



Cooling load optimization of an irreversible refrigerator with combined heat transfer

M. El Haj Assad

Aalto University School of Science and Technology, Department of Energy Technology, P. O. Box 14100, 00076 Aalto, Finland.

Abstract

In this work a mathematical model to study the performance of an irreversible refrigerator has been presented with the consideration of heat exchange by combined convection and radiation. The external irreversibility effects due to finite rate heat transfer as well as the effects of internal dissipations have been considered in the analysis. The relation between the cooling load and the coefficient of performance of the refrigerator has been derived. Furthermore an expression for the maximum cooling rate has been derived. The parameters that affect the cooling load have been investigated. The cooling load has been discussed and the effect of internal irreversibility has been investigated.

Copyright © 2013 International Energy and Environment Foundation - All rights reserved.

Keywords: Finite-time thermodynamics; Cooling load; Convection; Radiation.

1. Introduction

It is practical to design a heat engine or a refrigerator that operates at finite speed in order to produce power or provide cooling. The reversible performance limits of a heat engine to produce power and a refrigerator to provide cooling are not reached in reality and these limits have been considered as the upper performance limits of the real heat engines and refrigerators. The search for more realistic limits was the key to introduce the finite-time thermodynamics which was presented by Curzon and Ahlborn [1]. The heat engine studied in [1] produces not only work but also power and its efficiency (endoreversible efficiency) is less than the reversible Carnot efficiency. Finite-time thermodynamics has been used as efficient tool to analyze the performance of irreversible magnetohydrodynamic (MHD) power plants [2-4]. The general performance characteristics of a refrigerator which takes into account the internal irreversibility as well as the external irreversibility has been investigated [5] by using finite-time thermodynamics and nonlinear heat transfer law. Finite-time thermodynamics has been applied to study the irreversible heat engines with temperature dependent heat capacities of a working fluid [6], to obtain the optimal expansion of a heated working fluid with convective radiative heat transfer [7] and to investigate the effects of variable specific heat ratio of working fluid on the performance of an irreversible Diesel cycle [8]. The optimal performance of an endoreversible steady flow refrigerator cycle consisting of a constant thermal capacity heating branch, a constant thermal capacity cooling branch and two adiabatic branches with heat transfer loss has been analyzed using finite-time thermodynamics [9] in which analytical expressions have been derived for the cooling load and coefficient of performance. The finite-time exergoeconomic performance of an endoreversible Carnot refrigerator with a complex heat transfer law, including generalized convective and generalized radiative heat transfer laws has been

investigated [10]. The optimal ecological objective of an endoreversible Carnot refrigerator based on a new generalized heat transfer law including generalized convective heat transfer law and generalized radiative heat transfer law has been derived [11]. The heating load and coefficient of performance of a steady flow endoreversible heat pump cycle have been optimized using the theory of finite-time thermodynamics [12]. Fine-time exergoeconomic performance of a Newtonian heat transfer law system generalized irreversible combined heat pump with irreversibility of finite-rate heat transfer, heat leakage and internal irreversibility has been presented [13]. The performance of an air-standard Miller cycle has been analyzed using finite-time thermodynamics [14]. An endoreversible intercooled regenerative Brayton cogeneration plant model coupled to constant temperature heat reservoir has been established and the performance of the plant has been investigated using finite-time thermodynamics [15]. A generalized model of a real refrigerator which takes into account internal irreversibilities due to heat leak, friction and turbulence has been presented [16]. The relation between optimal cooling load and coefficient of performance has been derived based on a generalized heat transfer law [17] for the same internal irreversibilities considered in [16]. Generalized models of an endoreversible Carnot refrigerator [18] and irreversible Carnot refrigerator [19] have been developed in order to derive a relationship between cooling load and coefficient of performance based on a new generalized heat transfer law including generalized convective and generalized radiative heat transfer laws. Recently, a model of an irreversible quantum refrigerator with working medium consisting of many non-interacting spin-1/2 system has been presented [20] in which analytical expressions of cooling load and coefficient of performance for the irreversible spin quantum Carnot refrigerator have been derived. More recent works on using finite-time thermodynamics as an effective tool to optimize the performance of a chemical pump [21], a thermoacoustic cooler [22] and an endoreversible Meletis-Georgiou cycle [23] have been presented. From all the above mentioned works, it can be realized that finite-time thermodynamics is an effective and efficient powerful tool to determine and optimize the performance of heat engines, heat pumps and refrigerators.

