{18}$$

Then, the power output of single VAWT is given by:

$$P_S = \frac{1}{2}\rho(2RH)U^3 C_p \tag{19}$$

where B is the number of blades, c is the chord length of the blade airfoil, R is the rotor radius, λ is the tip speed ratio, H is the total blade length, ρ is air density, U is the free stream velocity and $C_{l,\alpha}$ is the lift curve slope for small angles of attack (below stall).

We assume a symmetrical airfoil; the lift coefficient is linearly related to the angle of attack, that is $C_l = C_{l,\alpha}\alpha$. $C_{l,\alpha}$ is calculated from the lift vs. angle of attack curve for NACA0015 from NACA report [8]. In this study, $C_{l,\alpha} \approx \frac{18}{\pi}$ and $C_{d,0}$ is assumed to be zero.

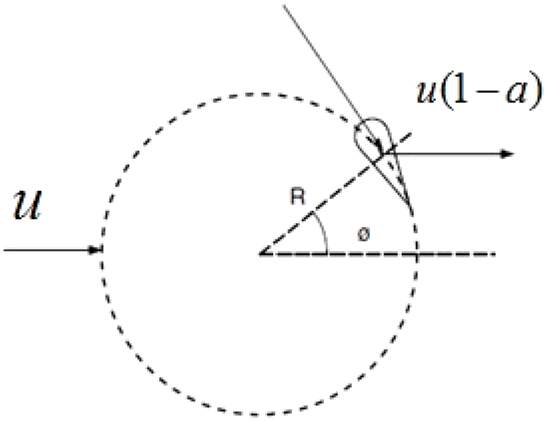


Figure 3. Single stream tube geometry of a VAWT

3.2 Wake model for a VAWT

Again, we assume the VAWT to be an actuator disc so the near field behind the wind turbine is neglected. We modify the Jensen's model [4] of the wake and apply it to determine the wake of a VAWT. Now, the cross-section area of the streamtube is a square of width 2*R* and height *H* instead of a circle. From the conservation of momentum,

$$2r_0 HV_0 + 2(r - r_0)HU = 2rHu \tag{20}$$

where r_0 and r are as shown in Figure 1.

Therefore, we obtain the following expression for the velocity downstream of a single VAWT:

$$u = U[1 - \frac{2a}{1+\alpha(x/r_0)}] \tag{21}$$

In equation (21), α is the same as given in equation (4). For wind turbine downstream encountering multiple wakes, again equation (13) is employed. The velocity $\bar{u}$ is then used to determine the power output of the wind turbine.

4. Brief description of genetic algorithm

In this section we provide a brief introduction to the genetic algorithm. Genetic algorithms (GA) are a class of stochastic optimization algorithms inspired by the biological evolution. The GA starts with an initial generation, which is a group of input vectors that are randomly generated possible solutions to the problem. Each solution or individual carries a set of information that is used to decide its fitness for achieving the optimization objective (or fitness function). The GA algorithm improves the fitness of the initial generation by going through a number of steps listed below. The procedure is repeated by going through many generations until the convergence criteria for fitness is met. The steps in implementation of GA are as follows [9, 10].

1. Evaluation: The quality or the fitness of each individual is evaluated by the fitness function.
2. Natural selection: Remove a subset of the individuals. In order to select a proportion of the current generation to create a new generation, those individuals with lowest fitness are removed according to the natural selection rate. The remaining individuals are called survivors and go to the next generation.
3. Reproduction: Two functions are used at this step to generate the next generation.
(1) *Crossover:* The fitness proportionate selection method is used to create individuals for the new generation from the last generation.
(2) *Mutation:* In order to maintain genetic diversity, some of the individuals in the group are randomly altered. After that, the new generation is finally created.
4. Termination: There are several ways to define the termination condition. In this work, a convergence criterion is applied; when most of the individuals reach an approximately the same fitness value which remains unchanged for many more generations, the solution is assumed to have converged and the optimized value of fitness is obtained as this converged value.

