



## **Numerical validation of the thermal quadrupoles method for a flow in a microchannel**

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### **Abstract**

The main objective of this work is to investigate and validate the thermal quadrupole method, used in this paper as a semi-analytical method to solve the equations of a falling film in a heated microchannel studied experimentally. In the experimental section, we created a temperature gradient within the liquid, while monitoring the temperature using an infrared camera. Then, a numerical model is established and solved by the thermal quadrupole method. Finally, we conclude with a comparison between the experiments and the numerical study.

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**Keywords:** Falling film; Micro-channel; Thermal quadrupole.

### **1. Introduction**

Miniaturization is currently a promising field of both theoretical and experimental research [1, 2]. It is to reduce the dimensions of some industrial equipment in order to reduce the technological constraints in favor of environmental chemistry and processes safety [3]. It is targeting, in general, refrigeration equipments, air conditioning and heat pumps which have their effects on the environment, the layer ozone and global warming [4]. Thus, came the idea of increasing processes. This means: developing and creating methods and devices offering improved production quality, reducing the size / capacity, lowering effluent quantities and / or catalysts, and reducing costs and energy losses with a greater respect for environmental constraints [5]. Micro-reactors operate on the principle of a continuous process and are highly different from those several traditional characteristics such that the increase of the exchange surface through the ratio  $S / V$  [6], the improvement of heat and mass transfer and a high temperature gradient and pressure [7, 8]. They help maintain better control of reaction conditions, improved security conditions, the gain in energy and reducing congestion [9]. The falling films are generally used as an intermediary for the transfer of heat and mass transfer [10, 11] in industrial equipment such as condensers and the evaporation towers [12, 13]. An extension of thermal quadrupole formalism is proposed for the modeling of heat conduction in heterogeneous environments, where the change in thermal properties is one-dimensional [14]. The main problem was to find an intrinsic relationship between generalized temperature and heat flux at the boundary of a heterogeneous medium.

## 2. Experimental set up and procedure

Our experiment consists in: creating a temperature gradient, in the back of the plate, and then managing the liquid flow (water and ethanol). An infrared camera is used in order to visualize the effect of the temperature gradient on the flow. Figure 1 illustrates the different parts of our setup.

### 2.1 Preparation

First of all, the plate was cleaned well and then sprayed with a black spray to create a black thin layer in order to maximize radiation, since an infrared camera is used for the visualization of the heat transfer on the surface of the falling film. Then, an electrical resistance is implanted in the back of the plate to generate heat. In order to create a temperature gradient, a heat exchanger is placed in contact with the back face of the plate to maintain a low temperature.

The infrared camera is a FLIR A320 (COMPACT range 19,600 pixels), which records the various infrared radiation emitted by the body and which vary with their temperature. This thermal imaging camera does not capture what is behind a wall or an obstacle. Its spectral range is from 12 to 8 microns, and since water is almost opaque in this field then it is certain that the radiations captured by this camera are those of the liquid surface.

To achieve the flow in the micro-channel, we used a syringe and a very thin tube. In this experiment we used two liquids: water and ethanol. The flow of ethanol was smoother because its surface tension is lower than that of water.

