Short term pumped storage scheduling using two proposed techniques

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Abstract

In this paper, a genetic algorithm and constriction factor based particle swarm optimization technique are proposed for solving the short term pumped storage hydro thermal scheduling problem. The performance efficiency of the proposed techniques is demonstrated on hydrothermal test system comprising of five thermal units and one pumped storage power plant. A wide range of thermal and hydraulic constraints are taken into consideration such as real power balance constraint, minimum and maximum limits of thermal units and pumped storage power plant, water discharge and water pumping rate limits and reservoir storage volume constraints. The simulation results obtained from the constriction factor based particle swarm optimization technique are compared with the outcomes obtained from the genetic algorithm in terms of cost saving and execution time to reveal the validity and verify the feasibility of the proposed methods. The test results show that the constriction factor based particle swarm optimization technique performs better than genetic algorithm in solving this problem in terms of cost saving and computational time.

Keywords: Hydrothermal generation scheduling; Pumped storage (PS); Genetic algorithm (GA); Constriction factor based particle swarm optimization (CFPSO).

1. Introduction

The hydrothermal generation scheduling plays an important role in the operation and planning of a power system. Since the operating cost of thermal power plant is very high compared to the operating cost of hydro power plant, the integrated operation of the hydro and thermal plants in the same grid has become the more economical [1]. The main objective of the short term pumped storage hydro thermal coordination problem is to determine the optimal generation schedules of both thermal units and pumped storage power plants that minimize the total operating cost of the system during a scheduling period, while satisfying various thermal and hydraulic constraints. The hydrothermal generation scheduling is mainly concerned with both hydro unit scheduling and thermal unit dispatching. The hydrothermal generation scheduling problem is more difficult than the scheduling of thermal power systems. Since there is no fuel cost associated with the hydro power generation, the problem of minimizing the total production cost of hydrothermal scheduling problem is achieved by minimizing the fuel cost of thermal power plants under the constraints of water available for the hydro power generation in a given period of time [2]. In short term pumped storage scheduling problem, the generating unit limits and the load...
demand over the scheduling interval is known. The pumped storage scheduling problem is a complex mixed integer, non-linear optimization problem. In the past, several traditional mathematical optimization techniques have been used to solve short term pumped storage hydro thermal scheduling problems [3-6]. In these techniques, the pumped storage scheduling problem is decomposed into thermal and hydro sub problems, which are usually coordinated by the Lagrange multipliers. In these conventional methods simplifying assumptions are made in order to make the optimization problem more tractable. Thus, most of conventional optimization techniques are unable to produce optimal or near optimal solution of this kind of problems. The computational time of these methods increases with the increase of the dimensionality of the problem. The most common optimization techniques based upon artificial intelligence concepts such as evolutionary programming [7, 8], simulated annealing [9, 10], differential evolution [11], artificial neural network [12], genetic algorithm [13-15] and particle swarm optimization [16-19] have been given attention by many researchers due to their ability to find an almost global or near global optimal solution for short term hydrothermal scheduling problems with operating constraints. Major problem associated with these techniques is that appropriate control parameters are required. Sometimes these techniques take large computational time due to improper selection of the control parameters.

A global optimization technique known as a particle swarm optimization (PSO) has a candidate for many optimization applications due to its high performance and flexibility. The PSO is a population based optimization technique first proposed by Kennedy and Eberhart in 1995. In PSO, each particle is a candidate solution to the problem. Each particle in PSO makes its decision based on its own experience together with other particles experiences. Particles approach to the optimum solution through its present velocity, previous experience and the best experience of its neighbors [20]. Compared to other evolutionary computation techniques, PSO can solve the problems quickly with high quality solution and stable convergence characteristic, whereas it is easily implemented.

The genetic algorithm (GA) is a stochastic global search and optimization method that mimics the metaphor of natural biological evolution such as selection, crossover and mutation. GA is started with a set of candidate solutions called population (represented by chromosomes). At each generation, pairs of chromosomes of the current population are selected to mate with each other to produce the children for the next generation. The chromosomes which are selected to form the new offspring are selected according to their fitness. In general, the chromosomes with higher fitness values have higher probability to reproduce and survive to the next generation. While the chromosomes with lower fitness values tend to be discarded. This process is repeated until a termination condition is reached (for example maximum number of generations). Most of the GA parameters are set after considerable experimentation and the major drawback of this method is the lack of a solid theoretical basis for their setting.

2. Modeling of pumped storage power plant
The main function of hydro power plant is to store cheap surplus electric energy that is available during low load time periods as hydraulic potential energy. This is achieved by pumping water from the lower reservoir to the upper reservoir. The stored potential energy is then used to generate electric energy during peak load periods to save fuel costs of thermal power plants. A schematic diagram of pumped storage hydro power plant is shown in Figure 1.

Pumped storage power plant can be operated in pumping mode or in generating mode as follows:

i. During low load periods: Water is pumped from the lower reservoir to the upper reservoir and the plant is used in pumping mode. Thus increasing overall system load. In this mode, the power generation of pumped storage power plant has a negative quantity.

ii. During high load periods: Water stored in the upper reservoir is released to generate power and the plant is used in generating mode. Thus, decreasing the overall system load that must be met by thermal power plants. This is beneficial because it avoids the need to start some of the expensive peaking plants. In this mode, the output power of pumped storage power plant has a positive quantity.

In generating mode, the input-output characteristic of pumped storage power plant is similar to that of conventional hydro electric power plants. The output power generation of pumped storage power plants is a function of water discharge rate through the turbines and the effective net head of the upper reservoir.

The general form of the output power generation can be represented as follows:
where $P^t_{hj,g}$ is the Power generation of pumped storage plant $j$ in the time interval $t$, $q^t_{hj,g}$ is the Water discharge of pumped storage plant $j$ in the time interval $t$ and $v^t_{hj,u}$ is the Water volume of the upper reservoir of plant $j$ at the time interval $t$.

Figure 1. Pumped storage power plant [16]

3. Objective function and operational constraints

The main objective of pumped storage hydro thermal coordination problem is to determine the optimal generation schedules of both thermal units and pumped storage power plants that minimize the total operating cost of the system during a scheduling period (Typically one day), while satisfying various thermal and hydro constraints.

