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Analytical and numerical thermal buckling analysis investigation of unidirectional and woven reinforcement composite plate structural

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Abstract
In this study, evaluated of the critical thermal effect caused the buckling of unidirectional and woven composite plate with different aspect ratio of plate combined from different types of long and woven reinforcement fiber and different resin material types. The thermal buckling analysis by using theoretical analysis with solution the general equation of motion of orthotropic composite simply supported plate with buckling thermal effect and evaluated the effect of reinforcement type and resin types on the buckling temperature with effect of volume fraction of reinforcement fiber and resin materials. In addition to, analysis the problem of thermal buckling by numerical study with using finite element method and compare the results of numerical analysis with theoretical results of thermal bucking plate and evaluated the agreement between the two methods used. The results are the critical thermal buckling temperature of orthotropic composite plate with effect of different reinforcement fiber as unidirectional or woven fiber and different resin materials with various volume fraction of reinforcement fiber and effect of aspect ratio of composite plate and compare of critical temperature with different reinforcement types. In addition to compare between the theoretical and numerical analysis and evaluated the maximum percent error about (3.5%). And, the results showed that the critical temperature buckling of unidirectional reinforcement fiber more than critical bucking temperature of woven reinforcement fiber and the buckling temperature increasing with increase the volume fraction of reinforcement fiber.

Keywords: Plate buckling; Thermal buckling; Thermal plate; Composite plate; Thermal composite plate; Thermal orthotropic plate.

1. Introduction
Composite structures, like beams, plates and shells, are used in many engineering applications because of different possibilities for design process. These structures are often subjected to severe thermal environments during launching and re-entry, and thermal loads become a primary design factor in specific cases. Geometrically perfect plates that are restrained from in-plane expansion when slowly heated generally develop compressive stresses and then buckle at a specific temperature, [1].
Composite plates when subjected to temperature environments the thermal stresses are developed at the edges of the plates due to constraint thermal expansion coefficients. These thermal stresses induce thermal buckling loads which may have affected the structural behavior of the plates consequently result

buckling of the plate. Therefore, it necessitates to understand the linear buckling behavior of the composite plates induced by thermal loading, [2].

Mostapha Raki et al. [3], presented the derive of equilibrium and stability equations of a rectangular plate made of functionally graded material (FGM) under thermal loads, based on the higher order shear deformation plate theory. A buckling analysis of a functionally graded plate under one type of thermal loads is carried out and results in closed form solutions, uniform temperature rise and gradient through the thickness are considered and the buckling temperatures are derived. The critical buckling temperature relations are reduced to the respective relations for functionally graded plates with a linear composition of constituent materials and homogeneous plates.

M.E. Fares et al. [4], a multi objective optimization problem is presented to determine the optimal layer thickness and optimal closed loop control function for a symmetric cross-ply laminate subjected to thermo-mechanical loadings. The optimization procedure aims to maximize the critical combination of the applied edges load and temperature levels and to minimize the laminate dynamic response subject to constraints on the thickness and control energy. The objective of the optimization problem is formulated based on a consistent first-order shear deformation theory without introducing a shear correction factor.

Ahmet Erkliğ and Eyüp Yeter [5], in this paper the effects of cut-outs on the thermal buckling behavior of hybrid composite plates in cross-ply and angle-ply laminate are presented. The effects of eccentric cut-out size in different plate aspect ratios and boundary conditions on the thermal buckling behavior of the cross-ply and angle-ply laminated hybrid composite plates are also investigated. Finite element analysis is also performed to calculate thermal buckling temperatures for Kevlar/Epoxy, Boron/Epoxy and E-glass/Epoxy.

A. R. Khorshidv and M. R. Eslami [6], in this paper, buckling of elastic, circular plates made of functionally graded material subjected to thermal loading have been investigated. Boundary condition of the plate as immovable clamped edge is considered. The Nonlinear equilibrium equations are derived based on the classical plate theory using variational formulations. Linear stability equations are used to obtain the critical buckling of solid FG circular plate under thermal load as uniform temperature rise, linear and nonlinear temperature distribution through the thickness.

In this study presented the analytical solution of critical buckling temperature of orthotropic unidirectional and woven composite plate with different aspect ratio of plate, volume fraction of reinforcement fiber and types of reinforcement fiber and resin materials. And compare the analytical results with numerical results evaluated by using finite element method with using Ansys program Ver. 14.

2. Theoretical study

The theoretical investigation of thermal buckling investigation included evaluated the mechanical and thermal properties of unidirectional and woven reinforcement fiber, then, evaluated of the thermal buckling of composite plate with analysis the general equation of motion of composite plate with thermal buckling effect for simply supported plate, as,

2.1 Mechanical and thermal properties of orthotropic composite materials

The mechanical properties evaluated of unidirectional and woven reinforcement composite plate are modulus of elasticity with longitudinal and transverse direction of unidirectional fiber and for woven reinforcement fiber in 1 and 2 directions, shear modulus of elasticity, and Poisson's ratio. And, the thermal properties of unidirectional and woven reinforcement composite materials are the thermal expansion with longitudinal and transverse direction of unidirectional fiber and for woven reinforcement fiber in 1 and 2 directions.

The mechanical properties of unidirectional reinforcement fiber composite materials plate can be evaluating as, [7],

$$E_1 = E_{l_f}.\forall_{l_f} + E_m.\left(1 - \forall_{l_f}\right),\ E_2 = E_m.\left(\frac{E_{l_f}}{\left(1-\forall_{l_f}\right).E_{l_f} + E_m.\forall_{l_f}}\right)$$

$$G_{12} = G_m.\left(\frac{G_{l_f}}{\left(1-\forall_{l_f}\right).G_{l_f} + G_m.\forall_{l_f}}\right),\ \nu_{12} = \nu_{l_f}.\forall_{l_f} + \nu_m.\forall_m$$

$$\alpha_1 = \frac{\alpha_f.E_f.\forall_f + \alpha_m.E_m.\forall_m}{E_f.\forall_f + E_m.\forall_m},\ \ \alpha_2 = \alpha_m.\forall_m + \alpha_f.\forall_f + \frac{\left([\nu_f.E_m - \nu_m.E_f].\forall_f.\forall_m\right)}{\left(E_m.\forall_m + E_f.\forall_f\right)}.\left(\alpha_f - \alpha_m\right) \tag{1}$$

where, $E_{l_f}, E_m, G_{l_f}, G_m, \nu_{l_f}, \nu_m$ are modulus of elasticity, shear modulus elasticity, and Poisson's ratio of unidirectional fiber and resin material, respectively, α_f, α_m are thermal expansions of unidirectional and resin materials, respectively. And, $\forall_f, \forall_m$ are volume fractions of reinforcement and resin materials, respectively.
And, the mechanical and thermal properties of Woven reinforcement fiber composite materials plate are, [7],

$$E_{1_w} = k.E_1 + (1-k)E_2 \ ,$$
$$E_{2_w} = (1-k)E_1 + k.E_2 \ ,$$
$$G_{12_w} = G_{12} \ , \nu_{12_w} = \frac{\nu_{12}.E_2}{(k.E_2 + (1-k)E_1)}$$
$$\alpha_{1_w} = k.\alpha_1 + (1-k).\alpha_2, \quad \alpha_{2_w} = (1-k).\alpha_1 + k.\alpha_2 \tag{2}$$

where, $k = \frac{n_1}{n_1+n_2}$, n_1 =number of warp yarns per meter, n_2 =number of fill yarns per meter.
And, E_{1_w}, E_{2_w}, G_{12_w}, and ν_{12_w} are mechanical properties of woven fabrics in 1 and 2-directions; and E_1, E_2, G_{12}, and ν_{12} as for unidirectional reinforcement composite materials shown in equation (1).

2.2 Buckling analysis of orthotropic composite materials plate

Thin plates of various shapes used in naval and aeronautical structures are often subjected to normal compressive and shearing loads acting in the middle plane of the plate (in-plane loads). Under certain conditions such loads can result in a plate buckling. Buckling or elastic instability of plates is of great practical importance. The buckling load depends on the plate thickness: the thinner the plate, the lower is the buckling load. In many cases, a failure of thin plate elements may be attributed to an elastic instability and not to the lack of their strength. Therefore, plate buckling analysis presents an integral part of the general analysis of a structure.
We can determine the expressions for the bending and twisting moments with the displacement and strain fields as in the following equations, [8],

$$U_x = -zw_{,x} \ ,$$
$$U_y = -zw_{,y} \ ,$$
$$U_z = w \tag{3}$$

$$\varepsilon_{xx} = -z\, w_{,xx} \ ,$$
$$\varepsilon_{yy} = -z\, w_{,yy} \ ,$$
$$\gamma_{xy} = -2z\, w_{,xy} \tag{4}$$

where, w is deflection of plate in z-direction.
The stresses-strain relation of orthotropic composite materials in 2-dimeansions can be written as the following, [8],

$$\sigma_{xx} = \frac{E_{xx}}{1-\nu_{xy}\,\nu_{yx}}\varepsilon_{xx} + \frac{\nu_{xy}\,E_{yy}}{1-\nu_{xy}\,\nu_{yx}}\varepsilon_{yy} \ ,$$
$$\sigma_{yy} = \frac{\nu_{xy}\,E_{yy}}{1-\nu_{xy}\,\nu_{yx}}\varepsilon_{xx} + \frac{E_{yy}}{1-\nu_{xy}\,\nu_{yx}}\varepsilon_{yy} \ ,$$
$$\tau_{xy} = G_{xy}\,\gamma_{xy} \tag{5}$$

Then, by Substituting for strain equations, equation (4), into stresses-strain relation, equation (5), get,

$$\sigma_{xx} = -z\left(\frac{E_{xx}}{1-\nu_{xy}\,\nu_{yx}}w_{,xx} + \frac{\nu_{xy}\,E_{yy}}{1-\nu_{xy}\,\nu_{yx}}w_{,yy}\right),$$
$$\sigma_{yy} = -z\left(\frac{\nu_{xy}\,E_{yy}}{1-\nu_{xy}\,\nu_{yx}}w_{,xx} + \frac{E_{yy}}{1-\nu_{xy}\,\nu_{yx}}w_{,yy}\right)$$
$$\tau_{xy} = -2G_{xy}\ z\, w_{,xy} \tag{6}$$

