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Development of a thermal resistance model to evaluate wellbore heat exchange efficiency

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Abstract

A new model is proposed to simulate conduction of heat between a pipe loop in a geoexchange system and the ground. The approach employs the thermal resistor technique coupled with a conduction shape factor modified by an occultation factor. The model is compared to available data and demonstrates suitable agreement with previous studies. The model facilitates a parametric study of borehole resistance as a function of geometry and thermal conductivity of the components. By spacing the legs of the loop against the borehole and increasing the pipe size, the study shows that one can maximize the wellbore heat transfer using a moderate (1.2 W/mK) thermal conductivity grout. This study further demonstrates that improved well construction techniques could increase the efficiency of most closed-loop geothermal systems by 10 percent.

Keywords: Ground-source heat pump; Borehole thermal resistivity; Heat transfer model.

1. Introduction

Contemporary ground-source heat pump (geothermal) systems rely on efficient transfer of heat between the borehole and surrounding geologic medium. Typical closed-loop systems in the United States employ a circuit of high-density polyethylene (HDPE) pipe embedded within a borehole filled with an aqueous grout mixture. Previous studies [1] have documented that grout thermal conductivity and the proximity of the pipe to the borehole wall strongly influence the rate of heat transfer. Increased heat transfer efficiency improves system performance and decreases installation costs due to a reduction in borehole length.

This study proposes a thermal resistor network model to evaluate the thermal resistance of geoexchange wells. The method is straight-forward, computationally efficient, and flexible. Previous studies have employed numerical techniques [2-4] that are rigorous but are time-consulting to construct. Other investigators have proposed analytical solutions [5-7] that are readily applied, but are restricted to a limited number of pipe geometries. The method proposed by this study uses a straight-forward analytical solution with the advantage of a lessrestrictive borehole geometry.

2. Model development

The transfer of heat between the geoexchange pipe and surrounding geologic medium is governed by the thermal properties of carrier fluid, pipe, grout, and geologic medium; pipe location, and the dimensions of pipe and the borehole (Figure 1). A model must consider the resistance contributions from the fluid (typically water) flowing inside the two pipes (supply and return joined at the base of the wellbore by a U-bend), through the pipe walls, and through the medium (typically grout) that fills the space between the pipes and surrounding bore wall.

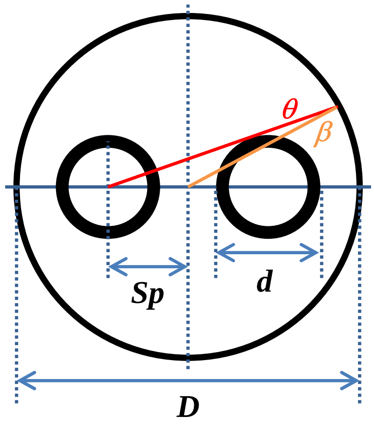

Figure 1. Cross-section geometry of a typical closed-loop heat exchange well. Dimensional parameters *D*, *d*, and *Sp* refer to the borehole diameter, pipe exterior diameter, and pipe spacing, respectively. The occultation angles θ and β are also shown

Thermal transport within the system may be represented by an equivalent network of resistors (Figure 2), where the temperatures are designated T_1, T_2, and T_b for the supply and return legs and borehole wall, respectively. The thermal resistance of each pipe, R_{p1} and R_{p2}, are composed of convection at the inside wall and conduction through the pipe wall. The shunt resistance, R_s, addresses heat transfer between the pipes through the medium, while R_b defines the thermal resistance between pipe surfaces to the surrounding wall.