In this study, the number of individuals in each generation is taken to be 40 and the total number of generations employed for converged solution is approximately 250. For both HAWT and VAWT wind farms, the size of the farm considered is 50D x 50D and a wind with uniform speed of 12 m/s is considered. Since each column of turbines has a width of 5D and the ground wind speed is uniform facing the turbine (in normal direction to the rotor plane), the wake behind the turbine in each column stays in the same column. Therefore, the optimization focuses only on one column of 10-column wind farm. The whole wind farm will have the same configuration for each column and the total power output of the farm will be 10 times that of one column.

5. Results

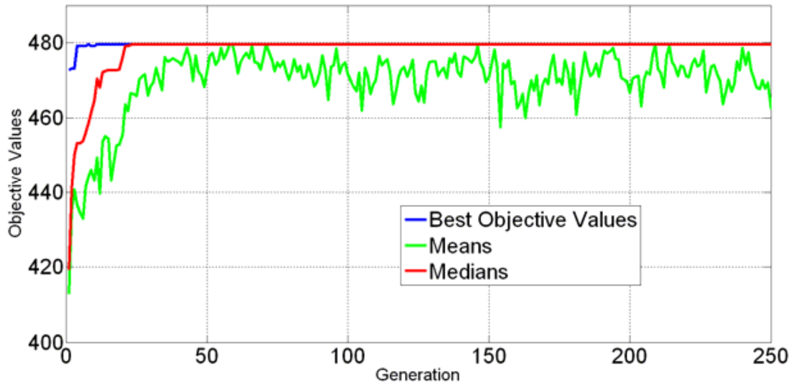
5.1 Layout optimization of HAWT wind farm

We consider two cases of HAWTs with the geometric parameters shown in Table 1. These parameters in the second column were also used by Grady et al. [2] in their study while the ones in the third column were cited in reference [11] as the size of the tallest wind turbine in the U.S. in 2005. The size of the farm considered is 50D x 50D and a wind with uniform speed of 12 m/s is considered. Here D is the rotor diameter of a HAWT. This assumption results in the following simplification: the optimization for one 50D x 50D wind farm equals to the optimization for one column of the size 5D x 50D since the largest wake in one column still stays in it. After knowing the layout in one column, the layout for the whole wind farm is composed with 10 column with the same pattern. Also, the best power output is 10 times the one for one column.

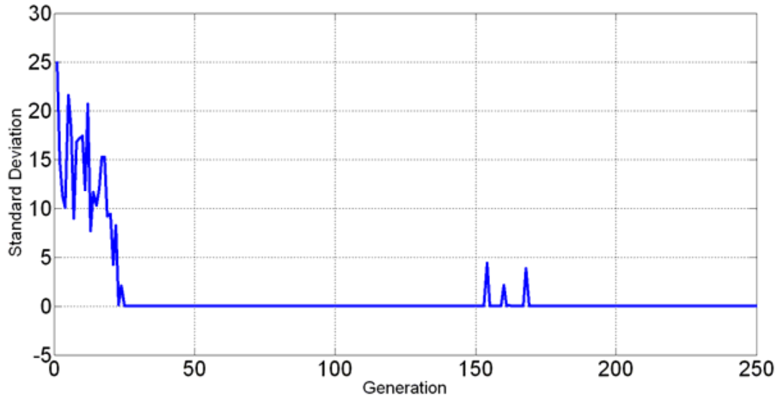
In this study, we first employ the wake model of Jensen [4] with case I in Table 1. Figure 4 shows the convergence history of the objective function given by equation (16). The calculation starts with a best objective function of approximately 473 and power output of about 1409 kW. It jumps quickly to a higher value and stays unchanged for the rest of the generations up to 250. The best objective function value obtained is 479.5 with cost equal to 2.98 and total power equal to 1431.2 kW. The history of medians follows the same trend and converges to the same value as the best objective value. The mean objective values also gets close to this value but stays unsteady due to the mutation in GA. Figure 5 shows the configuration for placement of wind turbines to get the optimal value of the objective function for the entire wind farm; identical results were obtained by Grady et al. [2]. Figure 5(a) is the optimal configuration for the farm with just one column with the same cell size. Figure 5(b) is the extended optimized result for a farm land with a size of 50D x 50D. The extended result for the whole farm has a total power output of 14312 kW.

