



## **Performance analysis of an endoreversible rectangular cycle with heat transfer loss and variable specific heats of working fluid**

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### **Abstract**

The performance of an air-standard rectangular cycle with heat transfer loss and variable specific heats of working fluid is analyzed by using finite-time thermodynamics. The relations between the work output and the compression ratio, between the efficiency and the compression ratio, and the optimal relation between work output and the efficiency of the cycle are derived by detailed numerical examples. Moreover, the effects of heat transfer loss and variable specific heats of working fluid on the cycle performance are analyzed. The results show that the effects of heat transfer loss and variable specific heats of working fluid on the cycle performance are obvious. The results may provide some guidelines for the application of the rectangular cycle.

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**Keywords:** Finite-time thermodynamics; Endoreversible rectangular cycle; Working fluid with variable specific heats; Performance analysis.

### **1. Introduction**

The application of Finite-time Thermodynamics [1-7] in performance analysis and optimization of thermal engine has achieved series of results. Rubin [8] defined the endoreversible cycle model earliest. Mozurkewich et al. [9] and Hoffman et al. [10] derived the optimal motion of the piston by using finite time thermodynamics and optimal control theory. Chen et al. [11] modeled the Diesel cycle with friction loss and studied the effect of friction loss on cycle performance. Klein et al. [12] and Chen et al. [13, 14] studied the performance of Diesel cycle and Otto cycle with heat transfer loss, and analyzed the effect of heat transfer loss on the performance. Al-Hinti et al. [15] studied the performance of Diesel cycle by using different heat transfer model. Qin et al. [16] and Ge et al. [17] derived the performance characteristics of Diesel cycle with friction loss and heat transfer loss. The works mentioned above were performed without considering the variable specific heats of the working fluid. Ghatak and Chakraborty [18] analyzed the performance of Dual cycle by considering the effect of heat transfer loss and variable specific heats of working fluid. Chen et al. [19] studied the performance characteristics of an irreversible Dual cycle with friction loss and linear variable specific heats of working fluid. Ge et al. [20-22] studied the performance of endoreversible and irreversible Otto cycle and Diesel cycle with variable specific

heats of the working fluid. Chen et al. [23] modeled a class of universal heat engine cycle with friction loss and heat transfer loss by considering the effect of variable specific heats of working fluid.

Rectangular cycle consists of four processes: an isochoric and an isobaric heat addition process, an isochoric and an isobaric heat rejection process. Ferreira Da Silva [24] derived the power output and the efficiency of rectangular cycle by using classical thermodynamics. Liu et al. [25] modeled endoreversible rectangular cycle with heat transfer loss and studied the performance characteristics of the cycle. Liu et al. [26] modeled irreversible rectangular cycle with friction loss and heat transfer loss by using finite time thermodynamics, and analyzed the effect of friction loss and heat transfer loss on cycle performance. Based on Refs. [24-26], this paper will study the performance characteristics of endoreversible rectangular cycle with heat transfer loss and variable specific heats of working fluid.

## 2. Cycle model

An air standard rectangular cycle is shown in Figure 1. The heat additions are an isochoric process 1-2 and an isobaric process 2-3; the heat rejections are an isochoric process 3-4 and an isobaric process 4-1.