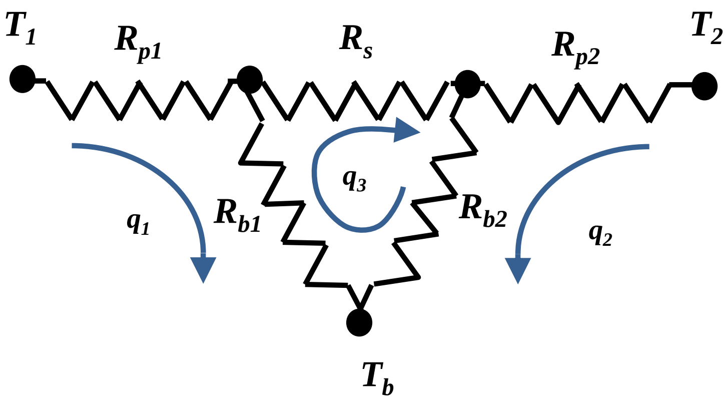

Figure 2. Resistance network model representing wellbore heat transfer

The temperature of the elements within the network model are considered unique, such that $T_1>T_2>T_b$ for the cooling season and $T_b>T_2>T_1$ during heating. The three governing equations that describe the resistive network are

$$(T_1 - T_b) - (q_1 - q_3)R_{b1} - q_1 R_{p1} = 0 \tag{1}$$

$$(T_2 - T_b) - (q_2 + q_3)R_{b2} - q_2 R_{p2} = 0 \tag{2}$$

$$q_3 R_s + (q_3 + q_2)R_{b2} + (q_3 - q_1)R_{b1} = 0 \tag{3}$$

where q is heat flux. Assuming $R_{p1} = R_{p2}$ and $R_{b1} = R_{b2}$ (symmetric case in which pipe is equidistant from the borehole center), Eq.(1) through (3) may be solved simultaneously assuming steady-state conditions to reveal

$$q_1 = \frac{\Delta T_{avg}}{R_p + R_b} + \frac{\Delta T}{2R_3} \tag{4}$$

$$q_2 = \frac{\Delta T_{avg}}{R_p + R_b} - \frac{\Delta T}{2R_3} \tag{5}$$

$$q_3 = \frac{\Delta T}{R_{shunt}} \tag{6}$$

where $\Delta T = T_1 - T_2$, $T_{avg} = (T_1 + T_2)/2$, $\Delta T_{avg} = (T_{avg} - T_b)$, $R_3 = (R_p + R_b) - 2R_b/(R_s/R_b + 2)$, and $R_{shunt} = (R_p + R_b)(R_s/R_b + 2) - 2R_b$.

The pipe thermal resistance includes convective heat transfer from the carrier fluid and heat conduction through the pipe wall, defined as

$$R_p = \frac{1}{\pi d_i h_{cv}} + \frac{\ln(d_o / d_i)}{2\pi k_p} \tag{7}$$

where subscripts o and i denote outside and inside pipe diameters, h_{cv} is the convection coefficient for water flowing inside the pipe, and k_p is the thermal conductivity of the pipe wall.

Thermal transport between the pipes and bore wall must consider the 2-dimensional geometry of the system and the thermal interference, or obscuration, created by the companion pipe. Applying the shape-factor method for 2-dimensional, steady-state heat conduction [8, 9], the thermal resistance between single pipe and borehole wall may be calculated as:

$$R_1 = \frac{\cosh^{-1}\left(\dfrac{D^2 + d^2 - 4x_1^2}{2Dd}\right)}{2\pi k_g f_{occ}} \tag{8a}$$

$$R_2 = \frac{\cosh^{-1}\left(\dfrac{D^2 + d^2 - 4x_2^2}{2Dd}\right)}{2\pi k_g f_{occ}} \tag{8b}$$

where D is borehole diameter, d is pipe diameter, and k_g is the grout thermal conductivity. The variables x_1 and x_2 are the distances between the borehole center and center of pipes 1 and 2, respectively. In the special case where $x_1 = x_2$, these parameters are equivalent to pipe spacing, Sp, and the distance between the pipes is $2Sp$.

Equation (8) may be used to simulate a single off-centered pipe conducting to an unobscured surrounding wall. In a two-pipe system, conductive interference increases the net thermal resistance. To address this obscuration effect, an occultation factor, f_{occ}, is applied to the equation that considers the spacing and dimensions of pipes within a cylindrical borehole:

$$f_{occ} = (1 - \beta / \pi) \tag{9}$$

$$\theta = \sin^{-1}\left(\frac{d}{2(x_1 + x_2)}\right) \tag{10}$$

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$$\beta = \theta + \sin^{-1}\left(\frac{d}{2D}\right) \tag{11}$$

The final thermal resistance, R_b, may be calculated by

$$R_b = \frac{R_1 R_2}{R_1 + R_2} + \frac{R_p}{2} \tag{12}$$

The borehole thermal resistance may be used to calculate the heat transfer efficiency of a wellbore and predict seasonal ground response to an operation geoexchange system.