Table 1. Geometric parameters of a HAWT

Size parameters	Case I	Case II
Hub height, z [m]	60	80
Rotor radius, r_r [m]	20	41
Thrust coefficient, C_T	0.88	0.88

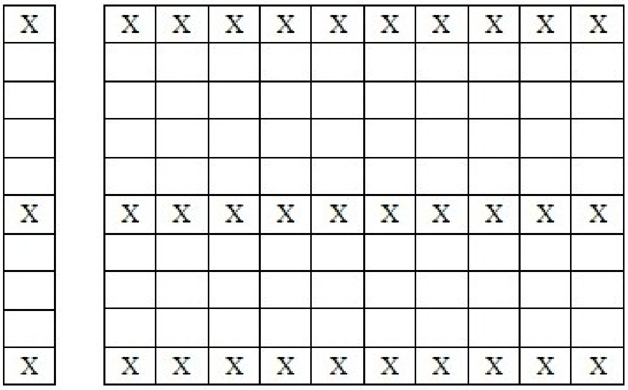


(a) Best, mean and median objective values



(b) Standard deviations

Figure 4. Convergence history of the objective function (total power/cost) for HAWT wind farm using GA for case I



(a) (b)

Figure 5. Optimal layout of HAWT in a 50D x 50D wind farm

Next, we study the effect on the optimization results by using a more complex but realistic wake model due to Werle [5] for both of the cases in Table 1 by assuming that the cost/year of two turbines stays unchanged. Figure 6 shows the changes in the wake velocity and wake growth behind a turbine. The curves in red are those obtained for the turbine parameters used in this study given in Table 1. The red curves calculated in this study for C_T = 0.88 fall accurately among those calculated by Werle [5] for different values of C_T. Using the wake model of Werle, we can perform the optimization study in Table 2. The convergence history is shown in Figure 7. The values in Table 2 are very close to those obtained by using Jensen's [4] model. The optimal layout configuration for the wind farm is the same as shown in Figure 5.

Table 2. Optimization results using Werle's wake model

Optimal values	Case I	Case II
Objective function	454.3	412.9
Cost/year	2.98	2.98
Total power [kW]	1355.7	1230.4