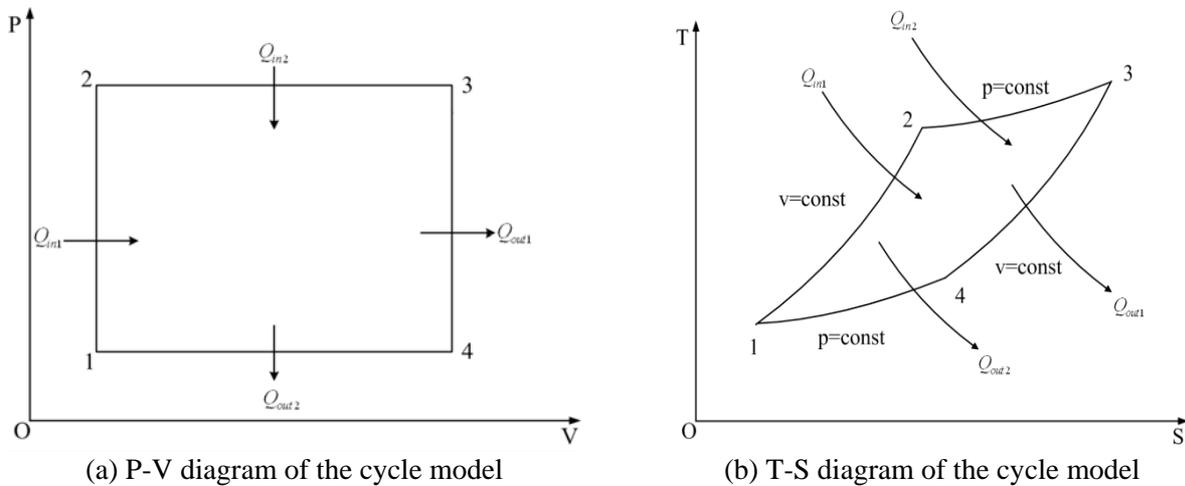


Figure 1. Endoreversible rectangular cycle model

The specific heats of working fluid are variable in practical cycle, and the performance of the cycle is affected greatly by the variation. According to Refs. [20, 24], it can be supposed that the specific heats of the working fluid are only related to its temperature, and over the temperature ranges generally encountered for gases in heat engines (300-2200K), the specific heats show a linear relationship with the temperature, which may be closely approximated in the following forms:

$$C_{pm} = a_p + kT \quad (1)$$

$$C_{vm} = b_v + kT \quad (2)$$

where  $a_p$ ,  $b_v$  and  $k$  are constants. According to the relation between  $C_{pm}$  and  $C_{vm}$

$$R = C_{pm} - C_{vm} = a_p - b_v \quad (3)$$

where  $R$  is the molar gas constant of the working fluid.

The heat added to unit mass of working fluid per cycle may be written as

$$\begin{aligned} Q_{in} &= Q_{in1} + Q_{in2} = Q_{12} + Q_{23} = \int_{T_1}^{T_2} C_{vm} dT + \int_{T_2}^{T_3} C_{pm} dT \\ &= b_v(T_2 - T_1) + 0.5k(T_2^2 - T_1^2) + a_p(T_3 - T_2) + 0.5k(T_3^2 - T_2^2) \\ &= b_v(T_2 - T_1) + a_p(T_3 - T_2) + 0.5k(T_3^2 - T_1^2) \end{aligned} \quad (4)$$

The heat rejected from unit mass of working fluid per cycle may be written as

$$\begin{aligned} Q_{out} &= Q_{out1} + Q_{out2} = Q_{34} + Q_{41} = \int_{T_4}^{T_3} C_{vm} dT + \int_{T_1}^{T_4} C_{pm} dT \\ &= b_v(T_3 - T_4) + 0.5k(T_3^2 - T_4^2) + a_p(T_4 - T_1) + 0.5k(T_4^2 - T_1^2) \\ &= b_v(T_3 - T_4) + a_p(T_4 - T_1) + 0.5k(T_3^2 - T_1^2) \end{aligned} \quad (5)$$

The work output of the cycle is

$$\begin{aligned} W &= Q_{in} - Q_{out} = b_v(T_2 + T_4 - T_3 - T_1) + a_p(T_3 + T_1 - T_2 - T_4) \\ &= (a_p - b_v)(T_3 + T_1 - T_2 - T_4) = R(T_3 + T_1 - T_2 - T_4) \end{aligned} \quad (6)$$