The pumped storage hydro thermal scheduling problem can be formulated as a mathematical constrained non linear optimization problem as follows:

$$F_T = \sum_{t=1}^{T} \sum_{i=1}^{N} n F^t_{gi}(p^t_{gi})$$

In general, the fuel cost function of thermal generating unit $i$ at time interval $t$ can be expressed as a quadratic function of real power generation as follows:

$$f^t_{gi}(p^t_{gi}) = a_i (p^t_{gi})^2 + b_i p^t_{gi} + c_i$$

where $p^t_{gi}$ is the real output power of thermal generating unit $i$ at time interval $t$ in (MW), $f^t_{gi}(p^t_{gi})$ is the operating fuel cost of thermal unit $i$ in ($$/hr), $F_T$ is the total fuel cost of the system in ($), $T$ is the total number of time intervals for the scheduling horizon, $n$ is the numbers of hours in scheduling time interval $t$, $N$ is the total number of thermal generating units, $a_i$, $b_i$ and $c_i$ are the fuel cost coefficients of thermal generating unit $i$.

The minimization of the objective function of short term pumped storage hydro thermal scheduling problem is subject to a number of thermal and hydraulic constraints. These constraints include the following:

1) Active power balance constraint:
   - In the generation mode: The total active power generation from the hydro and thermal plants must equal to the total load demand plus transmission line losses at each time interval over the scheduling period.
In the pumping mode: The pumped storage power plant consumes power from the electrical grid and the overall system load demand will be increased. The power balance equation becomes as follow:

\[
\sum_{i=1}^{N} P^i_{gl} + \sum_{j=1}^{M_p} P^t_{jg} = P_{Dt} + P_{Lt},
\]

(4)

For simplicity, the transmission power loss is neglected in this paper.

Where, \( P_{Dt} \) is the total load demand during the time interval \( t \) in (MW), \( P_{Lt} \) represents the total transmission line losses during the time interval \( t \) in (MW), \( M_p \) is the total number of pumped storage power plants, \( P^t_{jg} \) is the power generation of pumped storage plant \( j \) at time interval \( t \) in (MW), \( P^i_{gl} \) is the power generation of thermal generating unit \( i \) at time interval \( t \) in (MW) and \( P^t_{hj,p} \) is the pumping power of the pumped storage plant \( j \) at time interval \( t \) in (MW).

2) Thermal generator limits constraint:
The output power generation of thermal power plant must lie in between its minimum and maximum limits. The inequality constraint for each thermal generator can be expressed as:

\[
P_{gi}^{\min} \leq P^t_{gi} \leq P_{gi}^{\max}
\]

(6)

where \( P_{gi}^{\min} \) and \( P_{gi}^{\max} \) are the minimum and maximum power outputs of thermal unit \( i \) in (MW), respectively.

3) Pumped storage plant limits constraint:
- In the generation mode: The output power generation of pumped storage hydro plant must lie in between its minimum and maximum bounds. The inequality constraint for pumped storage plant in generation mode can be defined as:

\[
P_{hj,g}^{\min} \leq P^t_{hj,g} \leq P_{hj,g}^{\max}
\]

(7)

where \( P_{hj,g}^{\min} \) is the minimum power generation of pumped storage plant \( j \) in (MW) and \( P_{hj,g}^{\max} \) is the maximum power generation of pumped storage plant \( j \) in (MW).
- In the pumping mode: The pumping power of pumped storage hydro power plant must lie in between its upper and lower bounds. The inequality constraint for pumped storage plant in generation mode can be defined as:

\[
P_{hj,p}^{\min} \leq P^t_{hj,p} \leq P_{hj,p}^{\max}
\]

(8)

where \( P_{hj,p}^{\min} \) is the minimum power generation of pumped storage plant \( j \) in pumping mode and \( P_{hj,p}^{\max} \) is the maximum power generation of pumped storage plant \( j \) in pumping mode.

4) Water discharge rate limit constraint:
The water Discharge rate of pumped storage power plant must lie in between its minimum and maximum operating limits.

\[
q_{hj,g}^{\min} \leq q^t_{hj,g} \leq q_{hj,g}^{\max}
\]

(9)

where \( q_{hj,g} \) is the water discharge of pumped storage plant \( j \) at the time interval \( t \), \( q_{hj,g}^{\min} \) is the minimum water discharge rate of pumped storage plant \( j \) and \( q_{hj,g}^{\max} \) is the maximum water discharge rate of pumped storage plant \( j \).
5) Water pumping rate limit:
The water pumping rate of pumped storage power plant must lie in between its lower and upper operating limits.

\[ q_{hj,p}^{\text{min}} \leq q_{hj,p}^{t} \leq q_{hj,p}^{\text{max}} \]  \hspace{1cm} (10)

where \( q_{hj,p}^{t} \) is the water pumping of pumped storage plant \( j \) at the time interval \( t \), \( q_{hj,p}^{\text{min}} \) is the minimum water pumping rate of pumped storage plant \( j \) and \( q_{hj,p}^{\text{max}} \) is the maximum water pumping rate of pumped storage plant \( j \).

6) Reservoir storage volumes constraint:
The operating volume of upper and lower reservoir limits must lie in between the minimum and maximum capacity limits.

\[ V_{hj,u}^{\text{min}} \leq V_{hj,u}^{t} \leq V_{hj,u}^{\text{max}} \]  \hspace{1cm} (11)

\[ V_{hj,L}^{\text{min}} \leq V_{hj,L}^{t} \leq V_{hj,L}^{\text{max}} \]  \hspace{1cm} (12)

where \( V_{hj,u}^{t} \) is the water volume of the upper reservoir of plant \( j \) at the end of time \( t \), \( V_{hj,u}^{\text{min}} \) and \( V_{hj,u}^{\text{max}} \) are the minimum and maximum storage volume of upper reservoir of plant \( j \); \( V_{hj,L}^{t} \) is the water volume of the lower reservoir of plant \( j \) at the end of time \( t \), \( V_{hj,L}^{\text{min}} \) and \( V_{hj,L}^{\text{max}} \) are the minimum and maximum storage volume of lower reservoir of plant \( j \).

7) Water Dynamic Balance Constraint:
The water continuity equation relates the previous interval water storage in reservoirs with the current storage. The water continuity equation can be represented as:

\[ V_{hj,u}^{t} = V_{hj,u}^{t-1} + I_{hj}^{t} - q_{hj,p}^{t} + s_{hj}^{t} \]  \hspace{1cm} (13)

\[ V_{hj,L}^{t} = V_{hj,L}^{t-1} + q_{hj,g}^{t} - q_{hj,p}^{t} + s_{hj}^{t} \]  \hspace{1cm} (14)

where \( I_{hj}^{t} \) is the water inflow rate into the upper reservoir of pumped storage plant \( j \) at time interval \( t \) and \( s_{hj}^{t} \) is the water spillage from the upper reservoir of plant \( j \) at time interval \( t \). For simplicity, the spillage is neglected in this paper.