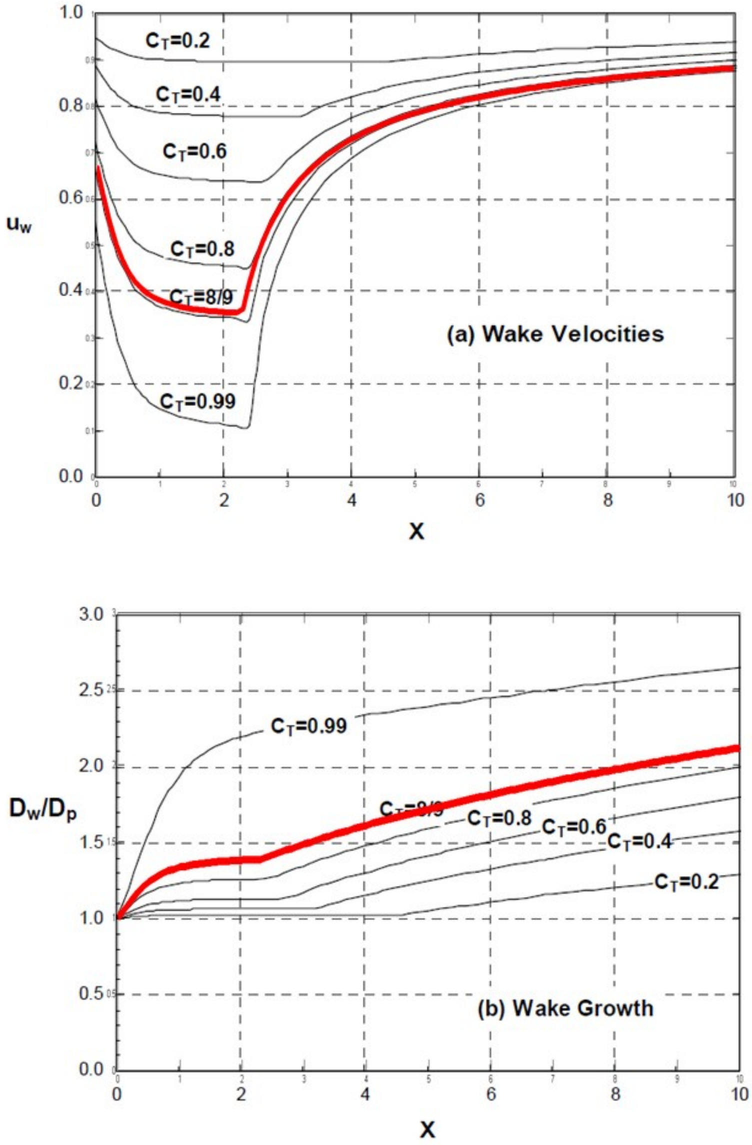
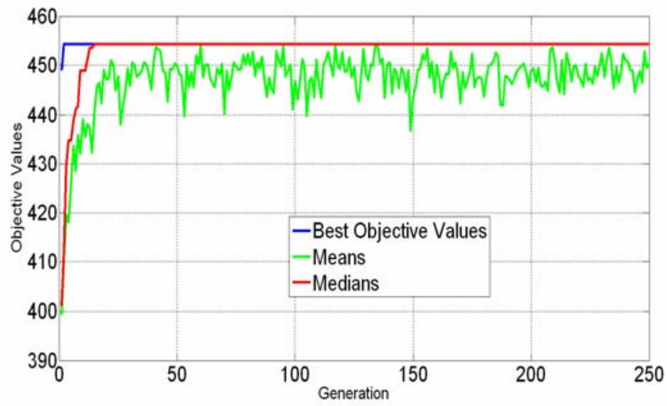
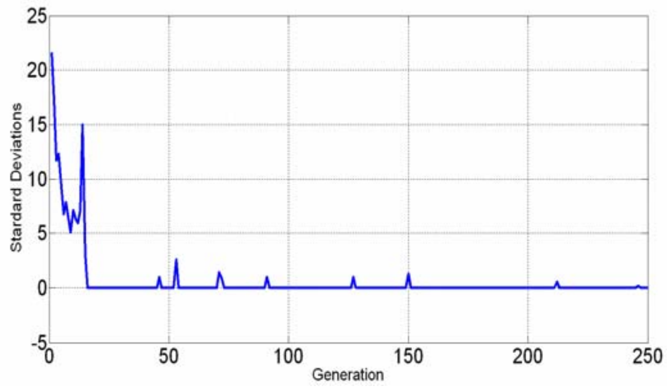


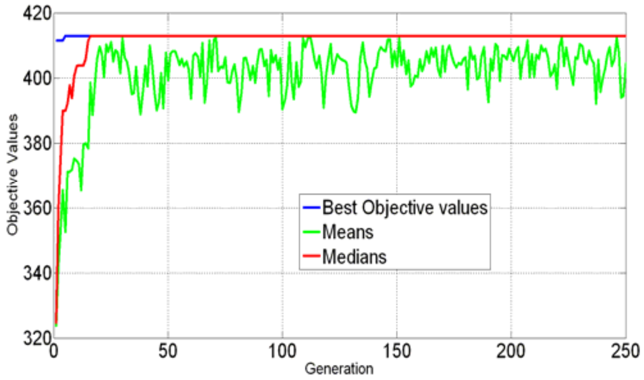
Figure 6. The variation in wake velocity and growth behind the HAWT using Werle's model [5]; the curves in red are present calculations for C_T=0.88