The objective of this work is to obtain the cooling load and the maximum cooling load of a refrigerator with internal and external irreversibilities working with combined heat transfer including convective and radiative heat transfer.

2. Analysis

The model of an irreversible refrigerator is shown in Figure 1. The cycle operates between the low temperature T_L and the high temperature T_H . The temperatures of the working fluid exchanging heat with the reservoirs at T_L and T_H are T_1 and T_2 , respectively. For simplicity, subscripts L, H, C and R are used for the parameters related to the sides of low and high temperature reservoirs, for the processes of convection and radiation, respectively. The heat Q_L absorbed from the low temperature reservoir by the working fluid per cycle by the combination of two different heat transfer mechanisms, which are Newtonian and radiative heat transfers and given as

$$Q_L = Q_{L_C} + Q_{L_R} \quad (1)$$

Equation (1) can be written as

$$Q_L = \left[A_{L_C} h_{L_C} (T_L - T_1) + A_{L_R} \sigma (\varepsilon_L T_L^4 - \alpha_L T_1^4) \right] t_L \quad (2)$$

where A is the heat transfer area, h is the convection heat transfer coefficient between the working fluid and the reservoir, σ is Stefan-Boltzmann coefficient, ε is the emittance coefficient of the heat source, α is the absorption coefficient of the heat source and t is the time of absorbing heat process. The emittance will be balanced uniformly by absorption for a thin slice of gas, i.e. $\varepsilon = \alpha$.

The radiation heat transfer coefficient between the low temperature heat source at T_L and the working fluid at T_1 can be written as

$$h_{L_R} = \varepsilon_L \sigma (T_L + T_1) (T_L^2 + T_1^2) \quad (3)$$

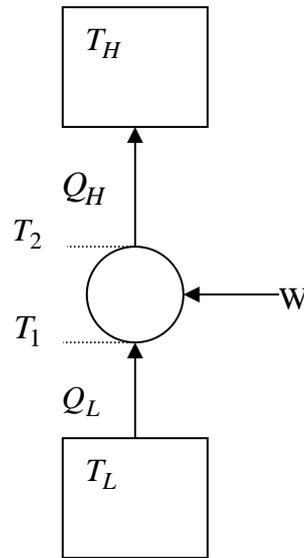


Figure 1. Schematic diagram of a refrigerator

In most engineering application the radiation heat transfer coefficient of equation (3) is preferably used and it can be obtained from $A_{L_R} h_{L_R} (T_L - T_1) = A_{L_R} \varepsilon_L \sigma (T_L^4 - T_1^4)$.

The heat Q_H released to the high temperature reservoir by the working fluid per cycle by the combined Newtonian and radiative heat transfers is

$$Q_H = Q_{H_C} + Q_{H_R} \quad (4)$$

Equation (4) can be rewritten in the following form as

$$Q_H = \left[A_{H_C} h_{H_C} (T_2 - T_H) + A_{H_R} \sigma (\varepsilon_H T_2^4 - \alpha_H T_H^4) \right] t_H \quad (5)$$

where A is the heat transfer area, h is the convection heat transfer coefficient between the working fluid and the reservoir, and t is the time of releasing heat process.

The radiation heat transfer coefficient of the hot side is

$$h_{H_R} = \varepsilon_H \sigma (T_H + T_2) (T_H^2 + T_2^2) \quad (6)$$

With $\theta = \frac{T_H}{T_L}$, $\omega = \frac{T_1}{T_L}$, $\gamma = \frac{T_2}{T_1}$, $\lambda = \frac{h_{L_R}}{h_{L_C}}$, $\delta = \frac{h_{H_R}}{h_{H_C}}$, $A_{L_C} = A_{L_R} = A_L$, $A_{H_C} = A_{H_R} = A_H$ and using

Equations (2), (3), (5) and (6), Q_L and Q_H are rearranged as

$$Q_L = A_L T_L h_{L_C} (1 - \omega)(1 + \lambda) t_L \quad (7)$$

$$Q_H = A_H T_L h_{H_C} (\omega \gamma - \theta)(1 + \delta) t_H \quad (8)$$

Since the refrigerator is internally irreversible due to the internal dissipation of the working fluid, the second law of thermodynamics requires

$$\oint \frac{\delta Q}{T} = \frac{Q_L}{T_1} - \frac{Q_H}{T_2} < 0 \quad (9)$$

Equation (9) can be written in exact form as

$$\frac{Q_L}{T_1} = R \frac{Q_H}{T_2} \quad (10)$$

where R is the irreversibility parameter which accounts of the internal dissipations.

