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An analytical approach for evaluating the flutter instability boundaries for cantilever pipes conveying fluid

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Abstract
A new approach for evaluating the flutter instability boundaries based on the analytical solution of the equation of motion of cantilever pipes conveying fluid has been attempted. This approach leads to a simple transcendental equations form which the critical speed of flutter instability and the associated natural frequencies of cantilever pipes can be determined for any pipe parameters. The stability and critical natural frequencies maps can be simply constructed. The results of the presented approach are carefully checked with published results. The presented results showed very good agreements.

Keywords: Flutter instability; FSI mass ratio; Damping; Coriolis force; Galerkin method.

1. Introduction

Cantilever pipes conveying fluid are non-conservative systems according to the effects of the fluid-structure interaction (FSI). Because of the FSI effects a significant amount of energy can either be added to or subtracted from the total system energy. In such non conservative systems the natural frequencies can become complex [1].

Flutter instability arises from the effect of the Coriolis force which results from the relative rotation of the fluid element as it vibrates laterally. At certain fluid velocities Coriolis force causes a positive damping mechanism and tends to decay the vibration. However, at certain higher velocities it causes a negative damping mechanism leading to exponential growth vibration or flutter. In contrast conservative pipes such as simply supported or clamped pipes the Coilolis force has no effect on vibration due to the symmetry of the mode shapes [2].

Flutter instabilities play a major role in the dynamics of cantilevers. It is of interest to split the region of fluid parameters at which they may occurs. At this region very large oscillations can exist in the pipe structure leading to fatigue stresses, these stresses may cause a failure or even rupture at the weak sections. In industrial or plant applications the damage may become cumulative since the oscillating pipe can keep a trigger of secondary failures for the surrounding components.

The first attempt to investigate flutter instability of cantilever pipes theoretically and experimentally was done by Gregory and Paidoussis [1, 3], later by Bishop and Fawzy [4] and also by Sugiyama and Noda [5]. These authors employed approximate analytical approaches to determine the flutter instability boundaries. Such approaches were made by using Galerkin method to desecretize the equation of motion to a few degrees of freedom (DOF). However, it had been demonstrated that the required number of DOF is 5 or more for accurate analysis [1].Hence a large matrix size will be arise for solving the resulting Eigen- value problem.

Finite element technique is an alternative approach for evaluating flutter instability boundaries, e.g., Osama [6] investigated a collar-stiffened and normal cantilever pipes conveying fluid by employing this method.

The mode exchange and flutter analysis which can be taken place as a result of the collision of Eigen-values was studied by Seyranian [7]. He made a mathematical description to discuss these phenomena in the complex plane.

The fundamental concepts of the dynamics of cantilever pipes conveying fluid such as modeling, vibration and flutter instability analysis and the comprehensive complete survey of this topics has been compiled in book form by Paidoussis [8].

Recently, Si-Ung et.al. [9] investigated the flutter problem graphically by considering the order of branches in root locus diagrams and the transferences of flutter-type instability from one Eigen-value branch to another, Wang [10] studied the non-linear dynamics for tubular cantilever including the effect of flutter instability. He used the Hopf-bifurcation diagram which constructed numerically from solving the non linear equation of motion using fourth order Runge–Kutta integration algorithm. He evaluated the critical velocities for flutter and the other non-linear behaviors such as the limit cycles of oscillations and chaotic.

As it can be seen from the work of the previous researches that the basic methods for evaluating the flutter instability were achieved either by using approximated analytical solution or numerical solutions. However, the analysis in the two cases will lead to numerical matrix solution with large number of DOF. Evidently this will lead to a hard computational task.

In the present work the same problem will be resolved analytically. The analytical solution is based on infinite number of DOF or continuous system analysis. The critical velocities and natural frequencies for flutter instability can be evaluated by simple solution of two transcendental equations derived directly from the General solution of the equation of motion.

2. Theoretical Considerations

The fluid conveying pipe under consideration is assumed to obey Euler–Bernoulli Beam theory. The structure of the pipe has small deformation, the conveyed fluid is assumed to be non-viscous and incompressible and the effect of gravity and internal damping are neglected. Now, for a pipe with uniform tubular section shown in Figure 1.

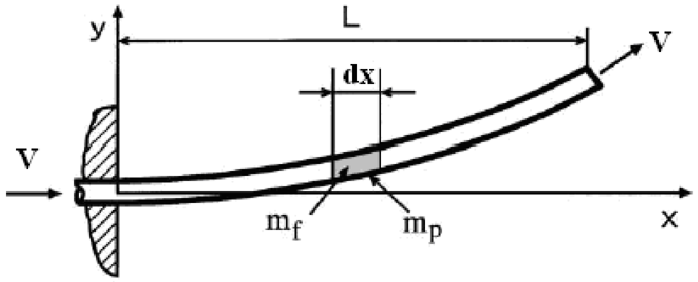

Figure 1. Cantilever pipe conveying fluid.