According to the ideal gas equation  $pV = nRT$ , one has

$$V_3/V_2 = T_3/T_2 \quad (7)$$

$$V_4/V_1 = T_4/T_1 \quad (8)$$

The compression ratio is defined as  $\gamma = V_3/V_2$ , therefore

$$T_3 = \gamma T_2 \quad (9)$$

$$T_4 = \gamma T_1 \quad (10)$$

$$Q_{in} = b_v(T_2 - T_1) + a_p T_2(\gamma - 1) + 0.5k(\gamma^2 T_2^2 - T_1^2) \quad (11)$$

$$Q_{out} = \gamma b_v(T_2 - T_1) + a_p T_1(\gamma - 1) + 0.5k(\gamma^2 T_2^2 - T_1^2) \quad (12)$$

$$W = R(\gamma - 1)(T_2 - T_1) \quad (13)$$

There are no losses in an ideal rectangular cycle, but the heat transfer loss can not be ignored in an endoreversible rectangular cycle. One can assume that the heat transfer loss through the cylinder wall is proportional to the temperature difference between the working fluid and the atmosphere, and that the wall temperature is a constant at  $T_0$ . One has the heat added to unit mass of working fluid by combustion as the following relation [14-16].

$$Q_{in} = \alpha - \beta(T_1 + T_3) = \alpha - \beta(\gamma T_2 + T_1) \quad (14)$$

where  $\alpha$  and  $\beta$  are two constants related to the combustion and heat transfer. The efficiency of the cycle is

$$\eta = \frac{W}{Q_{in}} = \frac{R(\gamma - 1)(T_2 - T_1)}{b_v(T_2 - T_1) + a_p T_2(\gamma - 1) + 0.5k(\gamma^2 T_2^2 - T_1^2)} \quad (15)$$

When  $\gamma$  and  $T_1$  are given,  $T_2$  can be obtained from Eq. (11) and Eq. (14).

$$T_2 = \frac{-[b_v + a_p(\gamma - 1) + \beta\gamma] + \sqrt{[b_v + a_p(\gamma - 1) + \beta\gamma]^2 + 2k\gamma^2(b_v T_1 + 0.5kT_1^2 + \alpha - \beta T_1)}}{k\gamma^2} \quad (16)$$

Defining

$$A = b_v + a_p(\gamma - 1) + \beta\gamma \quad (17)$$

$$B = b_v T_1 + 0.5kT_1^2 + \alpha - \beta T_1 \quad (18)$$

The work output and the efficiency of the cycle are as follows:

$$W = R(\gamma - 1) \left( \frac{-A + \sqrt{A^2 + 2k\gamma^2 B}}{k\gamma^2} - T_1 \right) \quad (19)$$

$$\eta = \frac{R(\gamma - 1)(\sqrt{A^2 + 2k\gamma^2 B} - A)}{[b_v + a_p(\gamma - 1)](\sqrt{A^2 + 2k\gamma^2 B} - A) + A^2 + k\gamma^2 B - A\sqrt{A^2 + 2k\gamma^2 B} - b_v k\gamma^2 T_1 - 0.5k^2\gamma^2 T_1^2} \quad (20)$$

### 3. Numerical examples and discussion

According to Refs. [14, 20], ranges of parameters are as follows:  $\gamma = 1.0 - 10.0$ ,  $\alpha = 60000 - 70000 \text{ J/mol}$ ,  $\beta = 20 - 30 \text{ J/mol} \cdot \text{K}$ ,  $k = 0.003844 - 0.009844 \text{ J/mol} \cdot \text{K}^2$ ,  $b_v = 19.868 - 23.868 \text{ J/mol} \cdot \text{K}$ ,  $T_1 = 300 - 400 \text{ K}$ .

Using the above ranges of parameters, the characteristics curves of  $W - \gamma$ ,  $\eta - \gamma$ , and  $W - \eta$  are plotted as in Figures 2-14.

Figures 2-13 show the effects of different parameters on cycle performance when  $T_1 = 300 \text{ K}$ . One can see that the work output versus compression ratio characteristics and the efficiency versus compression ratio characteristics are parabolic curves, and the work output versus efficiency is loop shaped.