Equation (10) shows that R is equal to one when the refrigerator is internally reversible less than one when the refrigerator is internally irreversible.

The cooling load of the refrigerator and the coefficient of performance are, respectively

$$q = \frac{Q_L}{\tau} \quad (11)$$

$$\beta = \frac{Q_L}{Q_H - Q_L} = \frac{R}{\gamma - R} \quad (12)$$

where τ is the cycle time and it is given by

$$\tau = t_H + t_L \quad (13)$$

With $z = \frac{1 + \beta}{\beta}$, $A = \frac{A_L}{A_H}$ and $H = \frac{h_{LC}}{h_{HC}}$, and using equations (10), (11) and (13), the dimensionless

cooling load q^* of the irreversible refrigerator is obtained as

$$q^* = \frac{q}{A_L T_L h_{LC}} = \frac{(1 - \omega)(1 + \lambda)(1 + \delta)(\omega R z - \theta)R}{R(1 + \delta)(\omega R z - \theta) + AH\lambda(1 - \omega)(1 + \lambda)} \quad (14)$$

The cooling load is maximized by solving $\left(\frac{dq^*}{d\omega}\right)_z = 0$ for a given coefficient of performance and the solution of this derivative gives the following optimum parameter as

$$\omega_{opt} = \frac{\theta + \sqrt{M}}{Rz + \sqrt{M}} \quad (15)$$

From the definition of ω , the optimum temperature T_1 of the working fluid on the cold side is obtained as

$$(T_1)_{opt} = \frac{\theta + \sqrt{M}}{Rz + \sqrt{M}} T_L \quad (16)$$

From the definition of γ and z , and using equation (12), the optimum temperature of the working fluid on the hot side is obtained as

$$(T_2)_{opt} = \frac{\theta + \sqrt{M}}{Rz + \sqrt{M}} R z T_L \quad (17)$$

where

$$M = AH\lambda z \frac{1 + \lambda}{1 + \delta} \quad (18)$$

Using equations (14) and (15), the optimum cooling load is expressed as

$$(q^*)_{opt} = \frac{(1 + \lambda)(1 + \delta)(Rz - \theta)R\sqrt{M}}{R(1 + \delta)\sqrt{M} + AH\lambda(1 + \lambda)} \cdot \frac{1}{Rz + \sqrt{M}} \quad (19)$$

The mathematical formulations required that $wRz - \theta > 0$ as a constraint in order to obtain the optimum cooling load given by equation (19).

3. Results and discussion

The dimensionless cooling load of equation (14) is differentiated with respect to the temperature ratio ω in order to obtain the optimum temperatures of the working fluid T_1 and T_2 . By running the refrigerator at the optimum temperature ratio ω_{opt} , the maximum cooling load of the refrigerator is achieved. The results show the relations between the thermal design parameters and are summarized by figures by considering different values of these thermal design parameters. Figure 2 shows the variation of the refrigerator cooling load with respect to the temperature ratio ω for different internal irreversibility parameter R values. As it is expected, the figure shows that the cooling load is highest when $R = 1$ at a given ω value, i.e. when refrigerator is internally reversible. In Figure 2, it is also shown that when $R = 1$, the cooling load monotonically decreases as ω increases but when $R \neq 1$ the cooling load increases to reach a maximum value and then decreases as ω increases. Figure 3 shows the variation of the cooling load with the temperature ratio ω for different values of the coefficient of performance η . Figure 3 shows that there exists a maximum values of cooling load for $\beta = 2.5$ and $\beta = 3$. For $\beta = 2$, the cooling load monotonically decreases as ω increases. The effect of the ratio of high temperature heat sink to low temperature heat source θ on the cooling load is presented in Figure 4. The figure shows that when $\theta = 1.1$, there exists a maximum cooling load but when $\theta = 1.02$ and $\theta = 1.05$, there is no maximum cooling load and the cooling load decreases as ω increases. Figure 4 shows that the refrigerator has better performance at lower θ values for a given ω .