3. Results and discussion

3.1 Typical wellbore

The proposed model was employed to simulate a geoexchange borehole similar to those typically installed in the United States. Common drilling techniques in the US produce a borehole with a diameter of 152 mm (6 inches) filled with a neat bentonite grout with a k of 0.78 W/m K, and round, high density polyethylene (HDPE) tubing with dimensions established in Table 1. In this simulation, the supply and return pipe legs are assumed to be equally-spaced within the borehole, resulting in an*Sp* of 32.3 mm. Applying equations (7) through (12) results in a calculated R_b of 0.17 m K/W.

Table 1. Calculation of thermal resistance for a typical closed-loop borehole

Variable	Value
Model input parameters	
D, borehole diameter	152 mm
Sp, pipe spacing	32.3 mm
d, pipe exterior diameter	42 mm
d_i, pipe interior diameter	34 mm
t, pipe wall thickness	4 mm
k_p, pipe thermal conductivity	0.4 W/m K
h_{cv}, thermal exchange coefficient	1100 W/m^2 K
k_g, grout thermal conductivity	0.78 W/m K
Model output, this study	
R_p, pipe thermal resistivity	0.084 m K/W
R_b, borehole thermal conductivity	0.17 m K/W

The model was compared to approaches employed by previous studies to validate results (Figure 3). All simulations included the resistive effect of fluid heat transfer and thermal conduction through the pipe wall according to eq. (7). Simulations demonstrate that the proposed model is in general agreement with previous approaches, showing a reduction in R_b as pipe spacing increases. Predicted R_b from eq. (12) is most similar to (within 5 percent of) the results from Sharqawy et al. [4], who employed a detailed 2-dimensional finite element solution. The Sharqawy et al. study, similar to the current approach, focused on heat transfer through the borehole and assumed a constant temperature boundary condition at the borehole wall. Hellström [6] applied a semi-infinite boundary condition that accounted for a variable thermal field along the borehole perimeter. This results in a predicted R_b that is 14-percent greater than this study's approach. Finally, the semi-empirical findings from Remund [1] are 25-percent higher than this study. However, Remund acknowledged his results were higher than predicted by models due to grout voids, uncertainty of pipe placement, and borehole wall irregularities that are inherent in the field.

3.2 Sensitivity analysis

Equations (7) through (12) were used to evaluate the causal relationship between the dependent variable R_b and independent variables k_g, *Sp*, t, and d (Figure 4a, 4b, 4c, and 4d, respectively). All other variables were consistent with the base case summarized in Table 1. Increasing the thermal conductivity of grout dramatically decreases R_b. However, increasing k_g above 1.2 W/m K appears to provide little additional benefit. Moreover, increasing k_g has the disadvantage of increasing thermal shunting between legs of the u-loop. Increasing pipe spacing can reduce R_b by half. Most geothermal boreholes installed in the United States do not utilize spacers, allowing the pipes to twist which minimizes pipe spacing. We recommend that pipes be pushed against the borehole wall to increase efficiency. Reducing pipe wall thickness improves thermal transfer across the pipe wall. However, reducing t below 3 mm would likely weaken the pipe and cause it to collapse at depth. Another option would be to increase the diameter of the pipe. An increase in pipe diameter increases the surface area and corresponding heat flux. Increasing d is limited by the physical properties of the pipe and the space available within the borehole. The combined benefit of optimizing these parameters yields a significant reduction in R_b. Simulation of the "best reasonable case" (k_g = 1.2 W/m K, Sp = 43 mm, t = 3 mm, and d = 60 mm) reveals an R_b of 0.05 m K/W.