The bending moments (per unit length) M_x, M_y and M_{xy} of orthotropic composite plate are then determined as, [8],

$$M_x = \int_{-h/2}^{h/2} \sigma_{xx} z\, dz ,$$
$$M_y = \int_{-h/2}^{h/2} \tau_{yy} z\, dz ,$$
$$M_{xy} = \int_{-h/2}^{h/2} \tau_{xy}\, z\, dz \qquad (7)$$

Then, by substitution equation (6) into equation (7), get the bending moments of orthotropic composite plate, as,

$$M_x = -\int_{-h/2}^{h/2} z^2 \left(\frac{E_{xx}}{1-v_{xy}\,v_{yx}} w,_{xx} + \frac{v_{xy}\,E_{yy}}{1-v_{xy}\,v_{yx}} w,_{yy}\right) dz = -\left(D_{11} w,_{xx} + D_{12} w,_{yy}\right)$$
$$M_y = -\int_{-h/2}^{h/2} z^2 \left(\frac{v_{xy}\,E_{yy}}{1-v_{xy}\,v_{yx}} w,_{xx} + \frac{E_{yy}}{1-v_{xy}\,v_{yx}} w,_{yy}\right) dz = -\left(D_{22} w,_{yy} + D_{12} w,_{xx}\right)$$
$$M_{xy} = -\int_{-h/2}^{h/2} 2G_{xy} z^2 w,_{xy}\, dz = 2D_{66} w,_{xy} \qquad (8)$$

where, $D_{11} = \frac{E_{xx}\, h^3}{12(1-v_{xy}\,v_{yx})}$, $D_{22} = \frac{E_{yy}\, h^3}{12(1-v_{xy}\,v_{yx})}$, $D_{12} = \frac{v_{xy}\,E_{yy}\, h^3}{12(1-v_{xy}\,v_{yx})}$, $D_{66} = \frac{G_{xy}\, h^3}{12}$ And, $v_{xy}\,E_{yy} = v_{yx}\,E_{xx}$

With using the general differential equation of orthotropic composite plate, as, [8],

$$\frac{\partial^2 M_x}{\partial x^2} - 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = -q \qquad (9)$$

where, q supplied bending load.

And, substituting for the bending and twisting moments from equation (8) into equation (9). So the above equations will be,

$$-\frac{\partial^2}{\partial x^2}\left(D_{11}\frac{\partial^2 w}{\partial x^2} + D_{12}\frac{\partial^2 w}{\partial y^2}\right) - 4\frac{\partial^2}{\partial x \partial y}\left(D_{66}\frac{\partial^2 w}{\partial x \partial y}\right) - \frac{\partial^2}{\partial y^2}\left(D_{22}\frac{\partial^2 w}{\partial y^2} + D_{12}\frac{\partial^2 w}{\partial x^2}\right) = -q$$

Or,

$$D_{11}\frac{\partial^4 w}{\partial x^4} + 2(D_{12} + 2D_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22}\frac{\partial^4 w}{\partial y^4} = q \qquad (10)$$

So the load supplied on the plate due to buckling effect is, $q = -\left(N_x w,_{xx} + N_y w,_{yy} + 2N_{xy} w,_{xy}\right)$, then, the general equation of buckling orthotropic plate will be as, [9],

$$D_{11}\frac{\partial^4 w}{\partial x^4} + 2(D_{12} + 2D_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22}\frac{\partial^4 w}{\partial y^4} + N_x\frac{\partial^2 w}{\partial x^2} + N_y\frac{\partial^2 w}{\partial y^2} + 2N_{xy}\frac{\partial^2 w}{\partial x \partial y} = 0 \qquad (11)$$

where, N_x, N_y, N_{xy} are buckling load in x, y, and xy-direction of plate.

The solution of buckling load in equation (11) needed the general behavior of deflection plate as a faction of x and y. So, to evaluate the deflection of plate as a faction of x and y of simply supported plate subjected the boundary conditions as, [8],

$$M_x = -\left(D_{11} w,_{xx} + D_{12} w,_{yy}\right) = 0, and, \mathrm{w} = 0 \text{ , On the edge } x = 0 \text{ and } x = a$$
$$M_y = -\left(D_{22} w,_{yy} + D_{12} w,_{xx}\right) = 0, and, \mathrm{w} = 0 \text{, On the edge y=0 and y=b} \qquad (12)$$

The solution of equation (11) satisfying the boundary conditions equation (12) can be written as,

$$w = A\, sin\frac{m\pi x}{a} sin\frac{n\pi y}{b} \qquad (13)$$

where, a, b are length and width of plate.

By substitution equation (13) in to equation (11), get the general equation of buckling load, as,

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$$\begin{bmatrix} D_{11}\left(\frac{m\pi}{a}\right)^4 + 2(D_{12}+2D_{66})\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2 + \\ D_{22}\left(\frac{n\pi}{b}\right)^4 - N_x\left(\frac{m\pi}{a}\right)^2 - N_y\left(\frac{n\pi}{b}\right)^2 \end{bmatrix} sin\frac{m\pi x}{a} sin\frac{n\pi y}{b} + 2N_{xy}\left(\frac{m\pi}{a}\right)\left(\frac{n\pi}{b}\right) cos\frac{m\pi x}{a} cos\frac{n\pi y}{b} = 0 \qquad (14)$$

Since no bukling load subjected on the plate, then, assuming the buckling load N_x, N_y, and N_{xy} are equal to the load resultant from the thermal effect, therfore the N_x, N_y, and N_{xy} are defined as, [10],

$$\begin{bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{bmatrix} = \begin{bmatrix} N_{xx}^T \\ N_{yy}^T \\ N_{xy}^T \end{bmatrix} = \begin{bmatrix} \left(\frac{E_{xx}}{1-\nu_{xy}.\nu_{yx}}\right) & \left(\frac{\nu_{xy}.E_{yy}}{1-\nu_{xy}.\nu_{yx}}\right) & 0 \\ \left(\frac{\nu_{xy}.E_{yy}}{1-\nu_{xy}.\nu_{yx}}\right) & \left(\frac{E_{yy}}{1-\nu_{xy}.\nu_{yx}}\right) & 0 \\ 0 & 0 & G_{xy} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ 0 \end{bmatrix} h.\Delta T \qquad (15)$$

Then, substation equation (15) in to equation (14), get the general equation of thermal buckling effect, as,

$$\begin{pmatrix} D_{11}\left(\frac{m\pi}{a}\right)^4 + 2(D_{12}+2D_{66})\left(\frac{m\pi}{a}\right)^2.\left(\frac{n\pi}{b}\right)^2 + D_{22}.\left(\frac{n\pi}{b}\right)^4 - \\ \left[\left(\frac{E_{xx}}{1-\nu_{xy}.\nu_{yx}}\right).\alpha_1 + \left(\frac{\nu_{xy}.E_{yy}}{1-\nu_{xy}.\nu_{yx}}\right).\alpha_2\right]\left(\frac{m\pi}{a}\right)^2.h.\Delta T - \\ \left[\left(\frac{\nu_{xy}.E_{yy}}{1-\nu_{xy}.\nu_{yx}}\right).\alpha_1 + \left(\frac{E_{yy}}{1-\nu_{xy}.\nu_{yx}}\right).\alpha_2\right].\left(\frac{n\pi}{b}\right)^2.h.\Delta T \end{pmatrix} = 0 \qquad (16)$$

Then, the critical different buckling in temperature can be evaluated, from equation (16), as,

$$\Delta T = \frac{\left[D_{11}\left(\frac{m\pi}{a}\right)^4 + 2(D_{12}+2D_{66})\left(\frac{m\pi}{a}\right)^2.\left(\frac{n\pi}{b}\right)^2 + D_{22}.\left(\frac{n\pi}{b}\right)^4\right]}{\left[\left(\frac{E_{xx}}{1-\nu_{xy}.\nu_{yx}}\right).\alpha_{xx} + \left(\frac{\nu_{xy}.E_{yy}}{1-\nu_{xy}.\nu_{yx}}\right).\alpha_{yy}\right]\left(\frac{m\pi}{a}\right)^2.h + \left[\left(\frac{\nu_{xy}.E_{yy}}{1-\nu_{xy}.\nu_{yx}}\right).\alpha_{xx} + \left(\frac{E_{yy}}{1-\nu_{xy}.\nu_{yx}}\right).\alpha_{yy}\right].\left(\frac{n\pi}{b}\right)^2.h} \qquad (17)$$

where, $E_{xx} = E_1$ – for unidirectional reinforcement fiber, and E_{1_w} – for woven reinforcement fiber.
$E_{yy} = E_2$ – for unidirectional reinforcement fiber, and E_{2_w} – for woven reinforcement fiber.
$G_{xy} = G_{12}$ – for unidirectional reinforcement fiber, and G_{12_w} – for woven reinforcement fiber.
$\alpha_{xx} = \alpha_1$ – for unidirectional reinforcement fiber, and α_{1_w} – for woven reinforcement fiber.
$\alpha_{yy} = \alpha_2$ – for unidirectional reinforcement fiber, and α_{2_w} – for woven reinforcement fiber.
It is evident that a minimum value of ΔT is reached for $n = 1$ and $m = 1$.
By using Fortran power station Ver. 4. Program to building program can be evaluated the buckling temperature from Eq. 14. The program was build evaluated the buckling temperature with different volume fraction of unidirectional and woven simply supported composite plate. The build program included two parts, first part: evaluated the mechanical and thermal properties of orthotropic composite plate, and second part: using the mechanical and thermal properties evaluated with first part of program to evaluating the buckling temperature of composite plate, the build program shown in flow chart, as in Figure 1. The input required to program are, mechanical and thermal properties of reinforcement unidirectional and woven fiber and resin material, and dimensions of composite plate. And the output program is buckling temperature with different volume fraction of reinforcement fiber and different reinforcement and resin materials types, in addition to buckling temperature with different aspect ratio of composite plate.