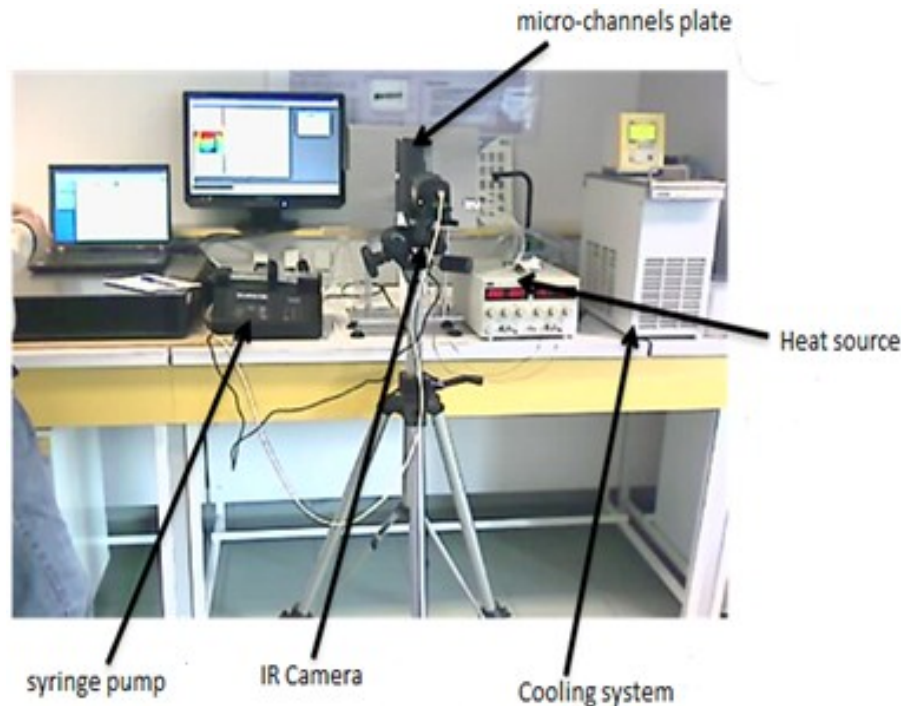


Figure 1. Experimental set up.

### 2.2 Results of the experiments

After processing the images, the results obtained are as follows: In Figures (2 and 3), we visualized the ethanol falling film flow in a micro-channel, wherein it is noted that initially the temperature of the ethanol at room temperature (20 °C), increases, once the flow reaches the heating zone to 32 °C and then begins to decrease through the cooling zone to 23 °C.

For the water (Figure 4 and Figure 5), the temperature reached 33°C, and then went down to 18 °C. In both experiments, we calculated the temperature gradient and found it 2.8 °C per centimeter which is close to 3°C/cm, the value we wanted to reach.

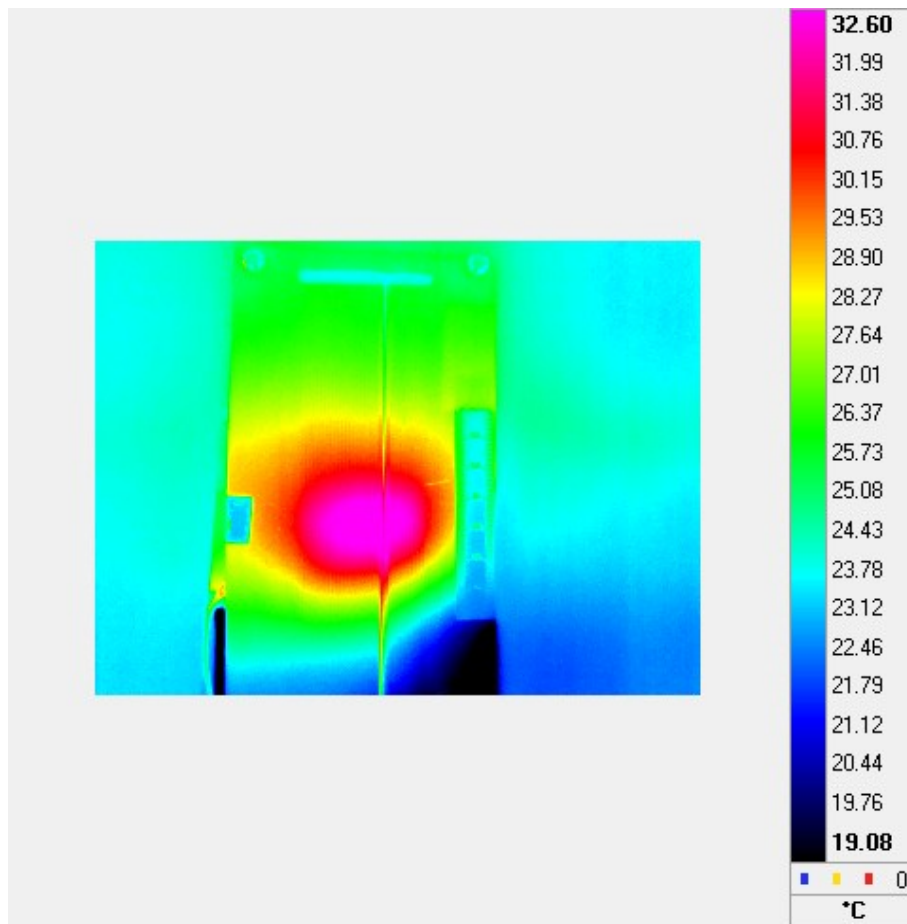


Figure 2. IR image: Ethanol falling film on a channel of the micro-structured plate.