Figures 5 and 6 show the effects of heat conductance ratio H and heat transfer area ratio A on the cooling load, respectively. Both parameters, H and A , have the same effect on the cooling load as can be seen in equation (14). In both figures there exists a maximum cooling load. Figures 5 and 6 show that the refrigerator has the best performance at low H and A values, in other words, it is more efficient to operate the refrigerator at higher heat conductance and higher heat transfer area on the hot working fluid side. Figure 7 shows the variation of the cooling load with respect to the temperature ratio ω for different δ (ratio of the radiation heat transfer coefficient to heat conductance on the hot working fluid side) values. It is shown in Figure 7 that higher cooling load is achieved at higher δ values, in other words, if the radiation heat transfer coefficient is higher than the heat conductance on the hot working fluid side, the cooling load increases.

Moreover Figure 7 shows that there is a maximum cooling load for all δ values considered in this study. The variation of the cooling load with respect to the temperature ratio ω for different values of λ (ratio of the radiation heat transfer coefficient to heat conductance on the cold working fluid side). Figure 8 shows that for $\omega \leq 0.945$, a decrease in λ yields in an increase in the cooling load but for $\omega \geq 0.96$,

$\lambda = 0.5$ yields the lowest cooling load at a given ω value. The parameters used in obtaining the results of this study are very close to real refrigerators design parameters available in practice.

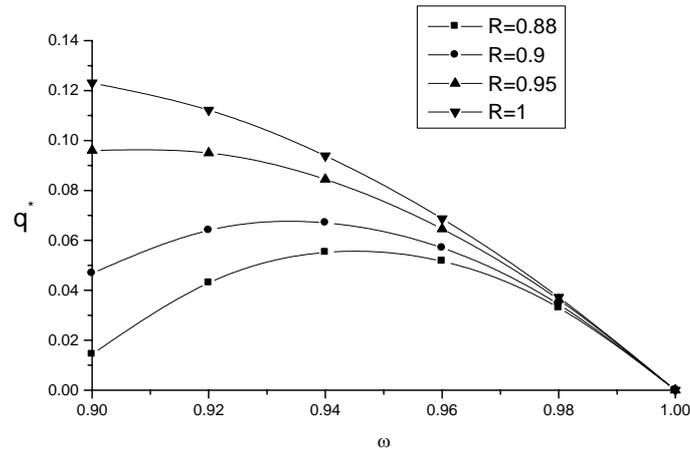


Figure 2. Variation of cooling load with respect to ω for different R values ($\beta = 2.5, \theta = 1.1, H = 1, A = 1, \delta = 1, \lambda = 1$)

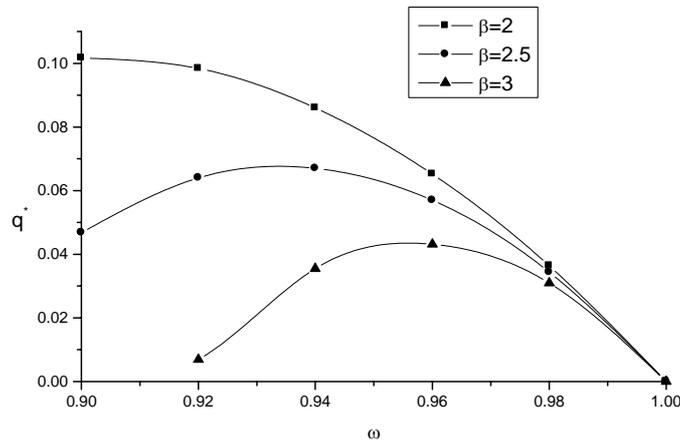


Figure 3. Variation of cooling load with respect to ω for different η values ($R = 0.9, \theta = 1.1, H = 1, A = 1, \delta = 1, \lambda = 1$)

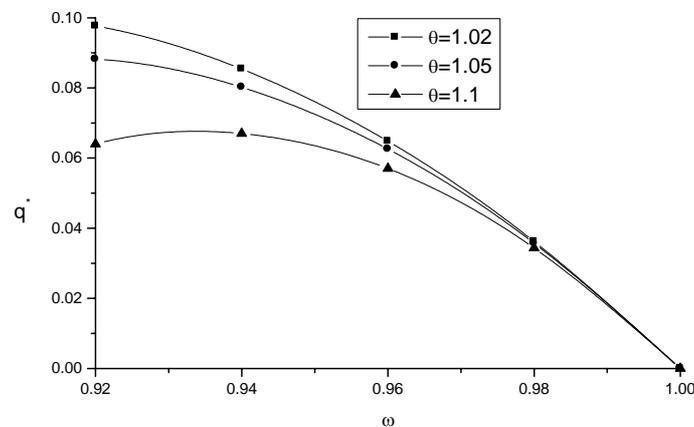


Figure 4. Variation of cooling load with respect to ω for different θ values ($R = 0.9, \beta = 2.5, H = 1, A = 1, \delta = 1, \lambda = 1$)