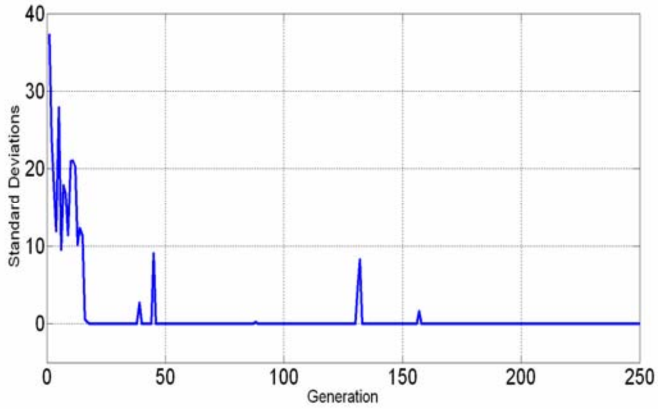
(a) Best, mean and median objective valuses for Case I



(b) Standard deviations for Case I



(c) Best, mean and median objective values for Case II



(d) Standard deviations for Case II

Figure 7. Convergence history of the objective function (total power/cost) for HAWT wind farm using GA with Werle's wake model [5] for cases I, II

5.2 Layout Optimization of VAWT Wind Farm

We consider VAWT with the geometric parameters shown in Table 3. Case III has been taken from the paper of Yan et al. [12] and Case III has a double rotor radius compared to Case IV so that the tip-speed ratio also doubles. Again, the size of the farm is 50D x 50D and a wind with uniform speed of 12 m/s is considered.

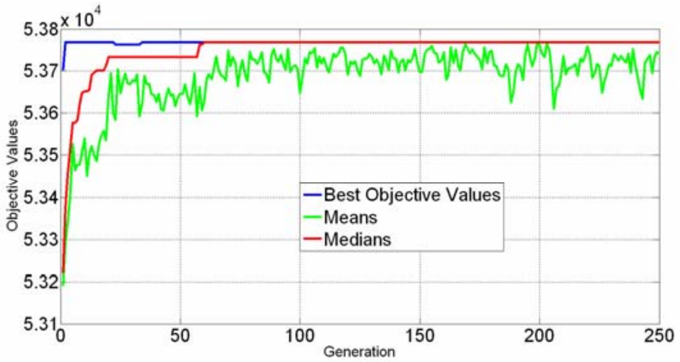
Table 3. Geometric parameters of a VAWT

Variable	Values	
	Case III	Case IV
Rotor radius, R [m]	6	3
Blade profile	NACA0015	
Blade chord, c [m]	0.2	
Blade length, H [m]	6	
Hub height, z [m]	6	
Rotational speed, ω [rad/s]	13.09	
Tip-speed ratio, λ	2.9	
Wind Speed, U [m/s]	12	
Air density, ρ [kg/m^3]	1.21	

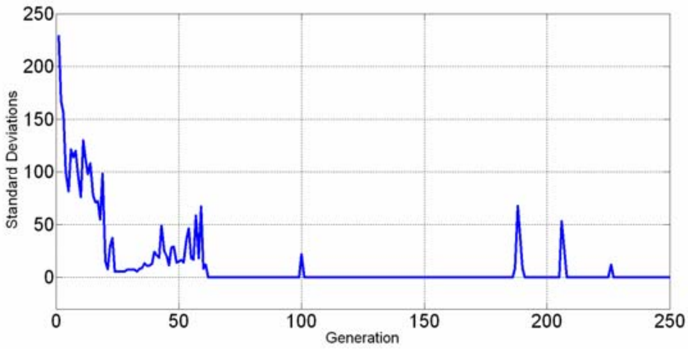
Figure 8 shows the convergence history of the GA optimization for VAWT wind farm. The results are given in Table 4. The optimal layout is the same as that of a HAWT as shown in Figure 9.