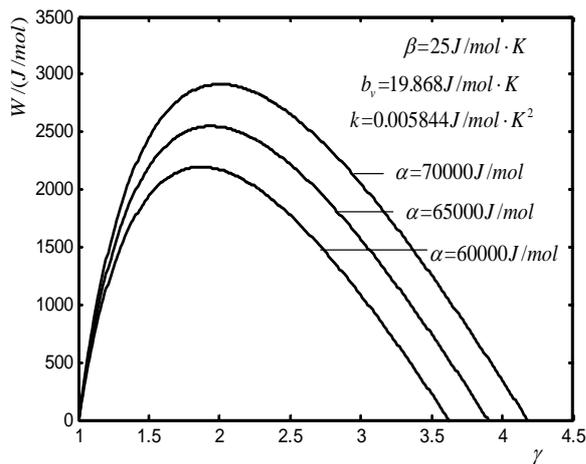


Figure 2. The influence of  $\alpha$  on cycle work output

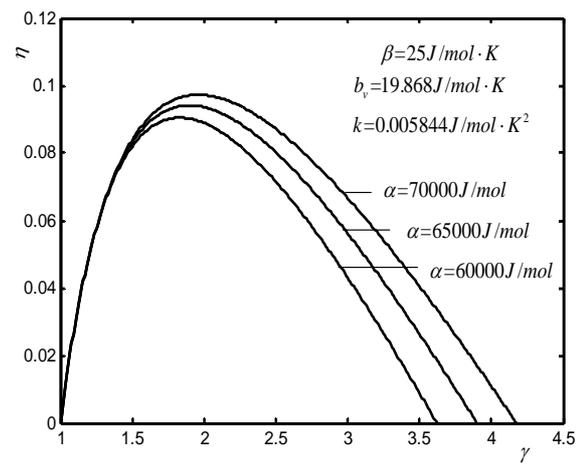


Figure 3. The influence of  $\alpha$  on cycle efficiency

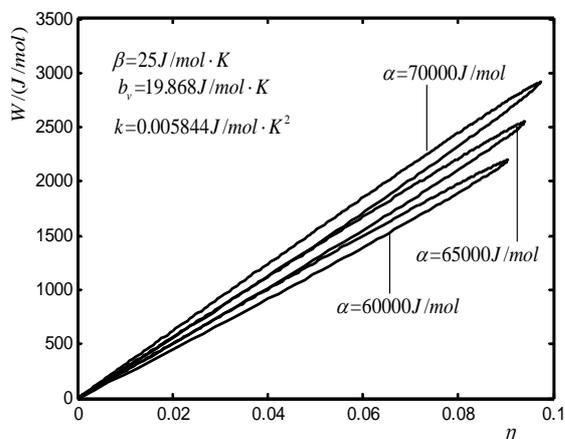


Figure 4. The influence of  $\alpha$  on cycle work output versus efficiency

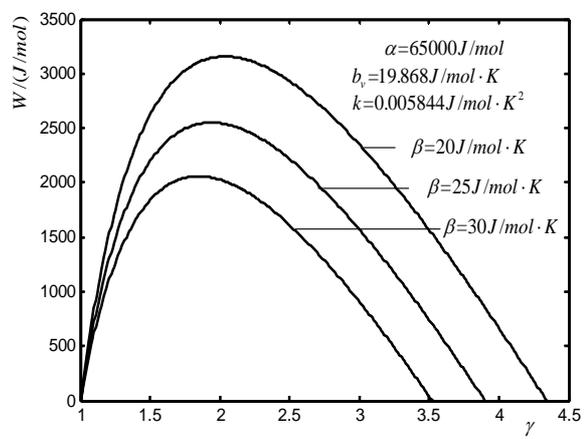


Figure 5. The influence of  $\beta$  on cycle work output

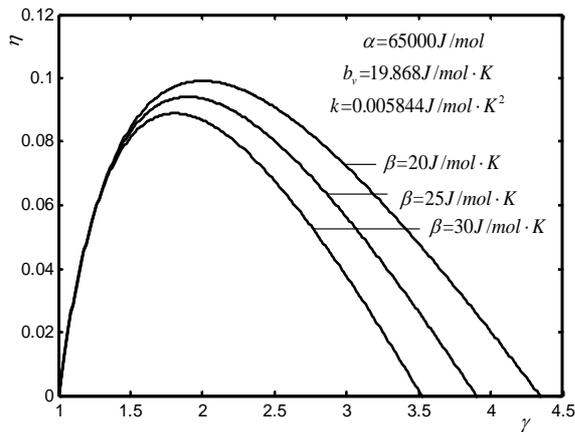


Figure 6. The influence of  $\beta$  on cycle efficiency

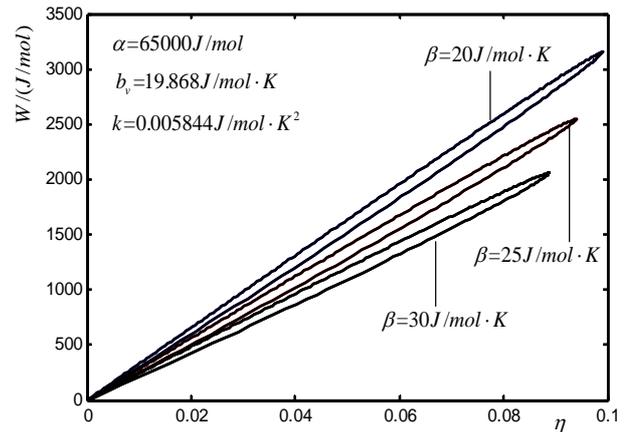


Figure 7. The influence of  $\beta$  on cycle work output versus efficiency

Figures 2-7 show the effects of combustion and heat transfer.  $\alpha$  is a parameter related to the combustion and it reflects the heating value of the fuel.  $\beta$  is a parameter related to heat transfer loss. In this case, when  $\alpha$  increases about 16.75%, the maximum work output increases about 32.73%, the efficiency at maximum work output increases about 7.52%, the compression ratio at maximum work output increases about 6.95%; the maximum efficiency increases about 7.51%, the work output at maximum efficiency increases about 32.75%, the compression ratio at maximum efficiency increases about 7.10%. And when  $\beta$  decreases about 33.3%, the maximum work output