3. Numerical modelling

The numerical study of different orthotropic composite plate with critical buckling temperature of plate evaluated by using the finite elements method was applied by using the ANSYS program (ver. 14).
The numerical procedure to evaluated the critical buckling temperature of composite plate are evaluate of critical mechanical buckling load evaluated with Ansys program by subjected to critical buckling temperature evaluated by theoretical analysis and compare the numerical results with mechanical

buckling load evaluated by using theoretical analysis with using of equation (15). by subjected to theoretical results of critical buckling temperature evaluated by equation (17).

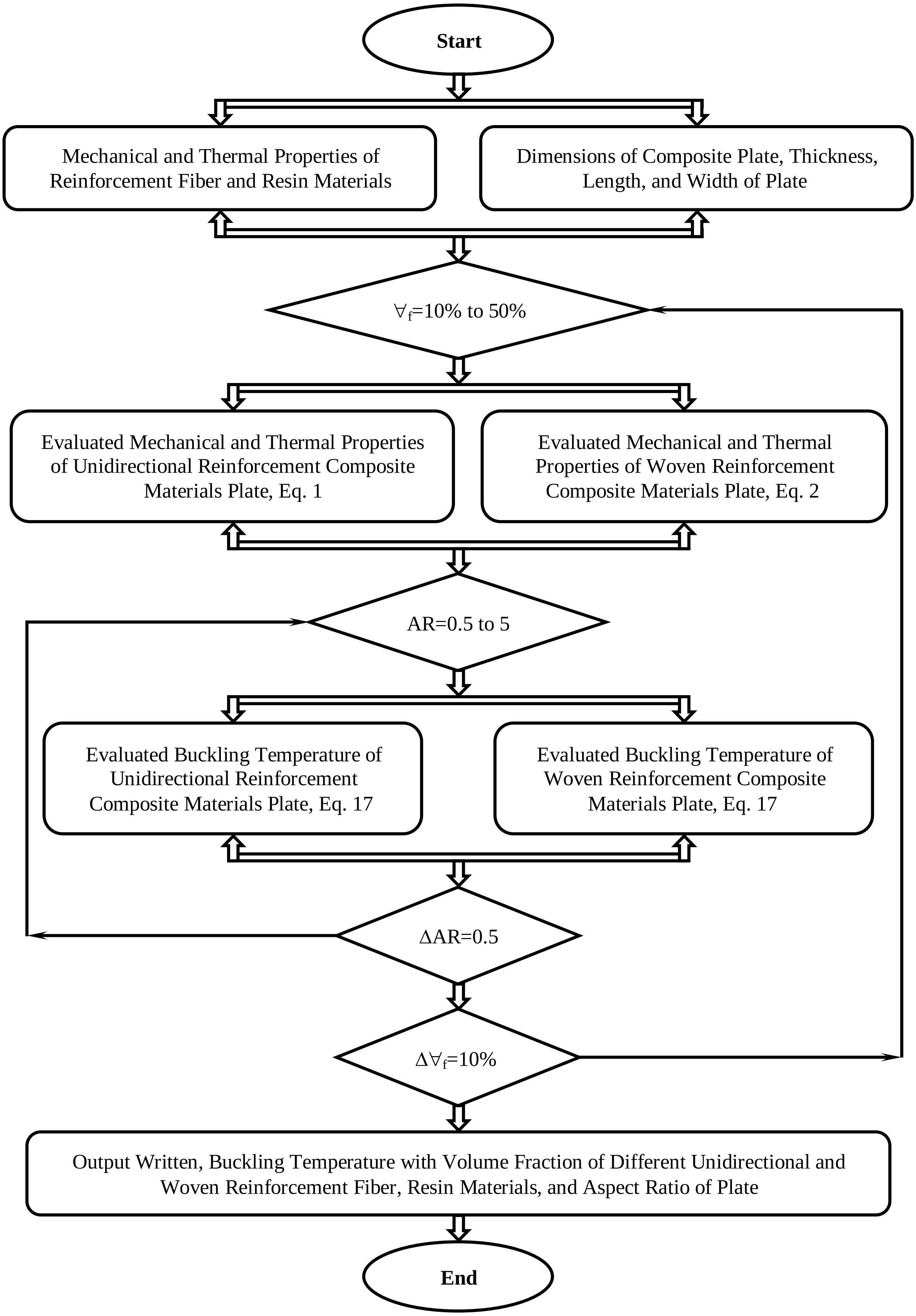

Figure 1. Flow chart program evaluated buckling temperature of orthotropic unidirectional and woven simply supported composite plate

The three dimensional model were built and the element (SHELL 8 node 281) were used. Shell 281 is suitable for analyzing thin to moderately-thick shell structures. The element has eight nodes with six degrees of freedom at each node: translations in the x, y, and z axes, and rotations about the x, y, and z-axes. (When using the membrane option, the element has translational degrees of freedom only.)
Shell 281 is well-suited for linear, large rotation, and/or large strain nonlinear applications. Change in shell thickness is accounted for in nonlinear analyses. The element accounts for follower (load stiffness) effects of distributed pressures. Shell 281 may be used for layered applications for modelling composite shells or sandwich construction. The accuracy in modelling composite shells is governed by the first-order shear-deformation theory (usually referred to as Mindlin-Reissner shell theory). The element formulation is based on logarithmic strain and true stress measures. The element kinematics allow for finite membrane strains (stretching). However, the curvature changes within a time increment are assumed to be small.
The Figure 2 shows the geometry, node locations, and the element coordinate system for this element. The element is defined by shell section information and by eight nodes (I, J, K, L, M, N, O and P).

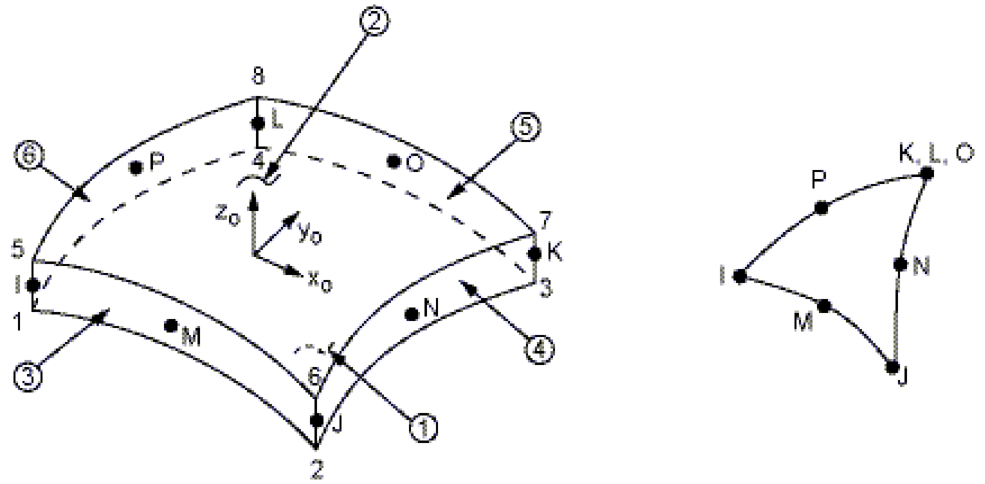

Figure 2. Shell 281 geometry

Shell 281 includes the effects of transverse shear deformation. The transverse shear stiffness of the element is,

$$E = \begin{bmatrix} E_{11} & E_{12} \\ E_{12} & E_{22} \end{bmatrix} \quad (18)$$

Shell 281 can be associated with linear elastic, elasto-plastic, creep, or hyper-elastic material properties. Only isotropic, anisotropic, and orthotropic linear elastic properties can be input for elasticity. Hyper-elastic material properties can be used with this element.
The solution output associated with the element is in two forms,

- Nodal displacements included in the overall nodal solution
- Additional element output as shown several items in Figure 3.

4. Results and discussion

The results are temperature can be supplied on the unidirectional and woven composite plate without occur buckling plate. The result of thermal buckling temperature was evaluated by theoretical analysis with solution equation of buckling simply supported composite plate with thermal buckling load effect, equation (17). And the theoretical results of thermal buckling load was get by equation (15) are comparing with numerical results of buckling load with buckling temperature effect evaluated by using ANSYS program Ver. 14, finite element methods, for different reinforcement fiber and materials types of reinforcement fiber and resin and different aspect ratio of simply supported composite plate.
The mechanical and thermal properties of reinforcement unidirectional and woven fiber and resin materials used to composite plate in this research are shown in Table 1, [7].