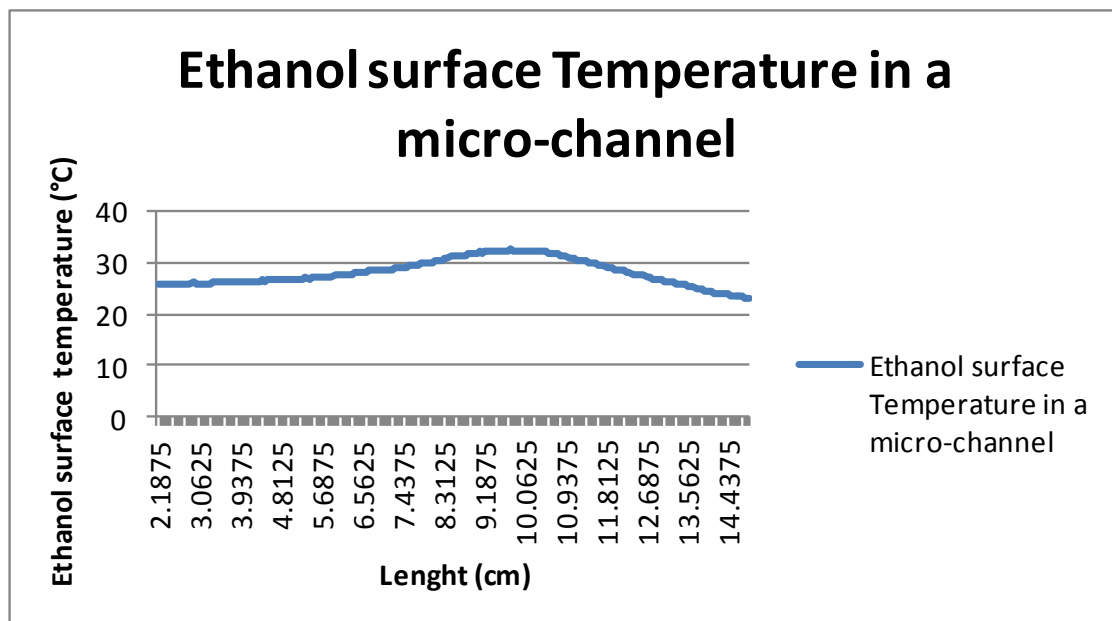


Figure 3. Curve of the surface temperature of the ethanol falling film in a single channel.