increases about 53.37%, the efficiency at maximum work output increases about 11.60%, the compression ratio at maximum work output increases about 10.27%; the maximum efficiency increases about 11.47%, the work output at maximum efficiency increases about 53.43%, the compression ratio at maximum efficiency increases about 10.50%.

Figures 8-10 show the effects of  $a_p$  and  $b_v$  on cycle performance. From Eqs. (1) and (2), one can see that when  $k=0$ ,  $C_{pm} = a_p$  and  $C_{vm} = b_v$ ,  $a_p$  and  $b_v$  are constant specific heats of working fluid. Because  $R = C_{pm} - C_{vm} = a_p - b_v = R = \text{constant}$ ,  $a_p$  and  $b_v$  must change synchronously. The results show that the maximum work output and the maximum efficiency of the cycle will decrease with the increases of  $a_p$  and  $b_v$ , and the values of compression ratios at maximum work output point and maximum efficiency point will decrease too. Furthermore, when  $b_v$  increases about 20.13%, the maximum work output decreases about 9.57%, the efficiency at maximum work output decreases about 14.26%, the compression ratio at maximum work output decreases about 2.06%; the maximum efficiency decreases about 14.24%, the work output at maximum efficiency decreases about 9.58%, the compression ratio at maximum efficiency increases about 2.10%.