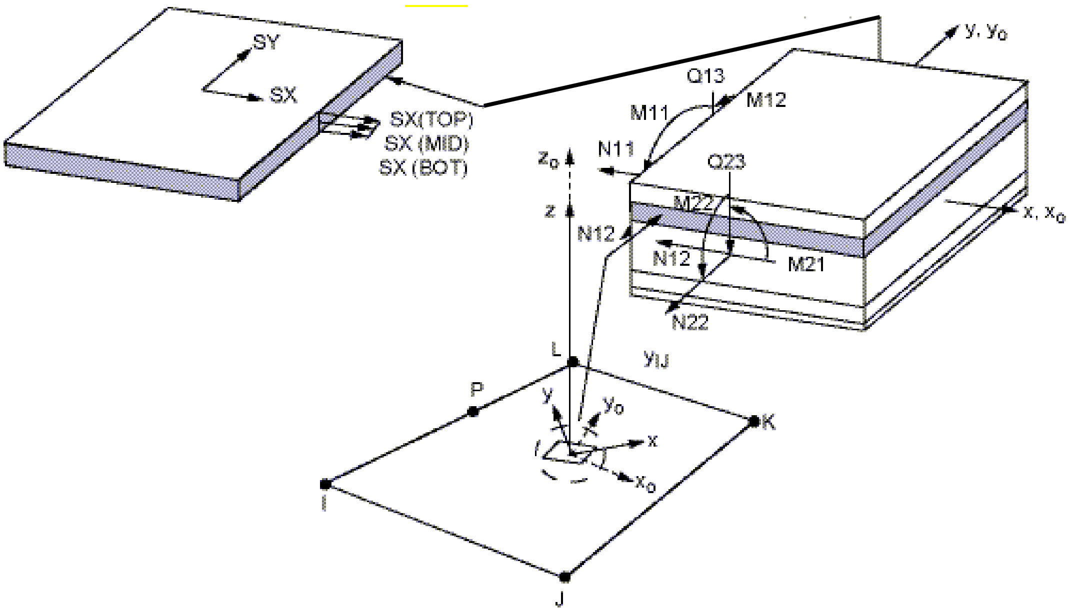

Figure 3. Shell 281 stress output

Table 1. Mechanical and thermal properties of different fibers and resin materials type

Materials	ρ (kg/m^3)	E (Gpa)	G (Gpa)	ν	α (^{o}C^{-1})	Tensile strength σ_{ult} (Mpa)	Temperature limit T_{max} (^{o}C)
Glass-E-Fibers	2600	74	30	0.25	0.5*10^{-5}	2500	700
Kevlar-49 Fiber	1450	130	12	0.4	- 0.2*10^{-5}	2900	>1500
Polyester Resin	1200	4	1.4	0.4	8*10^{-5}	80	60 to 200
Epoxy Resin	1200	4.5	1.6	0.4	11*10^{-5}	130	90 to 200

And the dimensions (length (a), width (b), and thickness (h)) of simply supported orthotropic composite plate using to evaluated the thermal buckling effect, with different aspect ratio (AR=0.5, 1, and 2) and volume fraction of reinforcement and resin materials, of different composite plate structure types, are,

- For, $AR = \frac{a}{b} = 0.5, \text{lenght} = a = 10\text{ cm}, \text{widht} = b = 20\text{ cm}, \text{thickness} = h = 5\text{ mm}$
- For, $AR = \frac{a}{b} = 1, \text{lenght} = a = 20\text{ cm}, \text{widht} = b = 20\text{ cm}, \text{thickness} = h = 5\text{ mm}$
- For, $AR = \frac{a}{b} = 2, \text{lenght} = a = 40\text{ cm}, \text{widht} = b = 20\text{ cm}, \text{thickness} = h = 5\text{ mm}$

The combine types of orthotropic composite plate studies are,

- Glass Unidirectional Reinforcements Fiber and Polyester Resin Materials.
- Glass Unidirectional Reinforcements Fiber and Epoxy Resin Materials.
- Kevlar Unidirectional Reinforcements Fiber and Polyester Resin Materials.
- Kevlar Unidirectional Reinforcements Fiber and Epoxy Resin Materials.
- Glass Woven Reinforcements Fiber and Polyester Resin Materials.
- Glass Woven Reinforcements Fiber and Epoxy Resin Materials.
- Kevlar Woven Reinforcements Fiber and Polyester Resin Materials.
- Kevlar Woven Reinforcements Fiber and Epoxy Resin Materials.

The mechanical and thermal properties of orthotropic unidirectional and woven composite materials types are shown in Table 2 and Table 3, for different unidirectional and woven reinforcement fiber and different resin materials types. From table shows that the mechanical properties of composite materials increasing with increase of volume fraction of reinforcement fiber, and thermal expansion properties of composite materials decrease with increases volume fraction of reinforcement finer. Also, shown that the thermal expansion in 2-direction more than thermal expansion in 1-direction of unidirectional composite materials and more than thermal expansion of woven composite materials.

Figures 4, 5, and 6, shown the compare between theoretical results, get form solution of general equation of thermal buckling plate, with numerical results, get from finite element methods by using ANSYS program Ver. 14, of buckling load with various volume fraction of unidirectional and woven reinforcement fiber for simply supported composite plate with different aspect ratio (AR=0.5, 1, and 2, respectively) and materials of reinforcement (Glass and Kevlar fiber) and resin (Polyester and Epoxy) matrix materials types, with buckling temperature effect (evaluated by theoretical study). From figures shows the good agreement between theoretical and numerical results with maximum error about (3.5%).

Table 2. Mechanical properties of unidirectional composite plate combined of different reinforcement fiber and different resin matrix materials

Combine of composite type	Volume fraction of resin	Volume fraction of fiber	Unidirectional reinforcement fiber and resin composite					
			E_1 (Gpa)	E_2 (Gpa)	G_{12} (Gpa)	ν_{12}	α_1 10^{-5} ($^oC^{-1}$)	α_2 10^{-5} ($^oC^{-1}$)
Glass fiber and polyester resin	90%	10%	11.00	4.418	1.548	0.385	2.955	9.005
	80%	20%	18.00	4.933	1.730	0.37	1.833	8.407
	70%	30%	25.00	5.585	1.961	0.355	1.340	7.552
	60%	40%	32.00	6.435	2.263	0.34	1.063	6.609
	50%	50%	39.00	7.590	2.675	0.325	0.885	5.625
Glass fiber and epoxy resin	90%	10%	11.45	4.966	1.767	0.385	4.214	12.300
	80%	20%	18.40	5.541	1.974	0.37	2.554	11.500
	70%	30%	25.35	6.265	2.235	0.355	1.805	10.327
	60%	40%	32.30	7.208	2.575	0.34	1.378	9.022
	50%	50%	39.25	8.484	3.038	0.325	1.102	7.654
Kevlar fiber and polyester resin	90%	10%	16.60	4.429	1.536	0.4	1.578	9.421
	80%	20%	29.20	4.962	1.700	0.4	0.699	8.625
	70%	30%	41.80	5.640	1.905	0.4	0.349	7.616
	60%	40%	54.40	6.533	2.165	0.4	0.162	6.543
	50%	50%	67.00	7.761	2.507	0.4	0.045	5.442
Kevlar fiber and epoxy resin	90%	10%	17.05	4.981	1.752	0.4	2.460	12.848
	80%	20%	29.60	5.577	1.935	0.4	1.162	11.799
	70%	30%	42.15	6.335	2.162	0.4	0.637	10.441
	60%	40%	54.70	7.331	2.449	0.4	0.353	8.987
	50%	50%	67.25	8.699	2.824	0.4	0.175	7.490

Figures 7 and 8, shown the critical (buckling) change temperature of different unidirectional and woven reinforcement fiber (glass, Kevlar reinforcement fiber), respectively, and different resin materials (polyester and epoxy resin) with various volume fraction of reinforcement fiber for different aspect ratio of composite plate (AR=0.5, a=0.1 m and b=0.2 m, AR=1, a=0.2 m and b=0.2 m, and AR=2, a=0.4 m and b=0.2 m). From figures shown that the buckling temperature increasing with increase the volume fraction of unidirectional or woven reinforcement fiber, due to decreasing of the thermal expansion of composite plate with increasing of volume fraction reinforcement fiber (as in Table 2 and Table 3), and the buckling temperature for composite plate with glass reinforcement less than the buckling temperature for composite plate with Kevlar reinforcement, since the thermal expansion of Kevlar reinforcement less than the thermal expansion of glass reinforcement. Also, the buckling temperature for composite plate with epoxy resin material less than the buckling temperature for composite plate with polyester resin material, since the thermal expansion epoxy resin material less than the thermal expansion of polyester resin materials. Then, can be see that the thermal buckling of composite plate increasing with decreasing the thermal expansion of composite plate with increasing the volume fraction of reinforcement, using reinforcement with low thermal expansion, or using resin with low thermal expansion.

Figure 9, shows the buckling temperature with various volume fraction of reinforcement fiber for different types of reinforcement composite plate effect (unidirectional and woven reinforcement types) with various types of reinforcement fiber and resin materials and different aspect ratio of composite plate. From figure shown the thermal buckling temperature for unidirectional reinforcement composite plate types more than the thermal buckling temperature for woven reinforcement composite plate types, since the thermal expansion of unidirectional reinforcement composite plate types less than the thermal expansion for the woven reinforcement fiber composite plate types.

Table 3. Mechanical properties of woven composite plate combined of different reinforcement fiber and different resin matrix materials

Combine of composite type	Volume fraction of resin	Volume fraction of fiber	Woven reinforcement fiber and resin composite					
			E_1 (Gpa)	E_2 (Gpa)	G_{12} (Gpa)	ν_{12}	α_1 10^{-5} ($^oC^{-1}$)	α_2 10^{-5} ($^oC^{-1}$)
Glass fiber and polyester resin	90%	10%	7.709	7.709	1.548	0.385	5.980	5.980
	80%	20%	11.467	11.467	1.730	0.37	5.120	5.120
	70%	30%	15.292	15.292	1.961	0.355	4.446	4.446
	60%	40%	19.217	19.217	2.263	0.34	3.836	3.836
	50%	50%	23.295	23.295	2.675	0.325	3.255	3.255
Glass fiber and epoxy resin	90%	10%	8.208	8.208	1.767	0.385	8.257	8.257
	80%	20%	11.970	11.970	1.974	0.37	7.027	7.027
	70%	30%	15.808	15.808	2.235	0.355	6.066	6.066
	60%	40%	19.754	19.754	2.575	0.34	5.200	5.200
	50%	50%	23.867	23.867	3.038	0.325	4.378	4.378
Kevlar fiber and polyester resin	90%	10%	10.515	10.515	1.536	0.4	5.499	5.499
	80%	20%	17.081	17.081	1.700	0.4	4.662	4.662
	70%	30%	23.720	23.720	1.905	0.4	3.983	3.983
	60%	40%	30.466	30.466	2.165	0.4	3.353	3.353
	50%	50%	37.381	37.381	2.507	0.4	2.743	2.743
Kevlar fiber and epoxy resin	90%	10%	11.015	11.015	1.752	0.4	7.654	7.654
	80%	20%	17.588	17.588	1.935	0.4	6.481	6.481
	70%	30%	24.242	24.242	2.162	0.4	5.539	5.539
	60%	40%	31.015	31.015	2.449	0.4	4.670	4.670
	50%	50%	37.974	37.974	2.824	0.4	3.832	3.832

Figures 10 to 17, shows the thermal buckling temperature of composite plate with effect of aspect ratio of composite plate for different unidirectional and woven reinforcement fiber and resin materials types. From the figures shows the buckling temperature of composite plate, with different reinforcement fiber materials, resin materials, or reinforcement types, decreases with increasing the aspect ratio of composite plate and the change in thermal temperature decrease with increase the aspect ratio of composite plate more than 2, since the increase in the length of plate causes decreasing the thermal strength of plate.