8) Initial and End Upper Reservoir Storage Volume Limit:
This constraint implies that the desired volume of water to be discharged by the upper reservoir over the scheduling period should be in limit.

\[ V_{hj}^{0} = V_{hj}^{\text{begin}} = V_{hj}^{\text{max}} \]  \hspace{1cm} (15)

\[ V_{hj}^{T} = V_{hj}^{\text{end}} \]  \hspace{1cm} (16)

where \( V_{hj}^{\text{begin}} \) is the initial stored water volume in the upper reservoir of plant \( j \) and \( V_{hj}^{\text{end}} \) is the final stored water volume in the upper reservoir of plant \( j \).

For the case study in this paper, the starting and ending water reservoir volume of the pumped storage power plant are the same, thus, the total amount of water used for generation must be equal to the total amount of water pumped. Hence the total net water amount used by the pumped storage power plant must be zero.
\[ q_{\text{spent}}^{\text{tot}} - q_{\text{pump}}^{\text{tot}} = q_{\text{net}}^{\text{tot}} = 0 \]  

\[ q_{\text{spent}}^{\text{tot}} = \sum_{t=1}^{T_g} q_{hj,g}^{t} \times nt \]  

\[ q_{\text{pump}}^{\text{tot}} = \sum_{t=1}^{T_p} q_{hj,p}^{t} \times nt \]

where \( q_{\text{spent}}^{\text{tot}} \) is the total water amount which is spent for generation, \( q_{\text{pump}}^{\text{tot}} \) is the total amount of pumped water, \( q_{\text{net}}^{\text{tot}} \) is the total water amount used by the pumped storage plant during operation cycle, \( T_g \) is sets which contains all time intervals where the pumped storage unit is operated in generation mode and \( T_p \) is sets which contains all time intervals where the pumped storage plant is operated in pumping mode.

4. Overview of genetic algorithm

The GA is a method for solving optimization problems that is based on natural selection, the process that drives biological evolution. The general scheme of GA is initialized with a population of candidate solutions (called chromosomes). Each chromosome is evaluated and given a value which corresponds to a fitness level in problem domain. At each generation, the GA selects chromosomes from the current population based on their fitness level to produce offspring. The chromosomes with higher fitness levels have higher probability to become parents for the next generation, while the chromosomes with lower fitness levels to be discarded. After the selection process, the crossover operator is applied to parent chromosomes to produce new offspring chromosomes that inherit information from both sides of parents by combining partial sets of genes from them. The chromosomes or children resulting from the crossover operator will now be subjected to the mutation operator in final step to form the new generation. Over successive generations, the population evolves toward an optimal solution. A schematic outline of simple genetic algorithm is illustrated in Figure 2.

![Figure 2. Schematic outline of simple genetic algorithm](image)

4.1 Genetic algorithm parameters

The performance of GA depends on choice of GA parameters such as:

i. Population size (Np): The population size affects the efficiency and performance of the algorithm. Higher population size increases its diversity and reduces the chances of premature converge to a local optimum, but the time for the population to converge to the optimal regions in the search space will also
increase. On the other hand, small population size may result in a poor performance from the algorithm. This is due to the process not covering the entire problem space. A good population size is about 20-30, however sometimes sizes 50-100 are reported as best.

ii. Crossover rate: The crossover rate is the parameter that affect the rate at which the process of crossover is applied. This rate generally should be high, about 80-95%.

iii. Mutation rate: It is a secondary search operator which increases the diversity of the population. Low mutation rate helps to prevent any bit position from getting trapped at a single value, whereas high mutation rate can result in essentially random search. This rate should be very low.

5. Genetic algorithm applied to short term pumped storage scheduling problem

Implementation of the pumped storage scheduling problem in a genetic algorithm starts from the parameter encoding of the control variables. In GA binary representation, the water discharge rate is used as a control variable rather than the output power generation of hydro units because the encoded parameter is more beneficial for dealing with water dynamic balance constraints.

5.1 Chromosome encoding of pumped storage power plant

Figure 3 presents the encoding chromosome that translated the control variable water discharge rate of pumped storage power plant into their binary representation. Each chromosome string contains 24 genes that represent the solution for hourly discharge/pumping schedules of the pumped storage power plant during 24 hours period. Each gene is assigned the same number of five bits. The first bit is used to identify whether the pumped storage plant is in generating or in pumping mode. The remaining four bits are used to represent the normalized water discharge in the generating mode or the number of pumping units in pumping mode.

![Figure 3. Chromosome encoding of pumped storage power plant](image)

5.2 Chromosome decoding of pumped storage power plant

The decoding procedure of the encoded chromosome string can be summarized in the following steps as follow:

Step 1: Decode the first bit of a gene to identify whether the pumped storage plant is in generating or pumping mode.
- If in pumping mode, go to step 2.
- If in generating mode go to step 5.

Step 2: Decode the remaining bits of each gene to calculate the number of pumping units. Each bit of "1" indicates on pumping unit. Figure 4 shows the decoding scheme of pumped storage power plant.
- If a0 =0: pumping mode.
- If a0 =1: generating mode.

Step 3: Evaluate the total volume of pumping water.

Step 4: Evaluate the pumping power of plant.

Step 5: Decode the remaining four bits of each gene and calculate the actual value of water discharge according to the following equation:

\[
q_{bj} = q_{bj}^{min} + \left( q_{bj}^{max} - q_{bj}^{min} \frac{2^{L-1}}{2^{L-1}} \right) \times d_i
\]  

(20)

where \( q_{bj}^{min} \) is the minimum value of discharge rate through pumped storage plant \( j \), \( q_{bj}^{max} \) is the maximum value of discharge rate through pumped storage plant \( j \), \( L \) is the String length (number of bits used for encoding water discharge rate of each pumped storage plant) and \( d_i \) is the binary coded value of the string (decimal value of string).

Step 6: Calculate the output power generation of pumped storage plant.
Step 7: Calculate the remaining thermal demand by subtracting the generation of hydro units from the total load demand. The thermal demand (total load – hydro generation) must be covered by the thermal units. The thermal generations are calculated from the power balance equation as follows:

$$\sum_{i=1}^{N} P_{g_i}^{t} = \sum_{j=1}^{M} P_{h_j}^{t} - D_{t}\quad (21)$$

Step 8: Repeat the above steps from hour 1 to hour 24.

Step 9: Calculate the output power of each thermal unit by solving economic load dispatch problem.