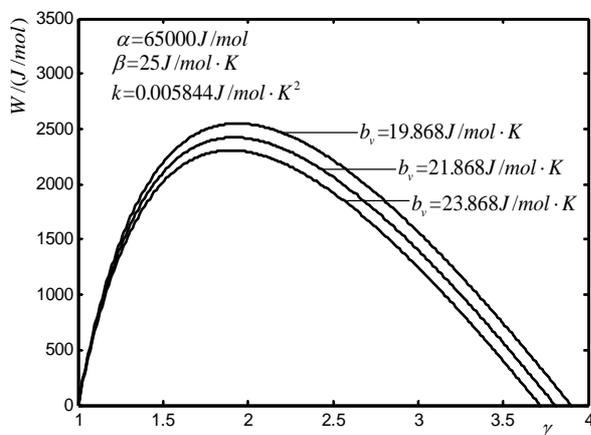


Figure 8. The influence of  $b_v$  on cycle work output

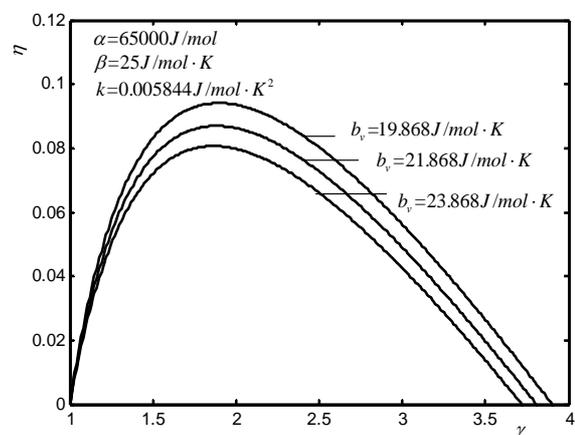


Figure 9. The influence of  $b_v$  on cycle efficiency

Figures 11-13 show the effect of  $k$  on cycle performance. The degree of variation of specific heats with temperature will be acute when  $k$  increases. One can see that the maximum work output and the maximum efficiency of the cycle will decrease with the increase of  $k$ , and the values of compression ratios at maximum work output point and maximum efficiency point will decrease too. Here, when  $k$  increases about 156%, the maximum work output decreases about 10.81%, the efficiency at maximum work output decreases about 16.37%, the compression ratio at maximum work output decreases about 2.05%; the maximum efficiency decreases about 16.35%, the work output at maximum efficiency decreases about 10.78%, the compression ratio at maximum efficiency decreases about 1.57%.

Figure 14 show the relationship between the work output and the efficiency of the cycle in different initial temperature  $T_1$ . One can see that the maximum work output and the maximum efficiency of the cycle will decrease with the increase of  $T_1$ . Furthermore, when  $T_1$  increases about 33.33%, the maximum work output decreases about 27.14%, the efficiency at maximum work output decreases about 19.04%, the compression ratio at maximum work output decreases about 12.89%; the maximum efficiency decreases about 19.12%, the work output at maximum efficiency decreases about 28.12%, and the compression ratio at maximum efficiency decreases about 12.10%.