5. Conclusion

Some concluding observations from the investigation of analytical and numerical study of thermal buckling of orthotropic composite plate are given below,

1. The suggested analytical solution is a powerful tool for thermal buckling analysis study of unidirectional and woven orthotropic composite plate with different volume fraction of reinforcement and resin materials with different materials types, by solution the general differential equations of thermal buckling analysis of orthotropic plated.
2. A comparison made between a suggested analytical solutions results from solved of general equation of thermal buckling analysis of orthotropic composite plate with numerical results from finite element method, solved by ANSYS program Ver. 14, shows a good approximation.
3. The temperature buckling increasing with increase the volume fraction of reinforcement fiber.
4. The temperature buckling for unidirectional reinforcement fiber more than temperature buckling for woven reinforcement fiber.
5. The temperature buckling increasing with decrease the thermal expansion of unidirectional and woven reinforcement fiber. And, the temperature buckling increasing with decrease the thermal expansion of resin materials.

The temperature buckling decrease with increases of aspect ratio plate. And, the decreases in temperature buckling are small for aspect ratio of plate more than 2.

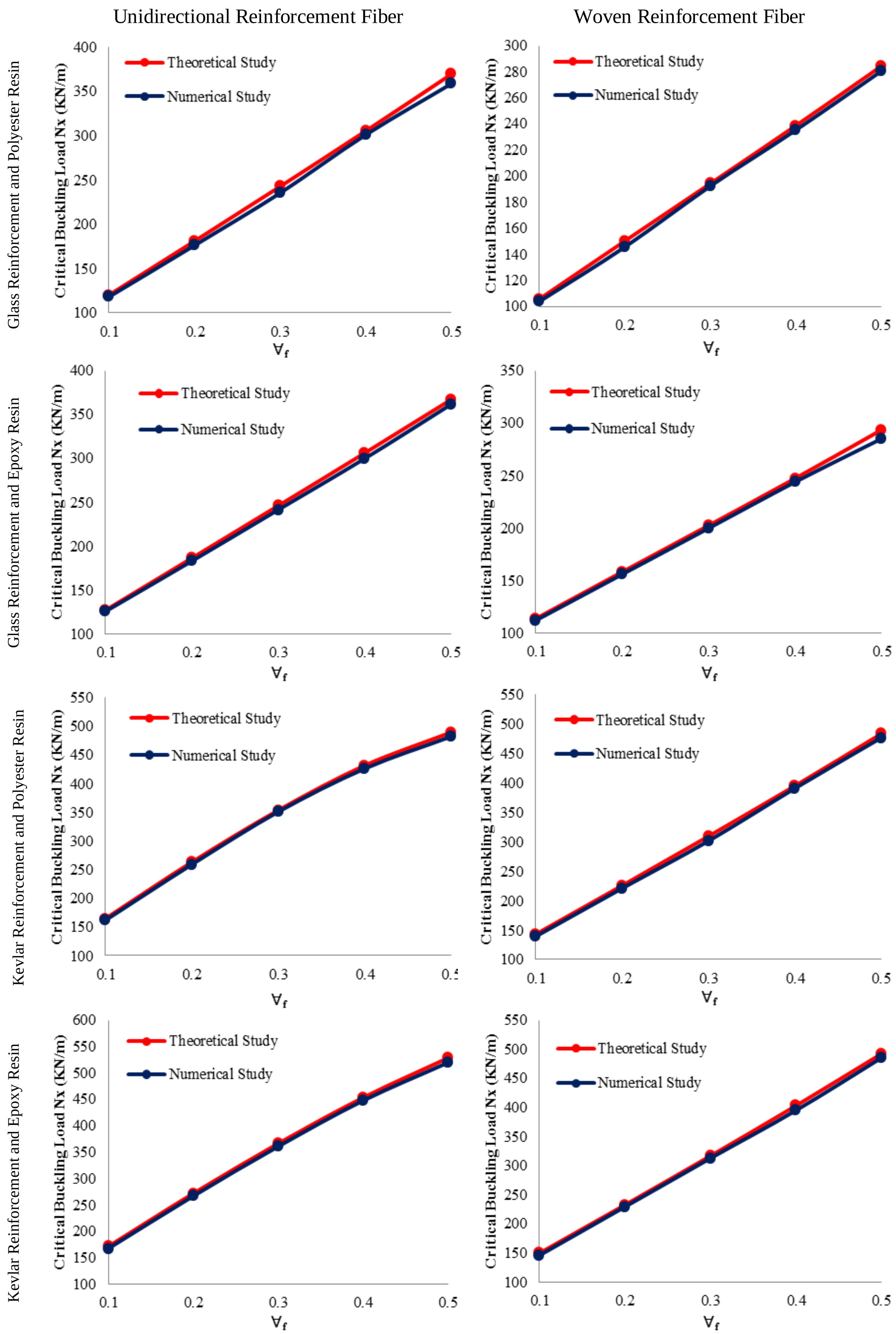

Figure 4. Compare between theoretical and numerical study for different unidirectional and woven reinforcement fiber and different resin materials, with aspect ratio of plate AR=0.5, b=0.2 m

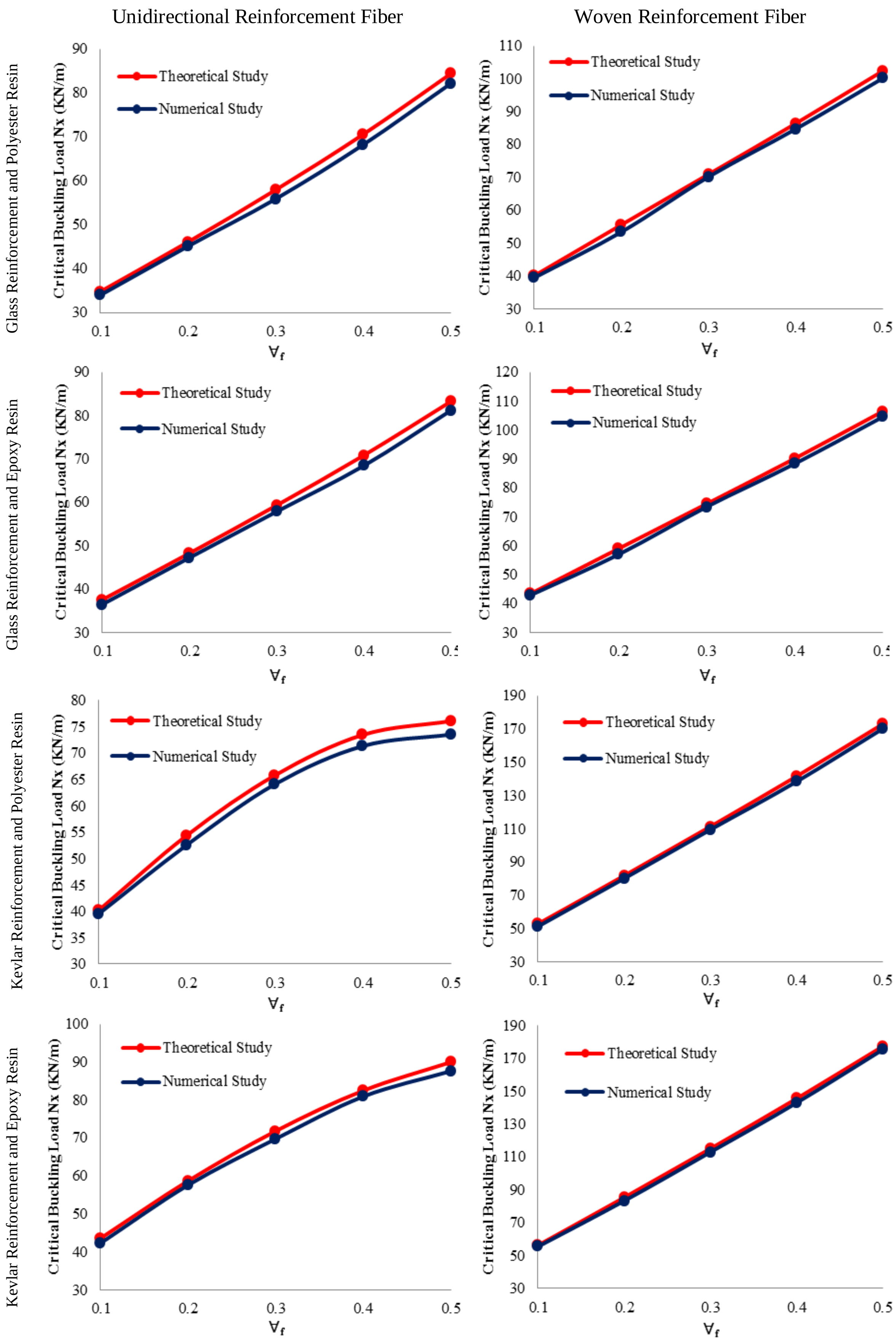

Figure 5. Compare between theoretical and numerical study for different unidirectional and woven reinforcement fiber and different resin materials, with aspect ratio of plate AR=1, b=0.2 m