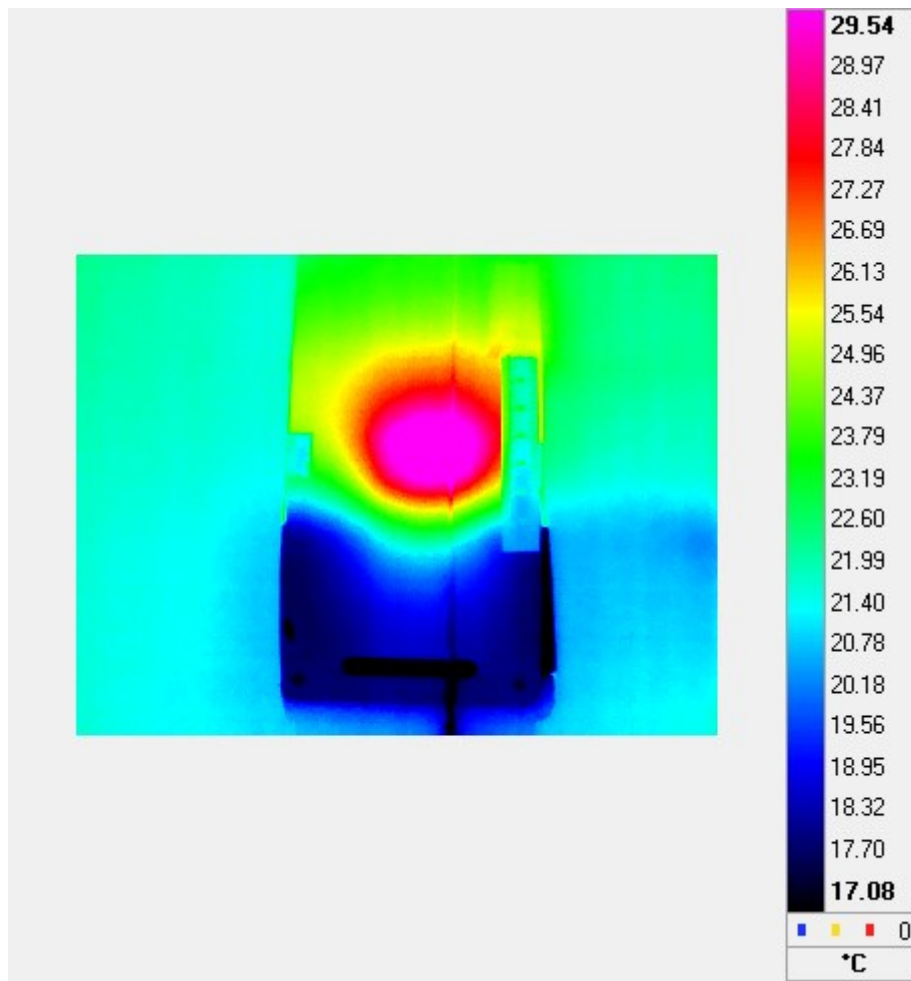


Figure 4. IR image: Water falling film on a channel of the micro-structured plate.

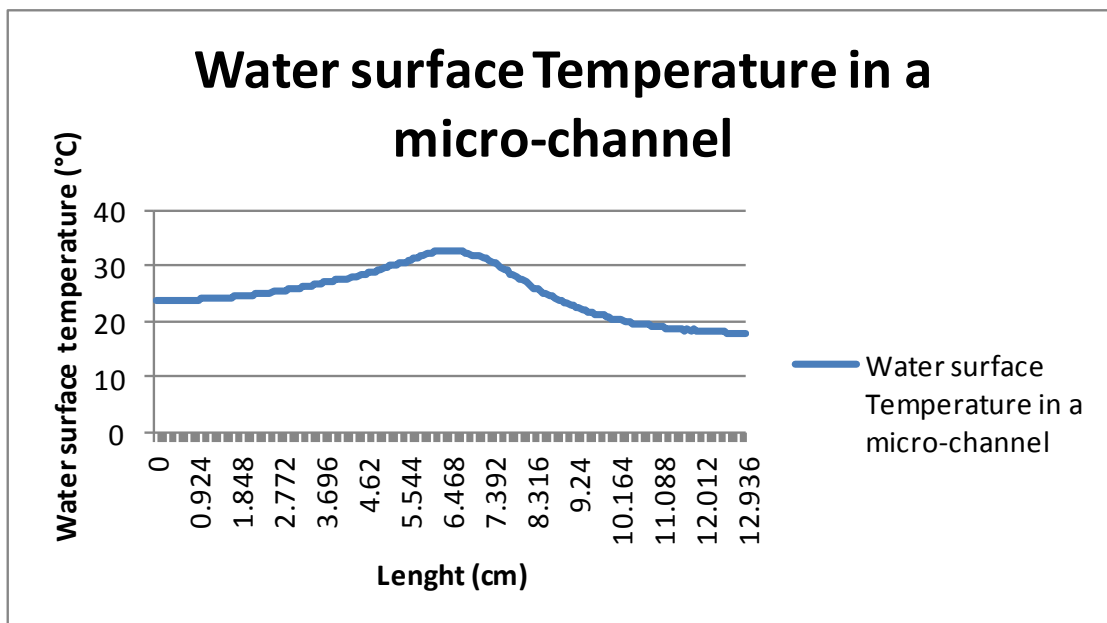


Figure 5. Curve of the surface temperature of the falling film of ethanol for one channel.

### 3. Mathematical model

#### 3.1 Governing equations

The plate we are using is a stainless micro-plate in which the micro-structured channels have the following dimensions:

Width: 1000 microns

Depth: 150 microns

Length: 15 cm

The fluid flow is in a free fall flow governed by gravity and in permanent regime. The flow assumptions are as follows:

- 2D permanent regime.
- Ideal fluid (no shear stresses).
- Constant physical properties.
- No effect of surface tension.
- Very thin liquid film.