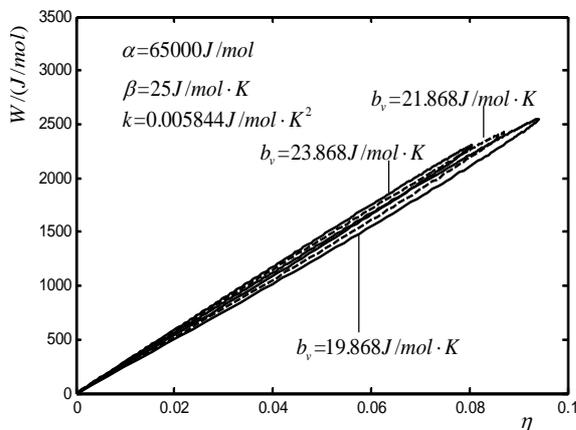


Figure 10. The influence of  $b_v$  on cycle work output versus efficiency

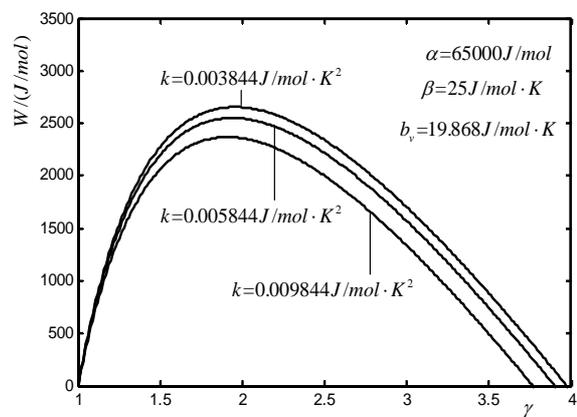


Figure 11. The influence of  $k$  on cycle work output

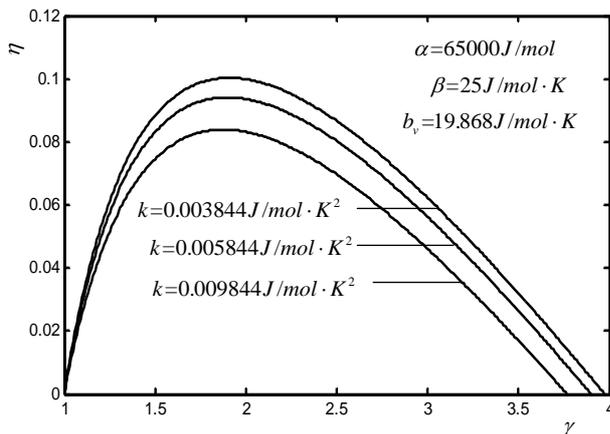


Figure 12. The influence of  $k$  on cycle efficiency

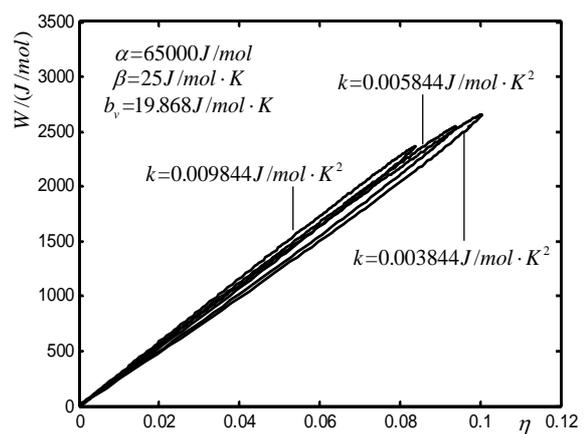


Figure 13. The influence of  $k$  on cycle work output versus efficiency

#### 4. Conclusion

In this paper, an air-standard rectangular cycle with heat transfer loss and variable specific heats of working fluid is analyzed by using finite-time thermodynamics. The analytical functions of the work output and the efficiency are derived, and the performance characteristics of the cycle are obtained by detailed numerical examples. The results show that the effects of heat transfer loss and variable specific

heats of working fluid on the cycle performance are obvious. The results may provide some guidelines for the application of the rectangular cycle.

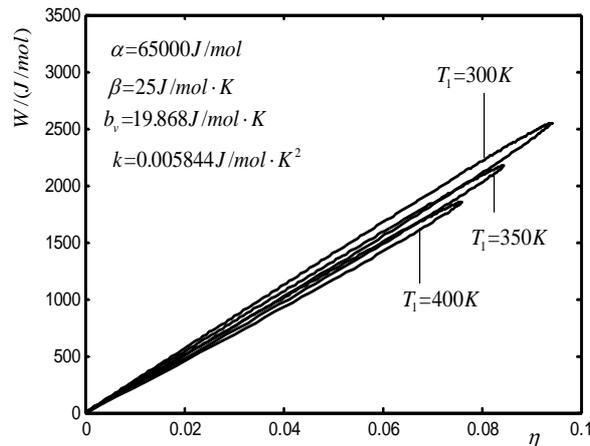


Figure 14. The influence of  $T_1$  on cycle work output versus efficiency

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