Step 10: Evaluate the fitness value for each string in the population by using the objective function as follow:

$$F_{i}(P_{gi}^{t})=a_{i}(P_{gi}^{t})^2+b_{i}P_{gi}^{t}+c_{i}\quad (22)$$

Step 11: Repeat the above steps for each chromosome in the population.

6. Algorithm for short term pumped storage scheduling problem using GA method

The sequential steps for solving short term pumped storage scheduling problem by using genetic algorithm are explained as follows:

Step 1: Read the system input data, namely fuel cost curve coefficients, power generation limits of pumped storage plants and thermal units, number of thermal units, number of pumped storage units, power demands, water discharge rate limits, starting and ending water volumes of the upper reservoir and water volume limits of the upper reservoir.

Step 2: Select genetic algorithm parameters such as population size, length of string, probability of crossover, probability of mutation and maximum number of generations to be performed.

Step 3: Generate the initial population randomly in the binary form (set of discharge values for each pumped storage plant over the scheduling period as shown in Figure 3). The initial population must be feasible candidate solutions that satisfy the practical operation constraints of all thermal and hydro units.

Step 4: Decode the first bit of a gene to identify whether the pumped storage plant is in generating or pumping mode.

• If in pumping mode, go to step 5.
• If in generating mode go to step 7.

Step 5: Calculate the total volume of water pumped to the upper reservoir according to the following equation:

$$V_{hj,u}^{t-1} = V_{hj,u}^{t} + Q_{hj,p}^{t}\quad (23)$$

Step 6: Calculate the pumping power of the pumped storage power plant.

Step 7: Calculate the water discharge rate of the pumped storage power plant by decoding the remaining bits of a gene according to equation (20).

Step 8: Calculate the output power generation of the pumped storage power plant.

Step 9: Calculate the total volume of water discharged from the upper reservoir according to the following equation:

$$V_{hj,g}^{t} = V_{hj,u}^{t-1} \cdot Q_{hj,g}^{t}\quad (24)$$
Step 10: Calculate the thermal demand by subtracting the generation of hydro units from the total load demand. The thermal demand (total load – hydro generation) must be covered by the thermal units. The thermal generations are calculated from the power balance equation defined in equation (21).

Step 11: If \( t \leq T \) go to step 4, otherwise go to step 12.

Step 12: Calculate the output power of each thermal unit by solving economic load dispatch problem.

Step 13: Evaluate the fitness value for each string in the population by using the objective function described in equation (22).

Step 14: Repeat the above steps for each chromosome in the population.

Step 15: The chromosomes with higher fitness values are selected to become parents for the next generation.

Step 16: Perform the crossover operator to parent chromosomes to create new offspring chromosomes.

Step 17: The mutation operator is applied to the new offspring resulting from the crossover operation to form the new generation.

Step 18: Update the population.

Step 19: If the number of iterations reached the maximum, then go to step 20. Otherwise go to step 4.

Step 20: The string that generates the minimum total fuel cost of the system is the optimal solution of the problem.

Step 21: Print the pumped storage scheduling results and stop.

7. Constriction factor based particle swarm optimization technique

7.1 Overview of particle swarm optimization

Particle swarm optimization (PSO) is a population based stochastic optimization technique, inspired by social behavior of bird flocking or fish schooling. It is one of the most modern heuristic algorithms, which can be used to solve non linear and non continuous optimization problems. PSO shares many similarities with evolutionary computation techniques such as genetic algorithm (GA). The system is initialized with a population of random solutions and searches for optima by updating generations. However, unlike GA, PSO has no evolution operators such as mutation and crossover. The PSO algorithm searches in parallel using a group of random particles. Each particle in a swarm corresponds to a candidate solution to the problem. Particles in a swarm approach to the optimum solution through its present velocity, its previous experience and the experience of its neighbors. In every generation, each particle in a swarm is updated by two best values. The first one is the best solution (best fitness) it has achieved so far. This value is called Pbest. Another best value that is tracked by the particle swarm optimizer is the best value, obtained so far by any particle in the population. This best value is a global best and called gbest. Each particle moves its position in the search space and updates its velocity according to the following equation:

\[
V_{i}^{k+1} = \omega \times V_{i}^{k} + c_1 \times r_1 \times (P_{best_{i}} - X_{i}^{k}) + c_2 \times r_2 \times (g_{best} - X_{i}^{k})
\]

where \( V_{i}^{k} \) is the velocity of particle \( i \) at iteration \( k \), \( X_{i}^{k} \) is the position of particle \( i \) at iteration \( k \), \( \omega \) is the inertia weight factor, \( c_1 \) and \( c_2 \) are the acceleration coefficients, \( r_1 \) and \( r_2 \) are positive random numbers between 0 and 1, \( P_{best_{i}}^{k} \) is the best position of particle \( i \) at iteration \( k \) and \( g_{best}^{k} \) is the best position of the group at iteration \( k \).

In the velocity updating process, the acceleration constants \( c_1 \), \( c_2 \) and the inertia weight factor are predefined and the random numbers \( r_1 \) and \( r_2 \) are uniformly distributed in the range of \((0,1)\). Suitable selection of inertia weight in equation (25) provides a balance between local and global searches, thus requiring less iteration on average to find a sufficiently optimal solution. A low value of inertia weight implies a local search, while a high value leads to global search. As originally developed, the inertia weight factor often is decreased linearly from about 0.9 to 0.4 during a run. It was proposed in [21]. In general, the inertia weight \( \omega \) is set according to the following equation:

\[
\omega = \omega_{\text{max}} - \frac{\omega_{\text{max}} - \omega_{\text{min}}}{\text{Iter}_{\text{max}}} \times \text{Iter}
\]

where \( \omega_{\text{min}} \) and \( \omega_{\text{max}} \) are the minimum and maximum value of inertia weight factor, \( \text{Iter}_{\text{max}} \) corresponds to the maximum iteration number and \( \text{Iter} \) is the current iteration number.
The current position (searching point in the solution space) can be modified by using the following equation:

\[ X_i^{k+1} = X_i^k + V_i^{k+1} \quad (27) \]

The velocity of particle i at iteration k must lie in the range:

\[ V_{imin} \leq V_i^k \leq V_{imax} \quad (28) \]

The parameter \( V_{max} \) determines the resolution or fitness, with which regions are to be searched between the present position and the target position. If \( V_{max} \) is too high, the PSO facilitates a global search and particles may fly past good solutions. Conversely, if \( V_{max} \) is too small, the PSO facilitates a local search and particles may not explore sufficiently beyond locally good solutions. In many experiences with PSO, \( V_{max} \) was often set at 10-20% of the dynamic range on each dimension.