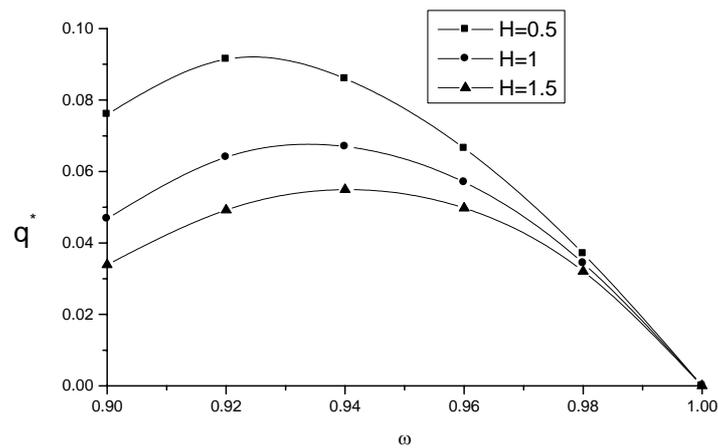


Figure 5. Variation of cooling load with respect to ω for different H values ($R = 0.9$, $\beta = 2.5$, $\theta = 1.1$, $A = 1$, $\delta = 1$, $\lambda = 1$)

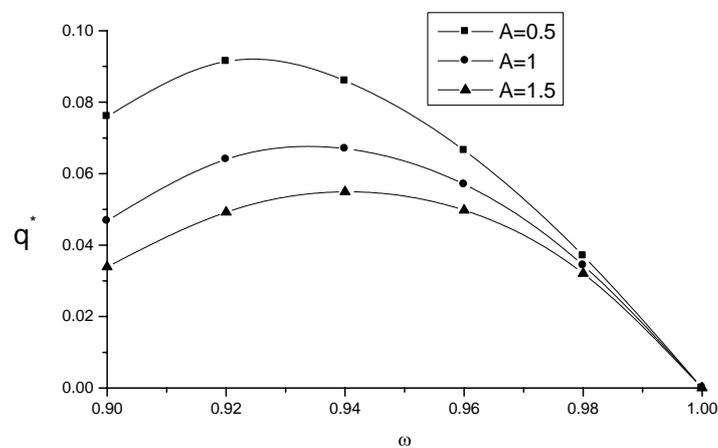


Figure 6. Variation of cooling load with respect to ω for different A values ($R = 0.9$, $\beta = 2.5$, $\theta = 1.1$, $H = 1$, $\delta = 1$, $\lambda = 1$)

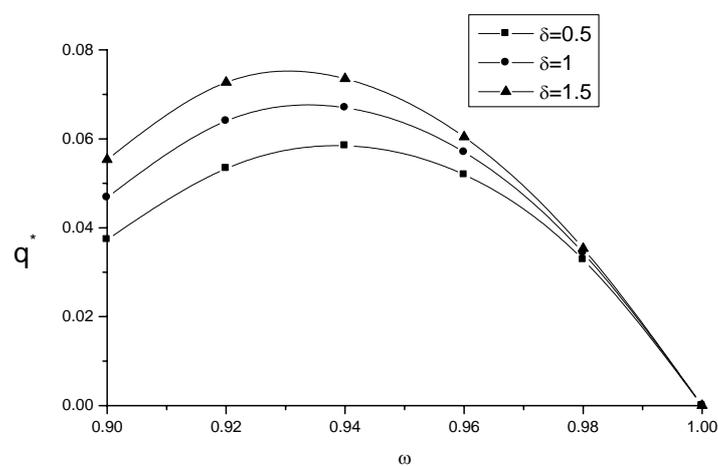


Figure 7. Variation of cooling load with respect to ω for different δ values ($R = 0.9$, $\beta = 2.5$, $\theta = 1.1$, $H = 1$, $A = 1$, $\lambda = 1$)