Table 4. Optimization results for VAWT wind farm

Optimization Objective	Case III	Case IV
Total Power Output [kW]	53.77	46.06

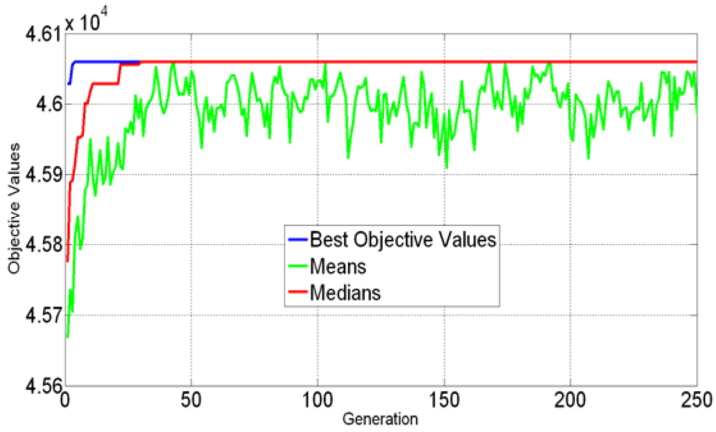


(a) Best, mean and median objective values for Case III

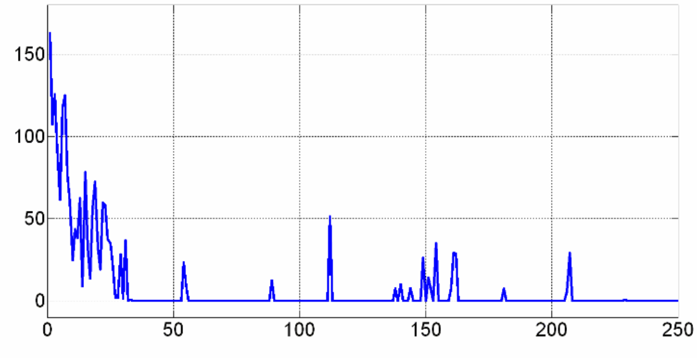


(b) Standard deviations for Case III

Figure 8. Continued

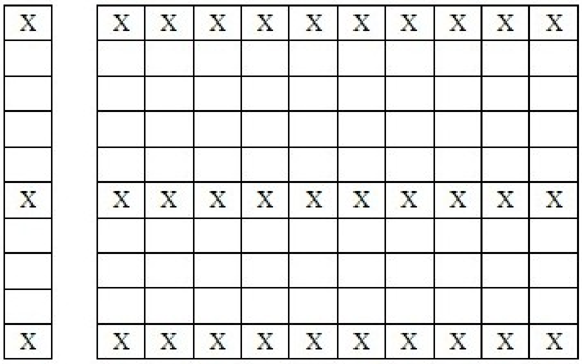


(c) Best, mean and median objective values for Case IV



(d) Standard deviations for Case IV

Figure 8. Convergence history of the objective function (total power/cost) for VAWT wind farm using GA



(a) (b)

Figure 9. Optimal layout of VAWT in a 50D x 50D wind farm

6. Conclusion and future work

The results shown in section 5 demonstrate that the layout optimization of a wind farm of HAWT using GA gives a uniform grid arrangement similar that obtained by Grady et al. [2]; this is different than that obtained by Mosetti [1] who obtained a somewhat random arrangement. The choice of the wake model has no effect on the layout; however there are small differences in the total power output. The use of a more realistic wake model due to Werle [5] is suggested for study of layout of larger HAWT and bigger wind farms. The results obtained for a VAWT wind farm should be considered preliminary because of a very simplified wake model. Additional work needs to be done by taking into account bigger size of VAWT and larger wind farms. In addition, a comparative cost and power study should be performed to determine the relative economics of HAWT and VAWT wind farms.

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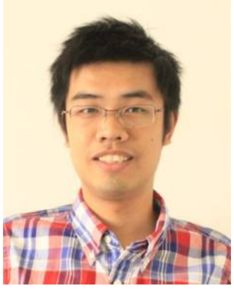

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