The constants \( c1 \) and \( c2 \) in equation (25) pull each particle towards \( P_{best} \) and \( g_{best} \) positions. Thus, adjustment of these constants changes the amount of tension in the system. Low values allow particles to roam far from target regions, while high values result in abrupt movement toward target regions. Figure 5 shows the search mechanism of particle swarm optimization technique using the modified velocity, best position of particle i and best position of the group.

![Figure 5. Updating the position mechanism of PSO technique [15]](image)

### 7.2 Constriction factor approach

After the original particle swarm proposed by Kennedy and Eberhart, a lot of improved particle swarms were introduced. The particle swarm with constriction factor is very typical. Recent work done by Clerc [22] indicates that the use of a constriction factor may be necessary to insure convergence of the particle swarm optimization algorithm. In order to insure convergence of the particle swarm optimization algorithm, the velocity of the constriction factor approach can be represented as follows:

\[ V_i^{k+1} = K \times [\omega \times V_i^k + c1 \times r_1 \times (P_{best}^k - X_i^k) + c2 \times r_2 \times (g_{best}^k - X_i^k)] \quad (29) \]

where \( K \) is the constriction factor and given by:

\[ K = \frac{2}{2 - \phi - \sqrt{\phi^2 - 4\phi}} \quad (30) \]

where: \( \phi = c1 + c2 \), \( \phi > 4 \)

The convergence characteristic of the particle swarm optimization technique can be controlled by \( \phi \). In the constriction factor approach, \( \phi \) must be greater than 4.0 to guarantee the stability of the PSO.
algorithm. However, as $\phi$ increases the constriction factor decreases and diversification is reduced, yielding slower response. Typically, when the constriction factor is used, $\phi$ is set to 4.1 (i.e. $c_1 = c_2 = 2.05$) and the constant multiplier $k$ is 0.729. The constriction factor approach can generate higher quality solutions than the basic PSO technique.

8. Algorithm for short term pumped storage scheduling problem using CFPSO technique

The solution methodology for solving short term pumped storage scheduling problem by using constriction factor based particle swarm optimization (PSO) are explained as follows:

**Step 1:** Read the system input data, namely fuel cost curve coefficients, power generation limits of pumped storage units and thermal units, number of thermal units, number of pumped storage units, power demands, water discharge rate limits, beginning and ending water volumes of the upper reservoir and water volume limits of the upper reservoir.

**Step 2:** Select particle swarm optimization parameters such as population size, initial and final weight factor, acceleration constants, constriction factor and maximum number of generations to be performed.

**Step 3:** Initialize a population of particles with random positions according to the minimum and maximum operating limits of each unit. These initial particles must be feasible candidate solutions that satisfy the practical operation constraints of all thermal units and pumped storage plants.

**Step 4:** Initialize the velocity of particles in the range between $[-V_i^{\text{max}}, +V_i^{\text{max}}]$.

**Step 5:** If the pumping storage plant is in pumping mode go to step 6, otherwise go to step 9.

**Step 6:** Calculate the water pumping rate to the upper reservoir.

**Step 7:** Calculate the volume of water pumped to the upper reservoir according to equation (23).

**Step 8:** Calculate the pumping power of the pumped storage power plant.

**Step 9:** Calculate the water discharge rate of the pumped storage power plant.

**Step 10:** Calculate the total volume of water discharged from the upper reservoir according to equation (24).

**Step 11:** Calculate the output power generation of the pumped storage power plant.

**Step 12:** Calculate the thermal demand by subtracting the generation of hydro units from the total load demand. The thermal demand (total load – hydro generation) must be covered by the thermal units. The thermal generations are calculated from the power balance equation defined in (21).

**Step 13:** If $t \leq T$ go to step 5, otherwise go to step 14.

**Step 14:** Calculate the output power of each thermal unit by solving economic load dispatch problem.

**Step 15:** Check the inequality constraint of thermal power generated according to the following equation:

$$
\begin{align*}
    P_{gi}^t &= \begin{cases} 
        P_{gi}^t & \text{if } P_{gi}^{\text{min}} \leq P_{gi}^t \leq P_{gi}^{\text{max}} \\
        P_{gi}^{\text{min}} & \text{if } P_{gi}^{\text{min}} \leq P_{gi}^t \leq P_{gi}^{\text{max}} \\
        P_{gi}^{\text{max}} & \text{if } P_{gi}^{\text{min}} \leq P_{gi}^t \leq P_{gi}^{\text{max}} 
    \end{cases}
\end{align*}
$$

(31)

**Step 16:** Evaluate the fitness value for each particle in the population by using the objective function defined in (22).

**Step 17:** Repeat the above steps for each particle in the population.

**Step 18:** If the evaluation value of each particle is better than the previous Pbest, then set Pbest equal to the current value.

**Step 19:** Select the particle with the best fitness value of all the particles in the population as the gbest.

**Step 20:** Update the velocity of each particle according to equation (29).

**Step 21:** Check the velocity of each particle using the following equation:

$$
\begin{align*}
    V_{i}^{k+1} &= \begin{cases} 
        V_{i}^{k+1} & \text{if } V_{i}^{k+1} \leq V_{i}^{\text{min}} \leq V_{i}^{k+1} \\
        V_{i}^{\text{min}} & \text{if } V_{i}^{k+1} \leq V_{i}^{\text{min}} \leq V_{i}^{k+1} \\
        V_{i}^{\text{max}} & \text{if } V_{i}^{k+1} \leq V_{i}^{\text{max}} 
    \end{cases}
\end{align*}
$$

(32)

**Step 22:** The position of each particle is modified according to equation (27).
Step 23: If the stopping criterion is reached (i.e. usually maximum number of iterations) go to step 24, otherwise go to step 5.

Step 24: The particle that generates the latest gbest is the optimal generation power of each unit with minimum total fuel cost of the thermal power plants.

Step 25: Print the output results of the pumped storage scheduling problem and stop.