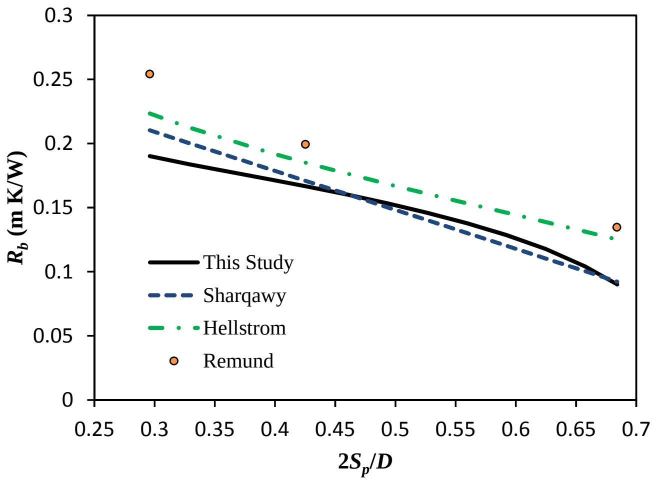

Figure 3. A comparison of effective borehole resistance (R_b) predicted by this study, a finite-element model (Sharqawy [4]), an semi-infinite analytical model (Hellström [6]), and empirical observations (Remund [1])

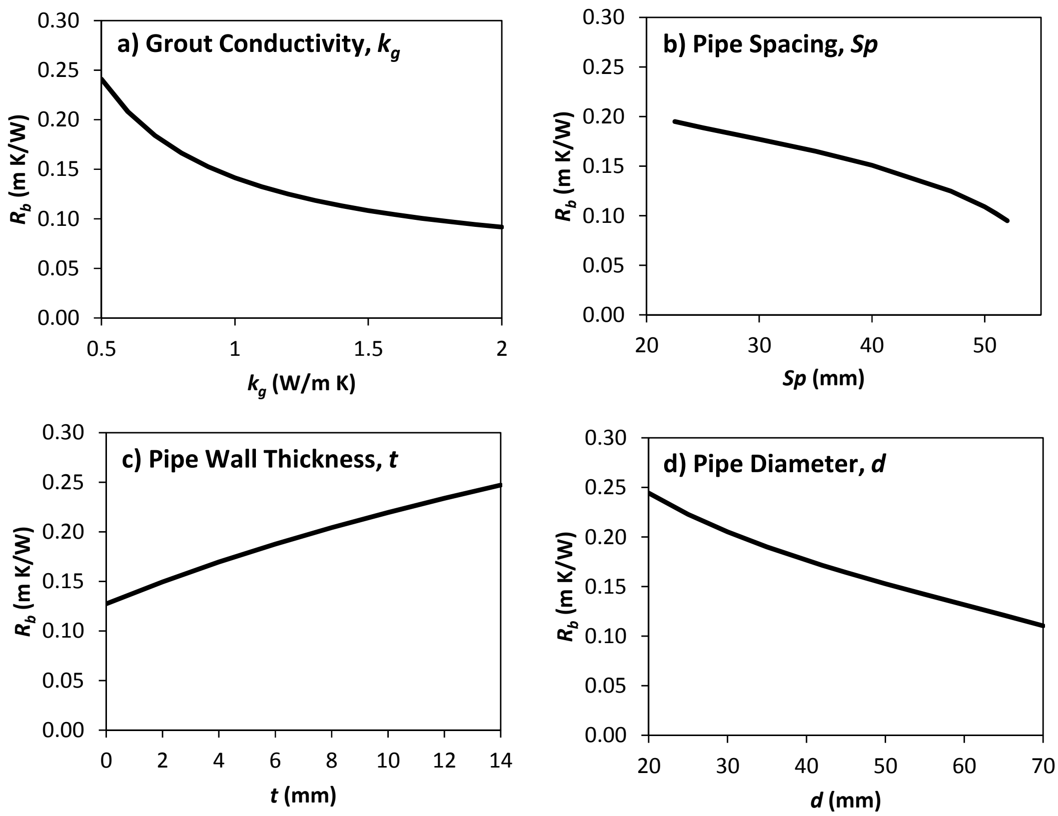

Figure 4. Borehole thermal resistance predicted by the model as a function of grout thermal conductivity, pipe spacing, pipe wall thickness, and pipe diameter