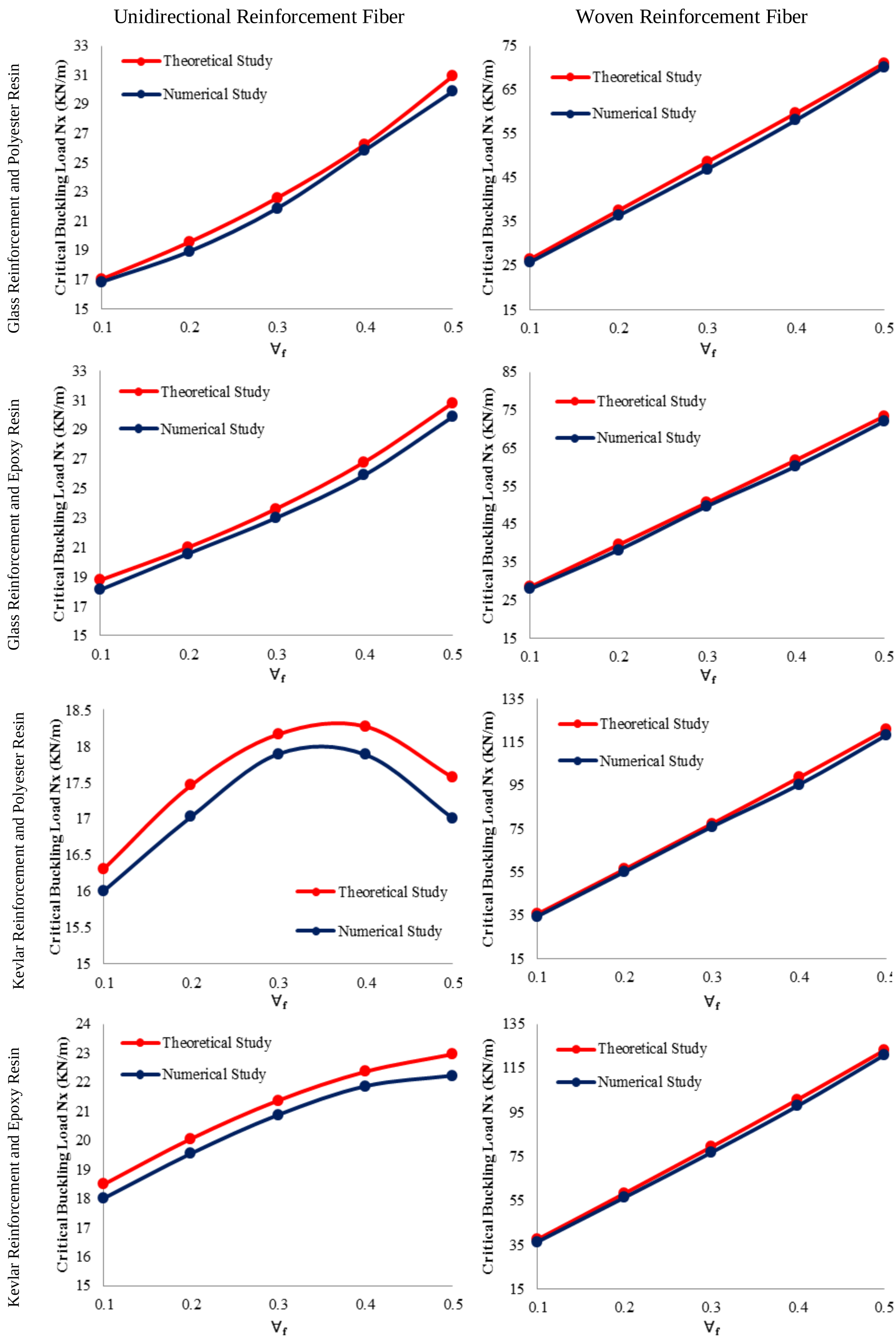

Figure 6. Compare between theoretical and numerical study for different unidirectional and woven reinforcement fiber and different resin materials, with aspect ratio of plate AR=2, b=0.2 m

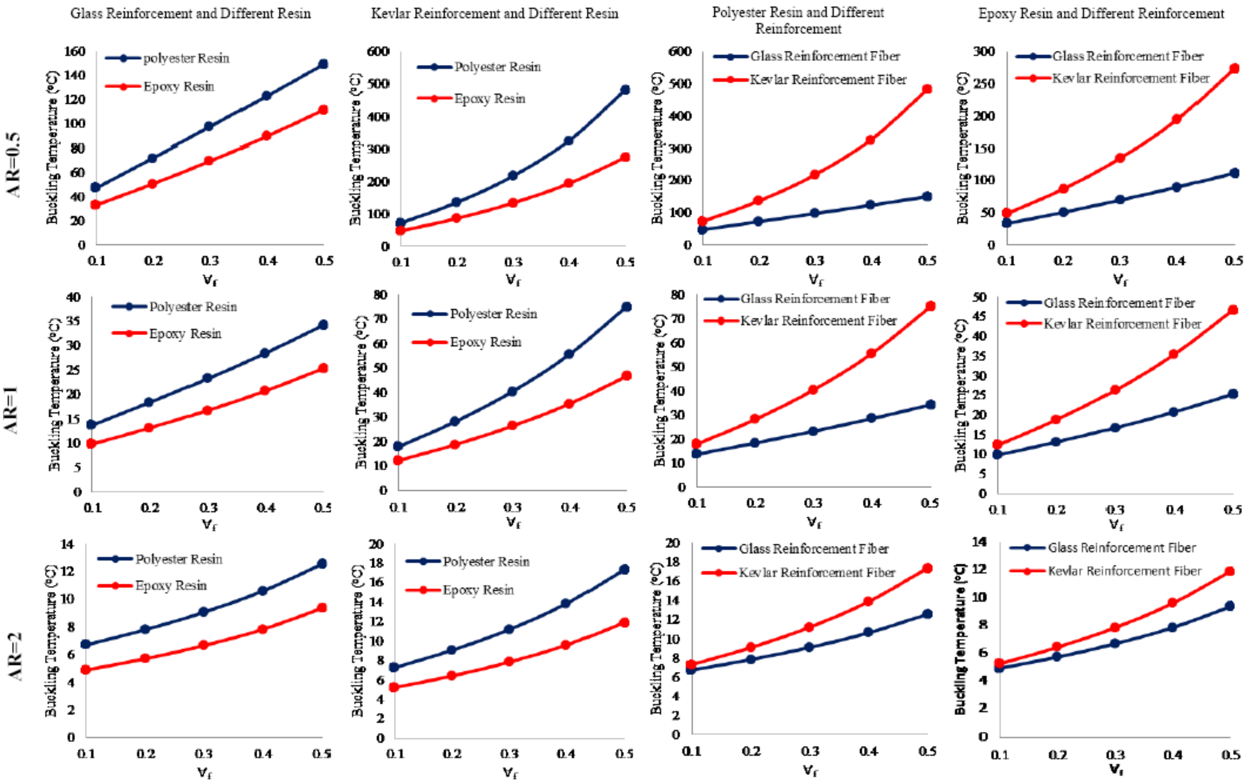

Figure 7. Critical (buckling) change temperature of different unidirectional reinforcement fiber and different resin materials, for b=0.2 m

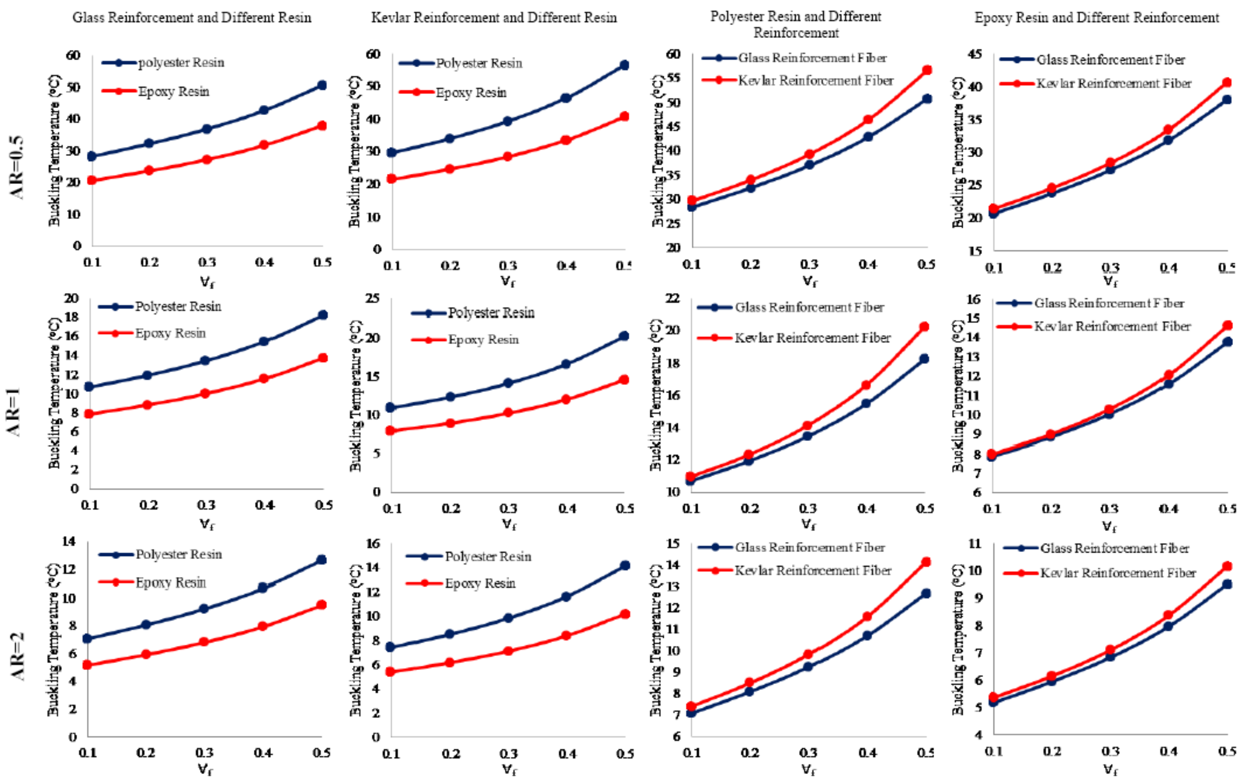

Figure 8. Critical (buckling) change temperature of different woven reinforcement fiber and different resin materials, for b=0.2 m