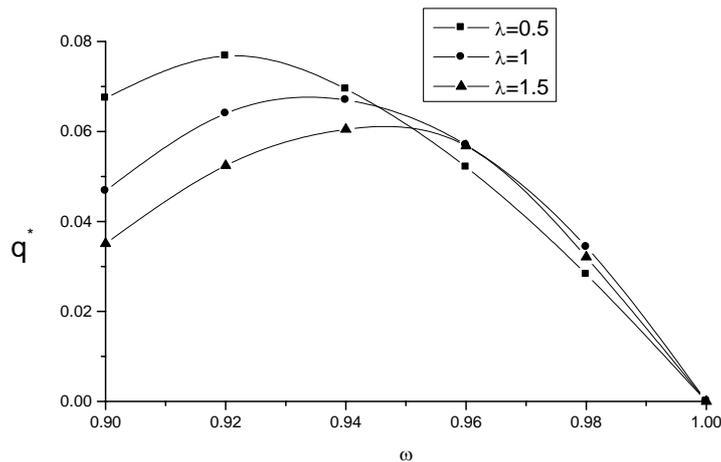


Figure 8. Variation of cooling load with respect to ω for different λ values ($R = 0.9$, $\beta = 2.5$, $\theta = 1.1$, $H = 1$, $A = 1$, $\delta = 1$)

4. Numerical example

Consider a real refrigerator that is running between $T_L = 263 \text{ K}$ and $T_H = 289.3 \text{ K}$. The refrigerator is set to operate at coefficient of performance (COP) $\beta = 2.5$. Consider that the refrigerator has the following design parameters: $A = 1$, $H = 1$, $\delta = 1$ and $\lambda = 1$. The internal irreversibility parameter is taken $R = 0.95$. Consider that the thermal conductance of the heat exchanger as $A_L h_{LC} = 20 \text{ kW/K}$. Applying the model equations of this paper yield $(T_1)_{opt} = 238.8 \text{ K}$ and $(T_2)_{opt} = 317.6 \text{ K}$ and $(q)_{opt} \cong 510 \text{ kW}$. The Carnot COP (coefficient of performance) can be obtained by using

$$\beta_{Carnot} = \frac{T_L}{T_H - T_L} \text{ as } \beta_{Carnot} = 10 \text{ which is much higher than the real COP } (\beta = 2.5). \text{ This explains}$$

why the real industrial refrigerators have lower COP than that of the Carnot refrigerator.

5. Conclusion

In this study, the effect of combined heat transfer on the performance of an irreversible refrigerator has been investigated and the effects of thermal design parameters have been studied. The effects of these thermal design parameters have been summarized in figures. Analytical expressions of the cooling load and the maximum cooling load have been obtained. It has been shown that the maximum cooling load of an irreversible heat transfer has strong dependence on the thermal design parameters beside the irreversibilities. It has been shown that the internal irreversibilities should be eliminated and the refrigerator should operate at low θ value in order to improve the cooling performance of the refrigerator. Moreover, it has been shown that the refrigerator should operate at low H and A values, and at high δ values. The refrigerator should also operate at $\lambda < 1$ for $\omega \leq 0.945$ and at $\lambda = 1$ for $\omega \geq 0.96$. The results of the numerical example show that the irreversible refrigerator give more realistic performance characteristics (lower COP) than that of the Carnot refrigerator since the former takes into account both the internal as well as the external irreversibilities. It is believed that the present study is very efficient and practical for engineers in the field of refrigeration systems design due to importance of the analytical expressions obtained in this study and illustrated in the figures.

References

- [1] Curzon F. L., Ahlborn B. Efficiency of a Carnot engine at maximum power output. American J. Physics, 1975, 43(1): 22-24.
- [2] El Haj Assad M. Thermodynamic analysis of an irreversible MHD power plant. Int. J. Energy Research, 2000, 24(10): 865-875.