Applying the Navier-Stokes equations on our system and taking the boundary conditions, we get these equations:

$$u = u_m \left(1 - \left(1 - \frac{y}{\delta}\right)^2\right) \quad (1)$$

$$u \frac{\partial T}{\partial x} = \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \quad (2)$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \quad (3)$$

Equation (2) is the result of the energy equation in the falling film and Equation (3) is the result of the energy equation in the micro-channel.

#### 3.2 Solving with the thermal quadrupoles method

We will adopt the method of thermal quadrupoles to solve this problem, which is mainly finding the relationship between flow and temperature at input and output by applying the Fourier Transform (Figure 6).

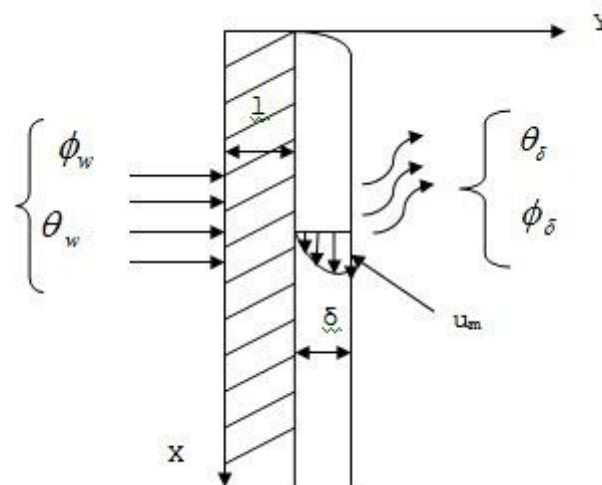


Figure 6. Input and output of the system.

Transforming all the equations previously obtained along with the boundary conditions gave us these results:

In the plate:

$$\begin{pmatrix} \theta_0 \\ \phi_0 \end{pmatrix} = \begin{pmatrix} \cos \sqrt{pl} & \frac{1}{\lambda \sqrt{p}} \sin \sqrt{pl} \\ -\lambda \sqrt{p} \sin \sqrt{pl} & \cos \sqrt{pl} \end{pmatrix} \begin{pmatrix} \theta_l \\ \phi_l \end{pmatrix} \tag{4}$$

In order to use the thermal quadrupoles method on the liquid, we proceed by dividing the film into N layers having each one the same “Eq. 1”. So the matrix in each layer is:

$$\begin{pmatrix} \theta_{i-1} \\ \phi_{i-1} \end{pmatrix} = \begin{pmatrix} \cosh K_i \frac{\delta}{N} & \frac{1}{\lambda K_i} \sinh K_i \frac{\delta}{N} \\ \lambda K_i \sinh K_i \frac{\delta}{N} & \cosh K_i \frac{\delta}{N} \end{pmatrix} \begin{pmatrix} \theta_i \\ \phi_i \end{pmatrix} \tag{5}$$

And finally we have the matrix of our system:

$$\begin{pmatrix} \theta_w \\ \phi_w \end{pmatrix} = \begin{pmatrix} \cos \sqrt{pl} & \frac{1}{\sqrt{pl}} \sin \sqrt{pl} \\ -\lambda \sqrt{p} \sin \sqrt{pl} & \cos \sqrt{pl} \end{pmatrix} * \prod_{i=1}^N \begin{pmatrix} \cosh K_i \frac{\delta}{N} & \frac{1}{\lambda K_i} \sinh K_i \frac{\delta}{N} \\ \lambda K_i \sinh K_i \frac{\delta}{N} & \cosh K_i \frac{\delta}{N} \end{pmatrix} \begin{pmatrix} \theta_\delta \\ \phi_\delta \end{pmatrix} \tag{6}$$

With the inverse Fourier transform, we find the expression of the surface temperature of the falling liquid film which is simulated then to make the profile:

$$T_\delta = \text{Re}(\theta_{\delta 0} + \sum \theta_\delta * \exp(px_m(ix))) / L \tag{7}$$

Figure 7 shows the curve of the temperature profile at the surface of the flow. We can see the increase in the temperature in the heating zone.