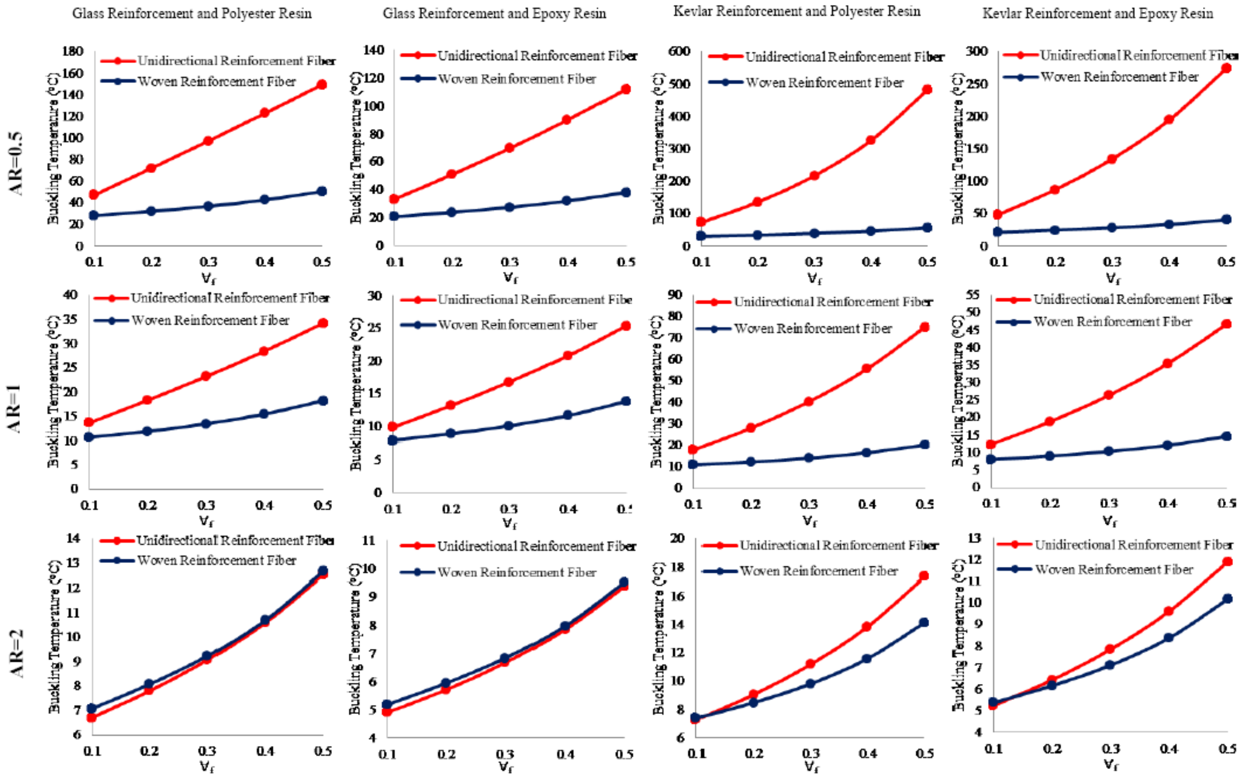

Figure 9. Critical (buckling) change temperature of different unidirectional and woven reinforcement fiber and different resin, for b=0.2 m

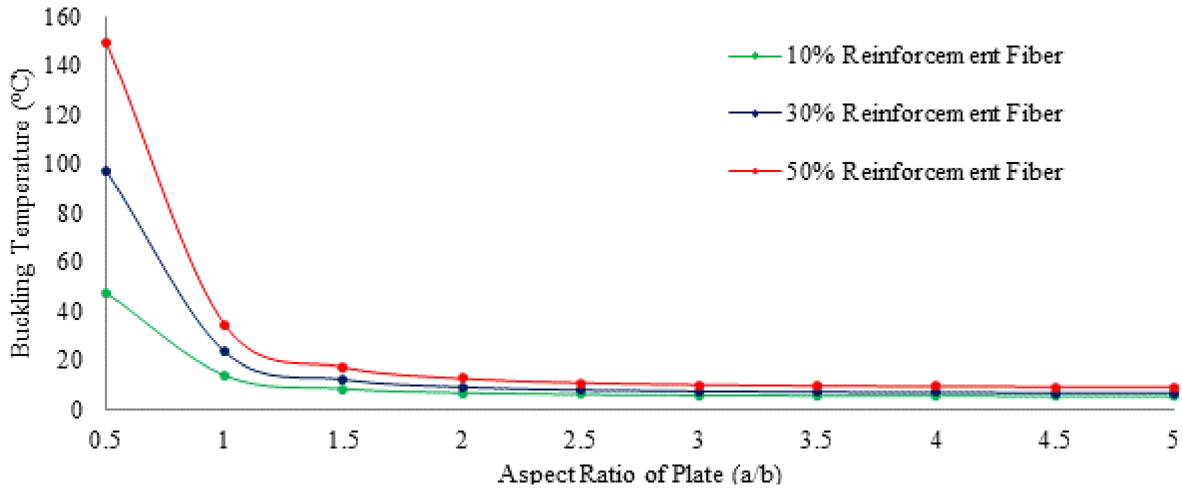

Figure 10. Critical (buckling) change temperature of different aspect ratio (a/b) of plate and different volume fraction of glass unidirectional fiber and polyester resin, for b=0.2 m

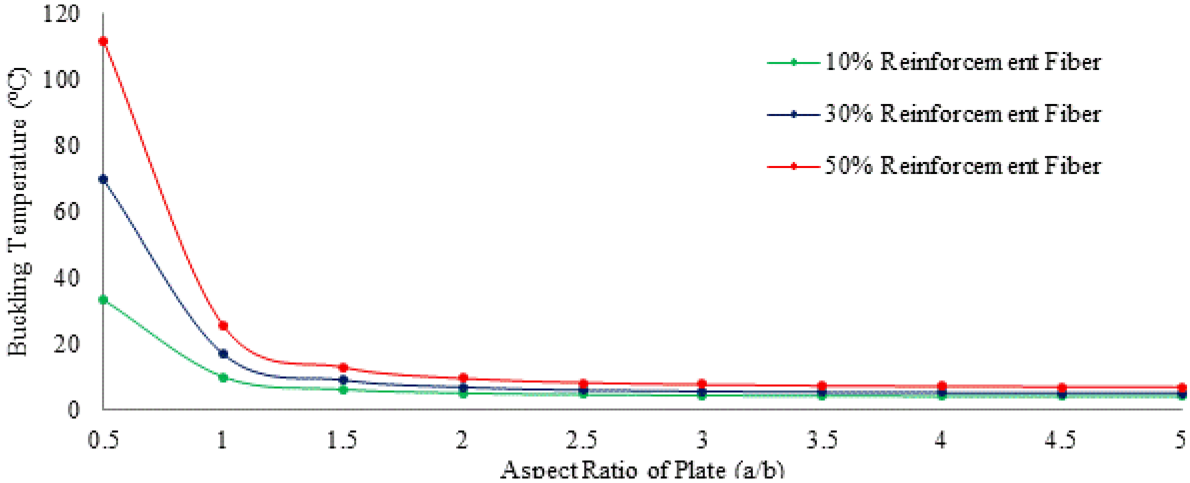

Figure 11. Critical (buckling) change temperature of different aspect ratio (a/b) of plate and different volume fraction of glass unidirectional fiber and epoxy resin, for b=0.2 m

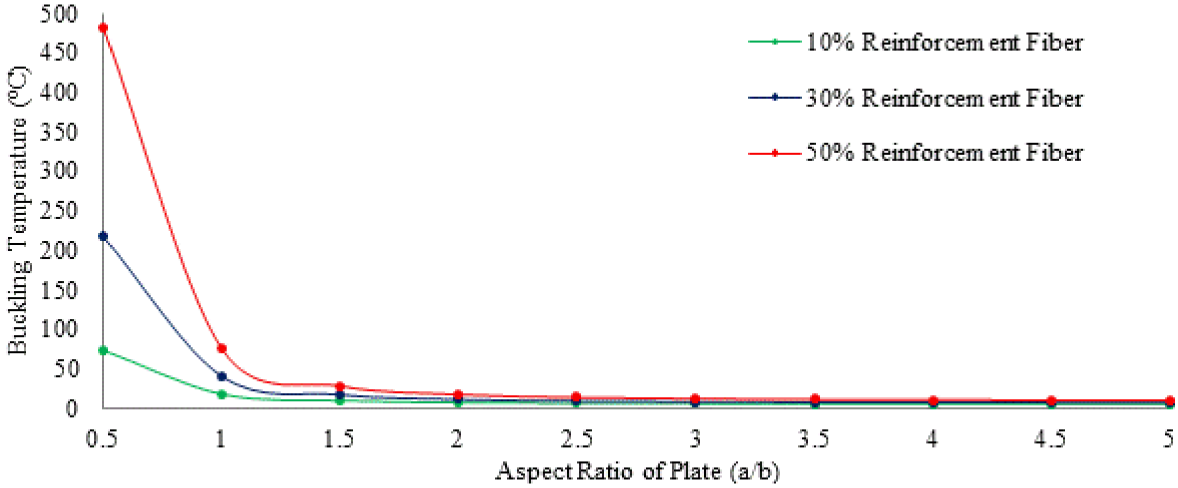

Figure 12. Critical (buckling) change temperature of different aspect ratio (a/b) of plate and different volume fraction of Kevlar unidirectional fiber and polyester resin, for b=0.2 m