9. Case study and simulation results

To verify the feasibility and effectiveness of the proposed algorithms to solve short term pumped storage scheduling problem, a hydrothermal power system consists of five thermal generating units and one pumped storage power plant were tested. The single line diagram of the test power system is shown in Figure 6. The data of test system are taken from [23]. The fuel cost data and the minimum and maximum limits of the thermal generating units are given in Table 1. The reservoir storage limits, starting and ending water volumes and generation limits of the pumped storage power plant are given in Table 2. The water discharge rate of pumped storage unit is given in equation (33) and the water pumping rate of pumped storage unit is given in equation (34). The scheduling time period is one day with 24 intervals of one hour each. In this case study, the 24 hours operation cycle having six equal time intervals is considered. The load demand for the six time intervals is given in Table 3. The thermal units connected to buses 9 and 11 are chosen as inefficient units with respect to the other thermal units. So, these units are expensive and generate active power only in the time interval where the peak load demand occurs. The proposed algorithms has been implemented in MATLAB language and executed on an Intel Core i3, 2.27 GHz personal computer with a 3.0 GB of RAM. The optimal control parameters used in genetic algorithm are listed in Table 4. The CFPSO control parameters selected for the solution are given in Table 5. The program is run 50 times for each algorithm and the best among the 50 runs are taken as the final solutions. The test system is solved when the pumped storage plant is off line and is solved again when the pumped storage unit is on line to determine the saving in fuel cost of thermal units. The resultant optimal schedule of thermal units obtained from the CFPSO technique when the pumped storage unit is offline is shown in Table 6. The fuel cost of each thermal unit and the total fuel cost of the thermal power system obtained from the CFPSO technique when the pumped storage plant is offline given in Table 7. The optimal schedules of pumped storage power plant and thermal units obtained from CFPSO method is presented in Table 8. The fuel cost of each thermal unit and total fuel cost over the day obtained from CFPSO algorithm when the pumped storage plant is online is given in Table 9. Table 10 shows the optimal power generation schedule of thermal units obtained from GA technique when the pumped storage unit is offline. The fuel cost of each thermal unit and the total fuel cost of the system obtained from the GA method when the pumped storage plant is offline are given in Table 11. The optimal power schedules of pumped storage power plant and thermal units obtained from GA method are presented in Table 12 while Table 13 shows the fuel cost of each thermal unit and the total fuel cost of the system obtained from GA approach when the pumped storage plant is online. Table 14 presents the performance comparison between the CFPSO technique and genetic algorithm in terms of the cost saving and execution time.

The water discharge rate curve of the pumped storage plant is given as follows:

\[
q_{h}(\text{Phg}) = \begin{cases} 
200 + 2.0 \times \text{Phg} \text{ (acre-ft/hr)} & \text{if } 0 < \text{Phg} \leq 130 \text{ MW} \\
0 \text{ (acre-ft/hr)} & \text{if } \text{Phg} = 0 \text{ MW}
\end{cases}
\]  

(33)

The water pumping rate curve of the pumped storage plant is given as follows:

\[
q_{p}(\text{Php}) = \begin{cases} 
200 + \frac{4}{3} \times |\text{Php}| \text{ (acre-ft/hr)} & \text{if } 0 < |\text{Php}| \leq 130 \text{ MW} \\
0 \text{ (acre-ft/hr)} & \text{if } |\text{Php}| = 0 \text{ MW}
\end{cases}
\]  

(34)
Figure 6. Single line diagram of the test power system

Table 1. Fuel cost data of thermal generating power plants

<table>
<thead>
<tr>
<th>Plant</th>
<th>Bus</th>
<th>$a_i$/($/MW^{2}hr$)</th>
<th>$b_i$/($$/MWhr$$)</th>
<th>$c_i$/($$/hr$$)</th>
<th>$P_{gi}^{\text{min}}$ (MW)</th>
<th>$P_{gi}^{\text{max}}$ (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0.001495</td>
<td>7.48</td>
<td>527</td>
<td>50</td>
<td>350</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>0.001562</td>
<td>7.92</td>
<td>561</td>
<td>45</td>
<td>180</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>0.001940</td>
<td>7.85</td>
<td>310</td>
<td>40</td>
<td>175</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td>0.004360</td>
<td>9.52</td>
<td>476</td>
<td>5</td>
<td>100</td>
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<tr>
<td>5</td>
<td>11</td>
<td>0.003970</td>
<td>9.40</td>
<td>460</td>
<td>3</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 2. Reservoir storage capacity limit, starting and ending water volumes and generation limits of pumped storage unit

<table>
<thead>
<tr>
<th>Plant</th>
<th>Bus</th>
<th>$V_{h}^{\text{min}}$ (acre-ft)</th>
<th>$V_{h}^{\text{max}}$ (acre-ft)</th>
<th>$V_{h}^{\text{ini}}$ (acre-ft)</th>
<th>$V_{h}^{\text{end}}$ (acre-ft)</th>
<th>$P_{h}^{\text{min}}$ (MW)</th>
<th>$P_{h}^{\text{max}}$ (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>5000</td>
<td>15000</td>
<td>10000</td>
<td>10000</td>
<td>0</td>
<td>130</td>
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</tbody>
</table>

Table 3. Load demand for six time intervals

<table>
<thead>
<tr>
<th>Interval</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load demand (MW)</td>
<td>200</td>
<td>600</td>
<td>800</td>
<td>600</td>
<td>300</td>
<td>200</td>
</tr>
</tbody>
</table>

Table 4. Control parameters of genetic algorithm

<table>
<thead>
<tr>
<th>Genetic algorithm parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population size</td>
<td>50</td>
</tr>
<tr>
<td>Maximum number of generations</td>
<td>300</td>
</tr>
<tr>
<td>Crossover probability</td>
<td>0.8</td>
</tr>
<tr>
<td>Mutation probability</td>
<td>0.05</td>
</tr>
</tbody>
</table>
Table 5. Control parameters of particle swarm optimization

<table>
<thead>
<tr>
<th>CFPSO parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population size</td>
<td>50</td>
</tr>
<tr>
<td>Maximum number of generations</td>
<td>300</td>
</tr>
<tr>
<td>Acceleration coefficients$(c_1/c_2)$</td>
<td>2.05</td>
</tr>
<tr>
<td>Minimum inertia weight $(\omega_{min})$</td>
<td>0.4</td>
</tr>
<tr>
<td>Minimum inertia weight $(\omega_{max})$</td>
<td>0.9</td>
</tr>
<tr>
<td>Constriction factor $(k)$</td>
<td>0.729</td>
</tr>
</tbody>
</table>

Table 6. Optimal generation schedule of thermal units obtained from CFPSO technique when pumped storage unit is offline