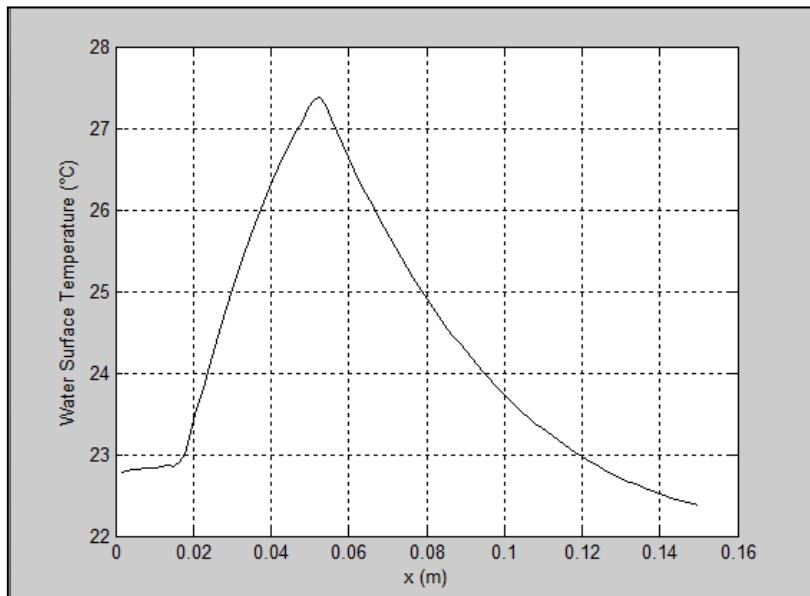


Figure 7. Simulated water surface temperature evolution.

Note that the temperature rises from 22.8 ° C, the input channel temperature and the beginning of the heating zone, at x = 0.018 m, reached its peak at x = 0.052 m with a temperature of 27.4 ° C, and begins to fall back to its initial temperature. This drop is due to the periodicity of the proper function (temperature), since the resolution was made by the integral Fourier transform. Same thing for the ethanol graph (Figure 8), rising

from a temperature close to 22°C, the liquid exceeded 28°C in the heating zone and then come down to the cooling temperature.

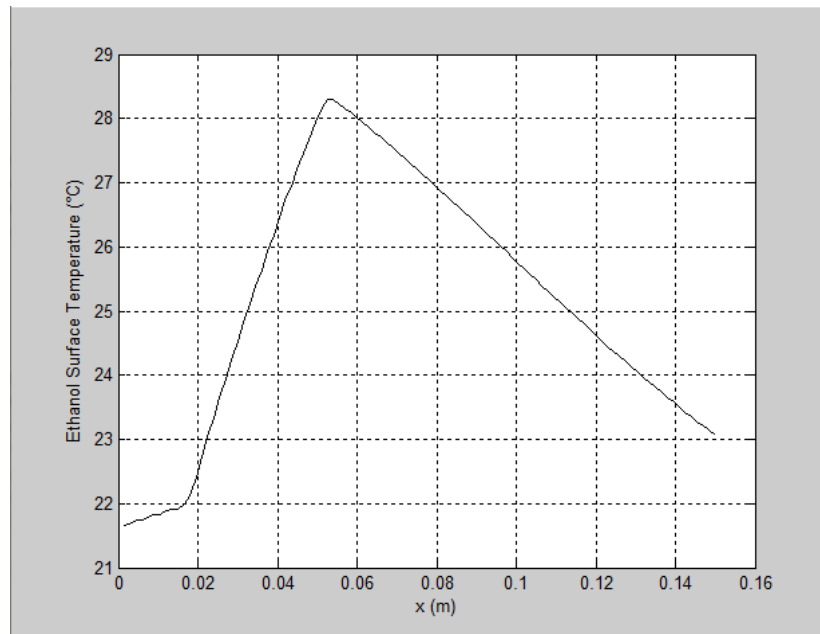


Figure 8. Simulated ethanol surface temperature evolution.

We studied also the changes in temperature within the liquid along the y axis. We note that there is not a great change throughout the y axis which means we can avoid the division of our multilayer medium, that is to say we can keep a single mesh. There is, also, a resemblance between the numerically determined temperature profile and that determined experimentally. From the figure that the temperature profile in the flow, we can conclude that the independence with respect to y means that the temperature can be considered as a single unit with one average speed.