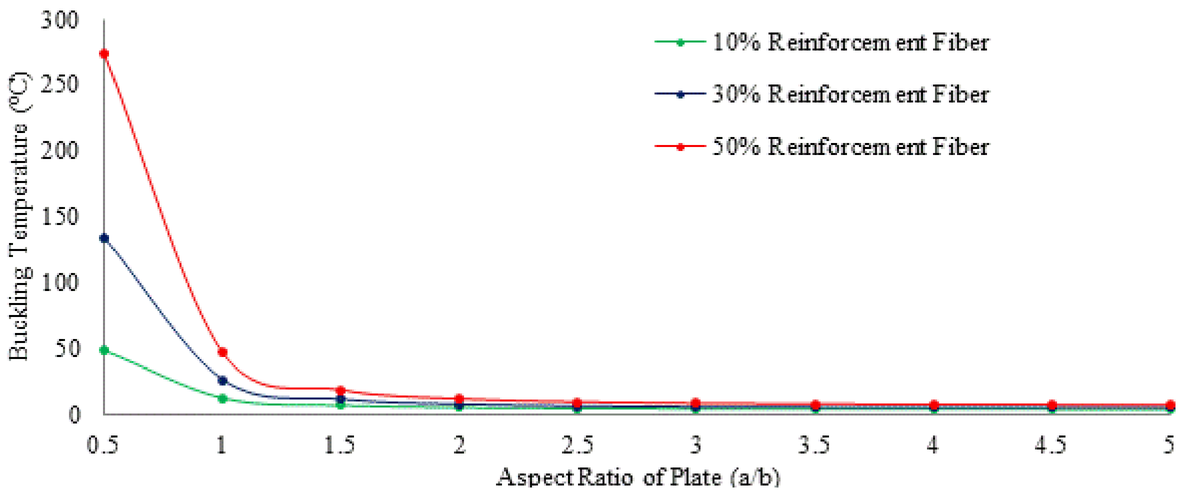

Figure 13. Critical (buckling) change temperature of different aspect ratio (a/b) of plate and different volume fraction of Kevlar unidirectional fiber and epoxy resin, for b=0.2 m

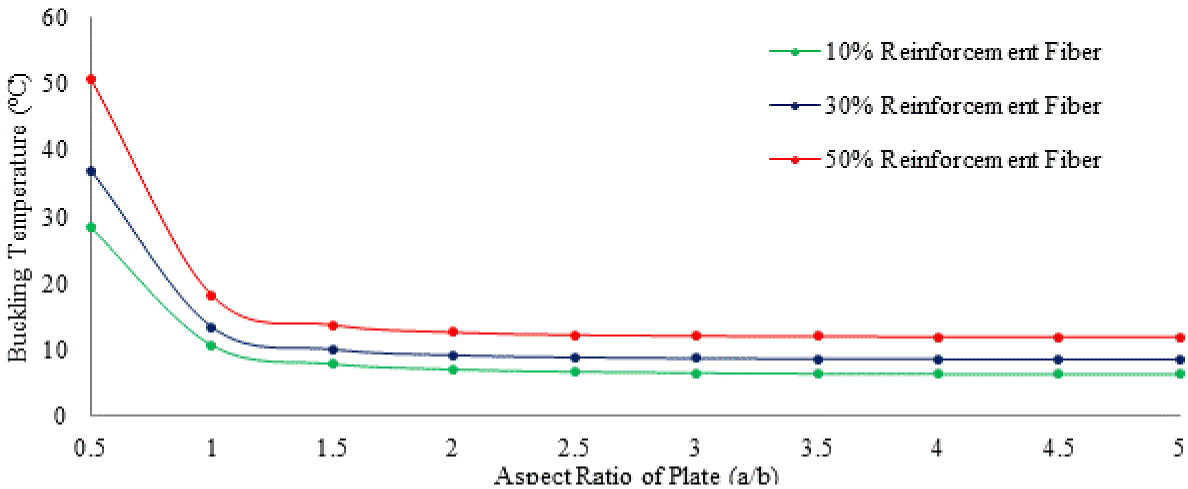

Figure 14. Critical (buckling) change temperature of different aspect ratio (a/b) of plate and different volume fraction of glass woven fiber and polyester resin, for b=0.2 m

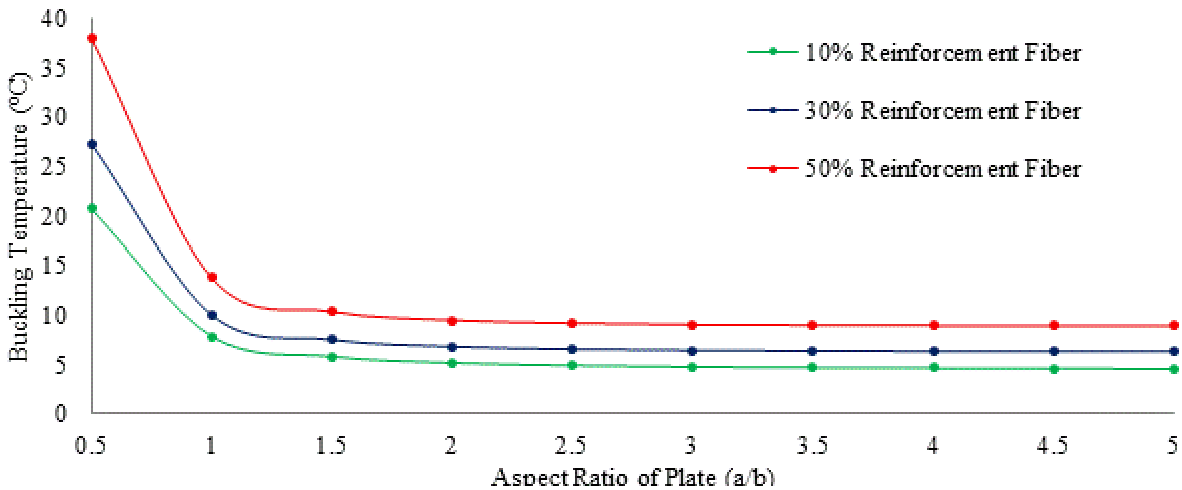

Figure 15. Critical (buckling) change temperature of different aspect ratio (a/b) of plate and different volume fraction of glass woven fiber and epoxy resin, for b=0.2 m

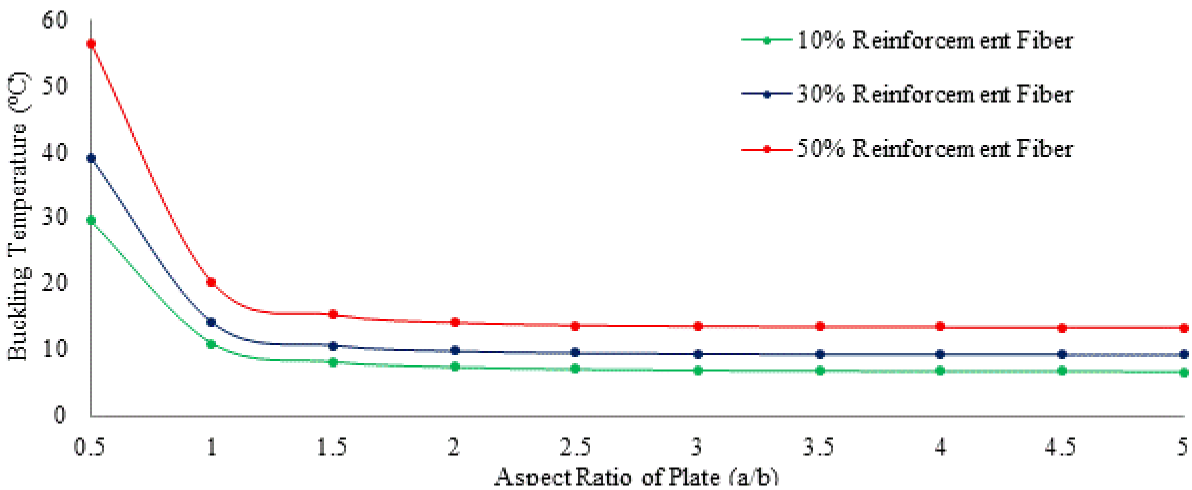

Figure 16. Critical (buckling) change temperature of different aspect ratio (a/b) of plate and different volume fraction of Kevlar woven fiber and polyester resin, for b=0.2 m

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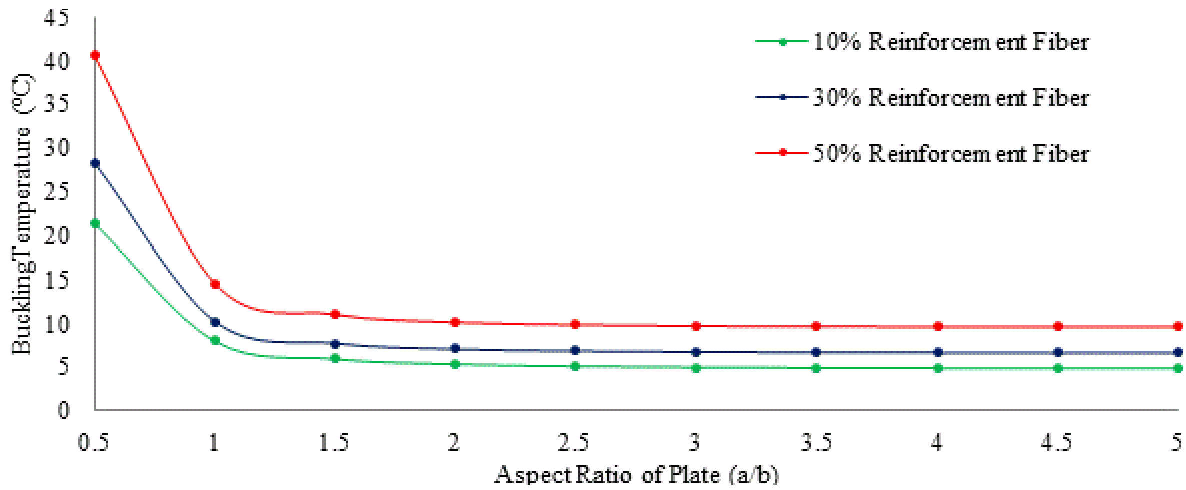

Figure 17. Critical (buckling) change temperature of different aspect ratio (a/b) of plate and different volume fraction of Kevlar woven fiber and epoxy resin, for b=0.2 m

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