<table>
<thead>
<tr>
<th>Interval</th>
<th>P_D (MW)</th>
<th>P_1 (MW)</th>
<th>P_2 (MW)</th>
<th>P_3 (MW)</th>
<th>P_4 (MW)</th>
<th>P_5 (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>200.00</td>
<td>115.00</td>
<td>45.0000</td>
<td>40.0000</td>
<td>-----</td>
<td>----</td>
</tr>
<tr>
<td>2</td>
<td>600.00</td>
<td>295.1932</td>
<td>158.9363</td>
<td>145.8702</td>
<td>-----</td>
<td>----</td>
</tr>
<tr>
<td>3</td>
<td>800.00</td>
<td>350.0000</td>
<td>180.0000</td>
<td>175.0000</td>
<td>38.0988</td>
<td>56.9012</td>
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<tr>
<td>4</td>
<td>600.00</td>
<td>306.5846</td>
<td>152.5488</td>
<td>140.8666</td>
<td>-----</td>
<td>----</td>
</tr>
<tr>
<td>5</td>
<td>300.00</td>
<td>197.2843</td>
<td>46.1123</td>
<td>56.6016</td>
<td>-----</td>
<td>----</td>
</tr>
<tr>
<td>6</td>
<td>200.00</td>
<td>115.00</td>
<td>45.0000</td>
<td>40.0000</td>
<td>-----</td>
<td>----</td>
</tr>
</tbody>
</table>

Table 7. Fuel cost of each thermal unit and total fuel cost obtained from CFPSO technique when pumped storage unit is offline

<table>
<thead>
<tr>
<th>Interval</th>
<th>F_1 ($/hr)</th>
<th>F_2 ($/hr)</th>
<th>F_3 ($/hr)</th>
<th>F_4 ($/hr)</th>
<th>F_5 ($/hr)</th>
<th>F_T ($/hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1406.971</td>
<td>920.563</td>
<td>627.104</td>
<td>-----</td>
<td>----</td>
<td>2954.638</td>
</tr>
<tr>
<td>2</td>
<td>2865.318</td>
<td>1859.233</td>
<td>1496.361</td>
<td>-----</td>
<td>----</td>
<td>6220.911</td>
</tr>
<tr>
<td>3</td>
<td>3328.138</td>
<td>2037.209</td>
<td>1743.163</td>
<td>845.029</td>
<td>1007.725</td>
<td>8961.263</td>
</tr>
<tr>
<td>4</td>
<td>2960.774</td>
<td>1805.536</td>
<td>1454.299</td>
<td>-----</td>
<td>----</td>
<td>6220.609</td>
</tr>
<tr>
<td>5</td>
<td>2060.874</td>
<td>929.531</td>
<td>760.538</td>
<td>-----</td>
<td>----</td>
<td>3750.942</td>
</tr>
<tr>
<td>6</td>
<td>1406.971</td>
<td>920.563</td>
<td>627.104</td>
<td>-----</td>
<td>----</td>
<td>2954.638</td>
</tr>
</tbody>
</table>

Total fuel cost over the day 124252.012

Table 8. Optimal generation schedule of pumped storage plant and thermal units obtained from CFPSO technique

<table>
<thead>
<tr>
<th>Interval</th>
<th>P_1 (MW)</th>
<th>P_2 (MW)</th>
<th>P_3 (MW)</th>
<th>P_4 (MW)</th>
<th>P_5 (MW)</th>
<th>Ps (MW)</th>
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<tbody>
<tr>
<td>1</td>
<td>201.1783</td>
<td>51.7040</td>
<td>59.6709</td>
<td>-----</td>
<td>----</td>
<td>-112.553</td>
</tr>
<tr>
<td>2</td>
<td>294.5450</td>
<td>141.0658</td>
<td>131.6210</td>
<td>-----</td>
<td>----</td>
<td>32.7682</td>
</tr>
<tr>
<td>3</td>
<td>350.0000</td>
<td>167.2947</td>
<td>152.7053</td>
<td>-----</td>
<td>----</td>
<td>130.0000</td>
</tr>
<tr>
<td>4</td>
<td>293.8398</td>
<td>140.3909</td>
<td>131.0776</td>
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<td>----</td>
<td>34.6917</td>
</tr>
<tr>
<td>5</td>
<td>225.6213</td>
<td>72.3618</td>
<td>79.8341</td>
<td>-----</td>
<td>----</td>
<td>-77.8172</td>
</tr>
<tr>
<td>6</td>
<td>198.7048</td>
<td>49.3366</td>
<td>57.7648</td>
<td>-----</td>
<td>----</td>
<td>-105.8062</td>
</tr>
</tbody>
</table>

Table 9. Fuel cost of each thermal unit and total fuel cost obtained from CFPSO technique when pumped storage unit is online

<table>
<thead>
<tr>
<th>Interval</th>
<th>F_1 ($/hr)</th>
<th>F_2 ($/hr)</th>
<th>F_3 ($/hr)</th>
<th>F_4 ($/hr)</th>
<th>F_5 ($/hr)</th>
<th>F_T ($/hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2092.320</td>
<td>974.671</td>
<td>785.324</td>
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<td>----</td>
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<tr>
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<td>2859.898</td>
<td>1709.324</td>
<td>1376.834</td>
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<td>5946.056</td>
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<tr>
<td>3</td>
<td>3328.138</td>
<td>1929.691</td>
<td>1553.975</td>
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<td>2854.003</td>
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<td>5929.976</td>
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<tr>
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<td>2290.750</td>
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<tr>
<td>6</td>
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<td>955.548</td>
<td>769.927</td>
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<td>3797.815</td>
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Total fuel cost over the day 122880.251
Table 10. Optimal generation schedule of thermal units obtained from GA technique when pumped storage unit is offline

<table>
<thead>
<tr>
<th>Interval</th>
<th>P₀ (MW)</th>
<th>P₁ (MW)</th>
<th>P₂ (MW)</th>
<th>P₃ (MW)</th>
<th>P₄ (MW)</th>
<th>P₅ (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>200.00</td>
<td>113.7802</td>
<td>45.6225</td>
<td>40.5973</td>
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<td>----</td>
</tr>
<tr>
<td>2</td>
<td>600.00</td>
<td>313.0117</td>
<td>144.3590</td>
<td>142.6292</td>
<td>----</td>
<td>----</td>
</tr>
<tr>
<td>3</td>
<td>800.00</td>
<td>350.0000</td>
<td>180.0000</td>
<td>175.0000</td>
<td>45.7869</td>
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<td>303.9290</td>
<td>162.0092</td>
<td>134.0618</td>
<td>----</td>
<td>----</td>
</tr>
<tr>
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<td>300.00</td>
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<td>----</td>
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</tr>
</tbody>
</table>

Table 11. Fuel cost of each thermal unit and total fuel cost obtained from GA technique when pumped storage unit is offline

<table>
<thead>
<tr>
<th>Interval</th>
<th>F₁ ($/hr)</th>
<th>F₂ ($/hr)</th>
<th>F₃ ($/hr)</th>
<th>F₄ ($/hr)</th>
<th>F₅ ($/hr)</th>
<th>F₅ ($/hr) (four intervals)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1397.430</td>
<td>925.581</td>
<td>631.886</td>
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<td>----</td>
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</tr>
<tr>
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<td>6220.782</td>
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<tr>
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<td>2954.713</td>
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</table>