3.3 Importance of R_b

Previous studies have emphasized the importance of minimizing R_b. Comparatively few, however, have quantified the overall benefit of reducing borehole thermal resistance in cases where heat transfer may be limited by the thermal conductivity of the surrounding geologic medium (k_r). To test the relative importance of R_b, we simulated a borehole using the spatial dimensions in Table 1 with a 2-dimensional finite difference model[10]. Well efficiency, E_c, was computed by comparing the response of borehole with resistance R_b to a borehole with zero thermal resistance as a function of k_r. Representative values of R_b ranged from 0 to 0.3 m K/W as suggested by this study. Thermal conductivity was adjusted to represent coal, limestone, saturated sand, granite, and quartzite (Table 2).

Table 2. Thermal conductivity, k_r, and the critical borehole thermal resistance, R_b, for a variety of representative geologic materials

Formation Type	Thermal conductivity, k_r (W/m K) [11]	Critical borehole thermal resistance, R_b (m K/W)
Coal	0.3	0.31
Limestone	1.3	0.07
Sand	2.6	0.04
Granite	4	0.02
Quartzite	8	0.01

Well efficiency diminishes as R_b and k_r increase (Figure 5). This results because a geoexchange well installed within a high-k_r formation is more likely to be limited by thermal resistance within the borehole. A significant (>1 percent) reduction in well efficiency is predicted if R_b increases above a critical value (column 3 in Table 2). This demonstrates that for most geologic media (with the notable exception of coal), thermal exchange is likely to be limited by borehole thermal resistance. This emphasizes the benefit of reducing R_b under most geologic situations. Moreover, the model shows that improved construction of the geoexchange piping could improve the overall efficiency of a system installed in typical geologic materials (limestone, sand, and granite) by 10 percent.

The relationship between well efficiency and the product of R_b and k_r is well-defined (Figure 5b). For the typical well geometry investigated by this study, E_c may be calculated from the relation

$$E_c = e^{(aR_b k_r)} \tag{13}$$

where a = -0.105. This equation fits the simulated data well, with an R^2coefficient of 0.9999.

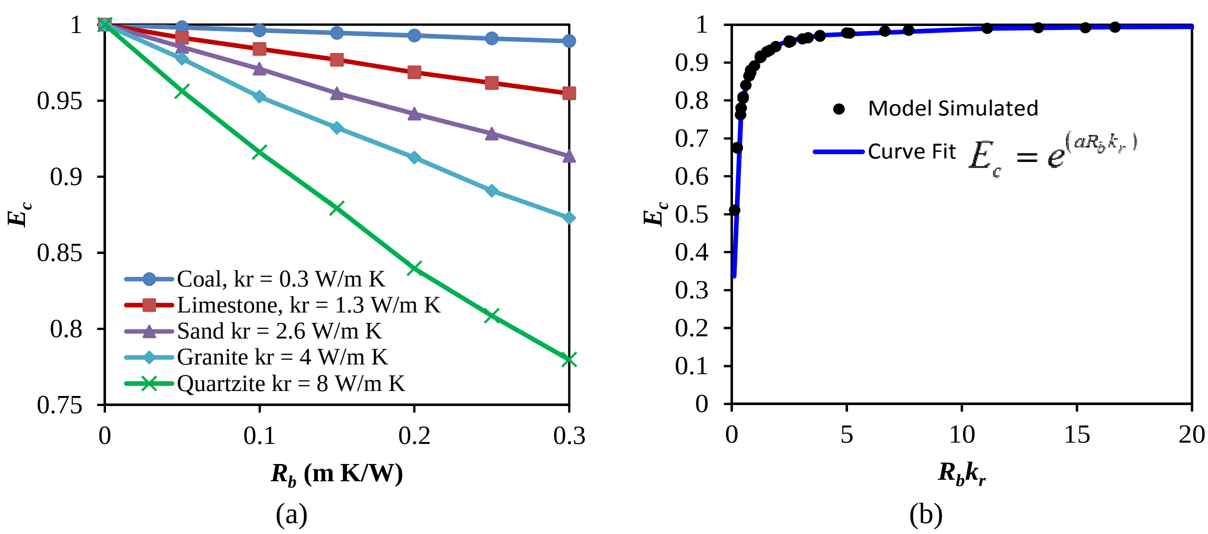

Figure 5.Thermal well efficiency (E_c) as a function of borehole thermal resistance (R_b) and the thermal conductivity of the surrounding geologic formation (k_r)