- [3] El Haj Assad M., Wu C. Thermodynamic performance of an irreversible MHD power cycle running at constant Mach number. *Int. J. Ambient Energy*, 2008, 29(1): 27-34.
- [4] El Haj Assad M., Wu C. Performance of a regenerative MHD power plant. *Int. J. Power and Energy Systems*, 2004, 24(2): 98-103.
- [5] El Haj Assad M. Performance characteristics of an irreversible refrigerator. *Recent Advances in Finite-Time Thermodynamics*, Editors: Chih Wu, Lingen Chen and Jincan Chen, New York: Nova Science Publishers, 1999.
- [6] Liu J., Chen J. Optimum performance analysis of a class of typical irreversible heat engines with temperature-dependent heat capacities of the working substance. *Int. J. Ambient Energy*, 2010, 31(2): 59-70.
- [7] Chen L., Song H., Sun F., Wu C. Optimal expansion of a heated working fluid with convective radiative heat transfer law. *Int. J. Ambient Energy*, 2010, 31(2): 81-90.
- [8] Ebrahimi R., Chen L. Effects of variable specific heat ratio of working fluid on the performance of an irreversible Diesel cycle. *Int. J. Ambient Energy*, 2010, 31(2): 101-108.
- [9] Kan X., Chen L., Sun F., Wu F., Wu C. Optimal performance for a steady endoreversible refrigerator model including five typical cycles. *Int. J. Ambient Energy*, 2011, 32(1): 18-24.
- [10] Chen L., Li J., Sun F., Wu C. Exergoeconomic performance of an endoreversible Carnot refrigerator with a complex heat transfer law. *Int. J. Ambient Energy*, 2011, 32(1): 25-30.
- [11] Li J., Chen L., Sun F., Wu C. Ecological performance of an endoreversible Carnot refrigerator with complex heat transfer law. *Int. J. Ambient Energy*, 2011, 32(1): 31-36.
- [12] Feng H., Chen L., Sun F., Wu C. Heating load and COP optimizations for a universal steady endoreversible heat pump model. *Int. J. Ambient Energy*, 2011, 32(2): 70-77.
- [13] Ma K., Chen L., Sun F., Wu C. Finite-time exergoeconomic performance analysis and optimization for generalized irreversible combined heat pump cycles. *Int. J. Ambient Energy*, 2011, 32(2): 78-86.
- [14] Chen L., Ge Y., Sun F., Wu C. Finite-time thermodynamic modeling and analysis for an irreversible Miller cycle. *Int. J. Ambient Energy*, 2011, 32(2): 87-94.
- [15] Chen L., Yang B., Sun F., Wu C. Exergetic performance optimization of an endoreversible Brayton cogeneration plant, Part 1: thermodynamic model and parametric analysis. *Int. J. Ambient Energy*, 2011, 32(3): 116-123.
- [16] Chen L., Sun F., Wu C., Kiang R.L. A generalized model of a real refrigerator and its performance. *Applied Thermal Engineering*, 1997, 17(4): 401-412.
- [17] Chen L., Sun F., Wu C. Effect of heat transfer law on the performance of a generalized irreversible Carnot refrigerator. *J. Non-Equilibrium Thermodynamics*, 2001, 26(3): 291-304.
- [18] Li J., Chen L., Sun F. Performance optimization for an endoreversible Carnot refrigerator with complex heat transfer law. *J. Energy Institute*, 2008, 81(3): 168-170.
- [19] Li J., Chen L., Sun F. Cooling load and coefficient of performance optimizations for a generalized irreversible Carnot refrigerator with heat transfer law. *Proceeding of the Institution of Mechanical Engineers, Part E: Journal of Process Mechanical Engineering*, 2008, 222(E1):55-62.
- [20] Liu X., Chen L., Wu F., Sun F. Cooling load and COP optimization of an irreversible Carnot refrigerator with spin-1/2 systems. *Int. J. Energy and Environment*, 2011, 2(5):797-812.
- [21] Xia D., Chen L., Sun F. Endoreversible four-mass-reservoir chemical pump with diffusive mass transfer law. *Int. J. Energy and Environment*, 2011, 2(6):975-984.
- [22] Chen L., Kan X., Wu F., Sun F. Finite time exergoeconomic performance of a thermoacoustic cooler with a complex heat transfer exponent. *Int. J. Energy and Environment*, 2012, 3(1):19-32.
- [23] Liu C., Chen L., Sun F. Endoreversible Meletis-Georgiou cycle. *Int. J. Energy and Environment*, 2012, 3(2):305-322.



Mamdouh El Haj Assad is a teaching research scientist of Energy Technology at Aalto University School of Science and Technology. He received his Ph.D. in Applied Thermodynamics from Aalto University in 1998. Dr. Assad studied at Middle East Technical University, Turkey, where he obtained his B.Sc. and M.Sc. degrees. His research interests include irreversible thermodynamics, heat exchangers and energy conversion systems.
E-mail address: mamdouh.assad@aalto.fi