We managed to achieve a temperature gradient close to 3 °C/cm; this number was calculated by the laboratory of Nancy LRGP for diabatic distillation.

#### 4. Conclusion

In this work, we conducted experiments of flowing liquid on our micro-structured plate, then on a single channel, and using an infrared camera, we filmed this flow to compare the experimental curves with digital ones. In the second part, we have developed a computer code using Matlab software, which allowed us to describe the temperature profile on the surface of the flow based on the semi-analytical method of thermal quadrupoles.

The code developed will allow us to address several problems, such as the control problem, which is essentially to determine the thermal conditions necessary to achieve a desired temperature profile. That is to say, the flux or seek to impose the temperature from the temperature profile at the surface.

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#### References

- [1] Aiello G, Ciafalo M (2009) Natural convection cooling of a hot vertical wall wet by a falling liquid film. *International Journal of Heat and Mass Transfer* 52: 5954-5961. doi: 10.1016/j.ijheatmasstransfer.2009.08.007.
- [2] Foerster M, Lam K F, Sorensen E, Gavriilidis A (2013) In situ monitoring of microfluidic distillation. *Chemical Engineering Journal* 227: 13-21. doi: 10.1016/j.cej.2012.11.125.

- [3] Wiesegger L E, Knauss R P, Guntching G E, Brandner J J (2013) Vapor-liquid phase separation in micro-/ministructured devices. *Chemical Engineering Science* 93: 32-46. doi: 10.1016/j.ces.2013.01.030.
- [4] Verma N, Mewes D, Luke A (2010) Lattice Boltzman study of velocity, temperature, and concentration in micro-reactors. *International Journal of Heat and Mass Transfer* 53: 3715-3185. doi: 10.1016/j.ijheatmasstransfer.2010.03.009.
- [5] Mhiri N (2006) Etude d'un micro-absorbeur Gaz/Liquide à film tombant, Récents progrès en génie des procédés, Dissertation, Institut national polytechnique de Lorraine.
- [6] Perry I, Jannot Y, Maillet D, Fiers B (2010) Effect of velocity distribution on external wall temperature field for a flat microchannel, *Experimental Heat Transfer* 23:27-43. doi: 10.1080/08916150903402724.
- [7] Baldassari C, Marengo M (2013) Flow boiling in micro-channels and micro-gravity. *Progress in Energy and Combustion Science* 39: 1-36. doi: 10.1016/j.pecs.2012.10.001.
- [8] Conti A, Lorenzini G, Jeluria Y (2012) Transient conjugate heat transfer in straight microchannels. *International Journal of Heat and Mass Transfer* 55: 7532-7543. doi: 10.1016/j.ijheatmasstransfer.2012.07.046.
- [9] Hung T C, Huang Y X, Yan W M (2013) Thermal performance analysis of porous-microchannel heat sink with different configuration designs. *International Journal of Heat and Mass Transfer* 66: 235-243. doi: 10.1016/j.ijheatmasstransfer.2013.07.019.
- [10] Zhang F, Wub YT, Gengb J, Zhangb ZB (2008) An investigation of falling liquid films on a vertical heated/cooled plate. *International Journal of Multiphase Flow* 34: 13-28. doi: 10.1016/j.ijmultiphaseflow.2007.08.003.
- [11] Bu X, Ma W, Huang Y (2011) Numerical study of heat and mass transfer of ammonia-water in falling film evaporator. *Heat Mass Transfer*, Vol. 48: 725-734. doi: 10.1007/s00231-011-0923-4.
- [12] Marenzana G, Perry I, Maillet D (2004) Mini and micro-channels: influence of axial conduction in the walls. *International Journal of Heat and Mass Transfer* 47: 3993-4004. doi: 10.1016/j.ijheatmasstransfer.2004.04.016.
- [13] Buonomo B, Manca O (2012) Transient natural convection in a vertical micro-channel heated at uniform heat flux. *International Journal of Thermal Sciences* 56: 35-47. doi: 10.1016/j.ijthermalsci.2012.01.013.
- [14] Fudym O, Ladevie B, Batsale JC (2002) A seminumerical approach for heat diffusion in heterogeneous media: one extension of analytical quadrupole method. *Numerical Heat Transfer, Part B*, 42: 325-348. doi: 10.1080/10407790190053978.