3.4 Shunting

The thermal shunt between the legs of the pipe loop is calculated from eq. (6), where R_s is developed from the pipe to pipe shape factor analysis in a surrounding cylinder from [8] where:

$$R_s = \cosh^{-1}\left[2\left(2Sp/d\right)^2 - 1\right] \tag{14}$$

As shown in Figure 6, the loop's ability to dissipate heat is compromised by the thermal shunt, which exists between supply and return legs of the loop. The vertical axis is a measure of the heat shunted between legs of the loop to the heat applied or extracted at the surface. When the pipes are in close proximity, the shunted heat transfer increases, and the performance of the loop is diminished. Alternatively, when the pipes are spread far apart (contacting the perimeter of the bore wall, for instance), shunting is minimized and the loop performance is maximized. Figure 6 shows that the thermal shunt is exacerbated for close spacing (small Sp) and for larger grout conductivity, k_g, while the least shunting is for legs that are separated widely and for low grout conductivity (i.e., neat bentonite).

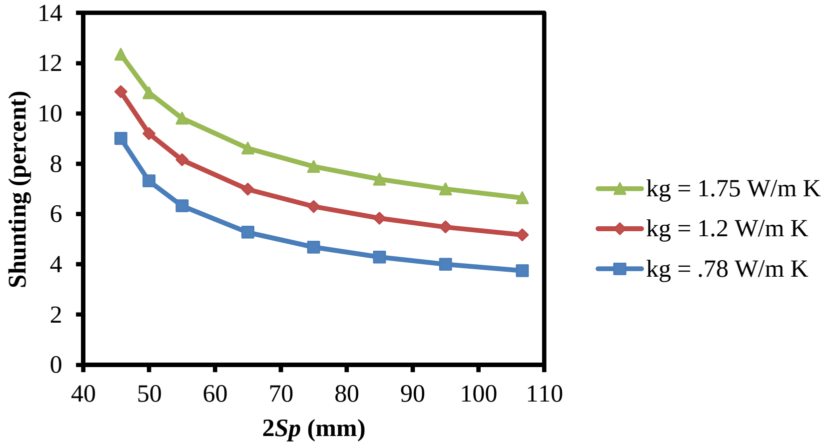

Figure 6. Loop thermal shunt between legs as function of spacing and grout conductivity

In practice, closely spaced U-bends and installation practices that allow the pipes to twist and contact one another over the pitch of the twist will suffer the consequences of greater thermal shunting. This has the effect of reducing the effective length of the loop, since the applied heat never makes it fully down hole, but instead raises the temperature of the upcoming water, so that (T_1-T_2), or the effective loop heat rejection, is smaller than what it would be with no shunting.

4. Conclusions

The analytical solution developed by this study to calculate borehole thermal resistance is computationally efficient, represents a variety of borehole geometries, and is readily accessible to installers because it requires no additional software. The model is particularly useful for predicting the benefits of optimizing borehole geometry and properties of the pipe and grout. The model compares well with previous studies, although actual R_b is likely to be somewhat greater than predicted by any model due to problems with grouting, pipe placement, and drilling techniques encountered in the field. Previous studies have emphasized the importance of increasing k_g as a means of reducing R_b; however, increasing k_g alone may cause unwanted thermal shunting between pipe legs. Numerical simulations demonstrate that reducing R_b could increase well thermal exchange efficiency by over 10 percent for most geologic materials. Future efforts to improve performance of geoexchange wells should consider maximizing pipe spacing, pipe surface area, and minimizing pipe wall thickness in addition to increasing k_g, which could significantly improve efficiency.

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