Total fuel cost over the day: 124255.853

Table 12. Optimal generation schedule of pumped storage plant and thermal units obtained from GA technique

<table>
<thead>
<tr>
<th>Interval</th>
<th>P₁ (MW)</th>
<th>P₂ (MW)</th>
<th>P₃ (MW)</th>
<th>P₄ (MW)</th>
<th>P₅ (MW)</th>
<th>P₅ (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<tr>
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<td>----</td>
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</tr>
<tr>
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<td>146.5988</td>
<td>144.1377</td>
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<td>32.6819</td>
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<td>65.9829</td>
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<td>-98.2442</td>
</tr>
</tbody>
</table>

Table 13. Fuel cost of each thermal unit and total fuel cost obtained from GA technique when pumped storage unit is offline

<table>
<thead>
<tr>
<th>Interval</th>
<th>F₁ ($/hr)</th>
<th>F₂ ($/hr)</th>
<th>F₃ ($/hr)</th>
<th>F₄ ($/hr)</th>
<th>F₅ ($/hr)</th>
<th>F₅ ($/hr) (four intervals)</th>
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<tbody>
<tr>
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<tr>
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<td>1755.683</td>
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<tr>
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<td>2188.341</td>
<td>1243.176</td>
<td>1012.550</td>
<td>----</td>
<td>----</td>
<td>4444.068</td>
</tr>
<tr>
<td>6</td>
<td>1980.139</td>
<td>920.563</td>
<td>836.412</td>
<td>----</td>
<td>----</td>
<td>3737.115</td>
</tr>
</tbody>
</table>

Total fuel cost over the day: 122948.414

Table 14. Comparison of total fuel cost, cost saving and execution time between GA and CFPSO techniques

<table>
<thead>
<tr>
<th>Method</th>
<th>Thermal cost without pumped storage plant ($)</th>
<th>Thermal cost with pumped storage plant ($)</th>
<th>Cost saving ($)</th>
<th>CPU time (second)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CFPSO</td>
<td>124252.012</td>
<td>122880.251</td>
<td>1371.761</td>
<td>7.36</td>
</tr>
<tr>
<td>GA</td>
<td>124255.853</td>
<td>122948.414</td>
<td>1307.439</td>
<td>13.21</td>
</tr>
</tbody>
</table>
10. Comparison of fuel cost and computation time between two proposes methods

The test results obtained when the pumped storage plant is offline are compared with those obtained when the pumped storage plant is online to determine the cost saving over the day. Cost saving is the thermal cost without pumped storage plant minus thermal cost with pumped storage plant.

The observations obtained from the test case study can be summarized as follows:

- When the pumped storage plant is offline
  The total fuel cost obtained from the CFPSO technique is nearly equal to the total fuel cost obtained from the genetic algorithm. From the tabulated results, it is seen that the thermal units connected to buses 9 and 11 are expensive units and generate power only during the peak load time interval (time interval 3). The total fuel cost obtained from the CFPSO algorithm over the day was found to be $124252.012 while the total thermal cost obtained from the genetic algorithm is found to be $124255.853.

- When the pumped storage plant is online
  From the tabulated results it is seen that, the expansive thermal units connected to buses 9 and 11 are not operated during all time intervals. When the CFPSO technique is applied to solve this problem it is found that, the pumped storage plant generates 197.4599 MWh during peak load periods and pumps up 296.1766 MWh during light load periods. The total fuel cost obtained from CFPSO method when the pumped storage unit is online is $122880.251, resulting in a cost saving of $1371.761 in one day. The thermal cost curve converges to the optimal solution in 7.36 seconds. The amount of water stored at the end of the operation cycle is found to be 9999.9293 acre-ft/h. Figure 7 shows the generation/pumping schedules obtained using the CFPSO approach. The water discharge/pumping pattern of pumped storage plant using the CFPSO method is given in Figure 8. Figure 9 gives the thermal cost without and with pumped storage power plant using the CFPSO technique. Figure 10 shows the thermal load profile without and with pumped storage plant using CFPSO algorithm.

When the genetic algorithm is used to solve this problem, it is seen that, the pumped storage power plant generates 201.4756 MWh during peak load time intervals and pumps up 302.1486 MWh during light load periods. The total thermal cost obtained from the genetic algorithm for getting optimal solution is 13.21 seconds. The amount of water stored at the end of the operation cycle was found to be 9999.654 acre-ft/hr. Figure 11 gives the generation/pumping profile obtained using genetic algorithm and Figure 12 presents the water discharge/pumping pattern of the pumped storage power plant by using genetic algorithm. Figure 13 shows the total fuel cost without and with pumped storage plant using the genetic algorithm and Figure 14 presents the thermal load profile without and with pumped storage plant using genetic algorithm.

From the tabulated results it is show that, the pumped storage power plant operates in pumping mode during the low load demand periods (i.e. time intervals 1, 5 and 6) and pumping power have higher value in time intervals 1 and 6 where the load demand is at its minimum value. The pumping storage power plant operates in generating mode during the peak load demand periods (i.e. time intervals 2, 3 and 4) and generate maximum power when the system load demand occurs (i.e. at time interval 3). From Table 14 it is observed that, the CFPSO technique has better cost saving and execution time than the genetic algorithm.

![Figure 7. Generation/Pumping schedules using CFPSO technique](image-url)
Figure 8. Water discharge/pumping pattern using CFPSO technique

Figure 9. Total thermal cost using CFPSO technique

Figure 10. Thermal load profile using CFPSO technique

Figure 11. Generation/Pumping schedules using genetic algorithm

Total water discharge: 994.920 acre-ft/hr
Total water pumping: 994.902 acre-ft/hr

Total generation: 201.4756 MW
Total pumping: 302.1486 MW
11. Conclusions

In this paper, two proposed approaches namely, genetic algorithm and constriction factor based particle swarm optimization technique are proposed for solving short term pumped storage scheduling problem. To demonstrate the performance efficiency of the proposed algorithms, they have been applied on test power system consists of five thermal units and one pumped storage power plant. The results obtained from the CFPSO technique are compared with the simulation results obtained from the GA to verify the feasibility of the proposed methods. The experimental results obtained when the pumped storage plant is offline are compared with those obtained when the pumped storage plant is online to determine the cost saving over the day. From the tabulated results it is seen that the CFPSO technique performs better than genetic algorithm in terms of cost saving and execution time.
References


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