For small displacement the x-component of the fluid velocity can be assumed to be V and the y-component is:

$$\frac{\partial y}{\partial t} + V\frac{\partial y}{\partial x} \tag{1a}$$

For a small pipe segment of length dx, the total kinetic energy for the pipe and fluid is:

$$dT = 1/2m_p(\frac{\partial y}{\partial t})^2 dx + 1/2m_f[V^2 + (\frac{\partial y}{\partial t} + V\frac{\partial y}{\partial x})^2]dx \tag{1b}$$

The strain energy is:

$$dS = 1/2EI(\frac{\partial^2 y}{\partial x^2})^2 dx + 1/2PA_p(\frac{dy}{dx})^2 dx \tag{2}$$

Using Hamilton's principle, one get:

$$\delta\int_0^t\int_0^x \{1/2m_p(\frac{\partial y}{\partial x})^2 + 1/2m_f[V^2 + (\frac{\partial y}{\partial t} + V\frac{\partial y}{\partial x})^2] - 1/2EI(\frac{\partial^2 y}{\partial x^2})^2 dx - 1/2PA_p(\frac{dy}{dx})^2 dx\}dxdt=0 \tag{3}$$

Performing the variation and integrating by parts yield to the following equation of motion [11]:

$$EI\frac{\partial^4 y}{\partial x^4} + (m_f V^2 + PA_p)\frac{\partial^2 y}{\partial x^2} + 2m_f V\frac{\partial^2 y}{\partial x \partial t} + (m_f + m_p)\frac{\partial^2 y}{\partial t^2} = 0 \tag{4}$$

Eq. (4) can be written in the following dimensionless form:

$$\eta^{IV} + (U^2 + \gamma)\eta'' + 2\beta^2 U\dot{\eta}' + \ddot{\eta} = 0 \tag{5}$$

where the notations (') and (.) represent $\frac{\partial}{\partial \xi}$ and $\frac{\partial}{\partial \tau}$, respectively.

$$\zeta = x/L, \eta = y/L, U = VL.\sqrt{m_f/EI}, \gamma = PA_pL^2/EI, \beta = mf/(mf+mp), \tau = (t/L^2).\sqrt{EI/(m_f+m_p)} \tag{6}$$

2.1 Analytical solution

Let the general solution of eq (5) takes in the following form:

$$\eta(\zeta,\tau) = \sum_{j=1}^{4} c_j e^{i\lambda j\zeta} e^{i\Omega\tau} \tag{7}$$

where

$$\Omega = \omega L^2.\sqrt{(m_f + m_p)/EI} \tag{8}$$

and ω is the circular frequency.

Let the complex natural frequency Ω for non conservative cantilever pipes, takes the following form:

$$\Omega = \Omega r + i\Omega i \tag{9}$$

where Ωr and Ωi are the real and imaginary components of the natural frequency, respectively

Substituting eq. (9) and (7) into eq. (5) gives the following polynomial equation in λ:

$$\lambda 4-(U2+\gamma)\lambda 2-2i\beta 2U(\Omega r+i\Omega i)\lambda-(\Omega r+i\Omega i)2=0 \tag{10}$$

Referring to eq. (7), when the imaginary part Ωi >0 this produces a decaying of vibration or stable vibration. And when Ωi<0 produces exponential growth or flutter. The limiting value at which the imaginary parts of the natural frequencies change from positive (stable) to negative (flutter) is when Ωi =0. At this condition the "neutral stability" can be evaluated.

Hence, to find the neutral conditions for flutter instability Ωi =0 is introduced in eq. (10) to get [13]:

$$\lambda 4-(U2+\gamma)\lambda 2-2i\beta 2U\Omega r\lambda-\Omega r2=0 \tag{11}$$

Equation (11) is a Quartic polynomial. Its four roots are given in a radical form [12], as the follows:

λ1,2= $-1/2\sqrt{\alpha} \pm i/2\sqrt{(4\beta^2 U\Omega_r/\sqrt{\alpha}) + \alpha - 2\rho}$

λ3,4= $1/2\sqrt{\alpha} \pm 1/2\sqrt{(4\beta^2 U\Omega_r/\sqrt{\alpha}) - \alpha + 2\rho}$ (12)

where:

ρ= U2 +γ,
α = 2 ρ /3+0.42S1/S2+0.265S2,
S1= ρ2 -12 Ωr 2,
S2=(S+ $\sqrt{S^2 - 4S_1^3}$)1/3, and
S=108(β2UΩ)2-72RΩ2-2R3 (13)

Eq. (12) can be written in complex form as follows:

λ1,2= $-a \pm ib_1$,
λ3,4= $a \pm b_2$ (14)

where:

a= $1/2\sqrt{\alpha}$,
b1= $1/2\sqrt{(4\beta^2 U\Omega_r/\sqrt{\alpha}) + \alpha - 2\rho}$,
b2= $1/2\sqrt{(4\beta^2 U\Omega_r/\sqrt{\alpha}) - \alpha + 2\rho}$ (15)

On substituting the values of λ, s given in eq. (14) into the solution given in eq. (7) with Ωi =0 and after making some algebraic and geometrical manipulations, the General solution of the equation the neutral flutter instability can be finally presented as the follows:

η(ζ,τ)= $e^{i(\Omega_r\tau - a\zeta)}[A\sinh b_1\zeta + B\cosh b_1\zeta] + e^{i(\Omega_r\tau + a\zeta)}[D\sin b_2\zeta + E\cos b_2\zeta]$ (16)

Where A,B,D and E are constants related to C1,C2,C3and C4.

2.2 Flutter analysis
The boundary conditions for cantilever pipes are;

η(0,τ)=0, $\eta'(0,\tau) = 0$, $\eta'''(1,\tau) = 0$, $\eta''(1,\tau) = 0$ (17)

Imposing the boundary conditions given in eq.(17) in the general solution given by eq.(16) results in the following set of algebraic equations:

B+E= 0
-iaB+b1A+iaE+b2D=0
Ae-iasinhb1+B(e-iacoshb1-eiacosb2)+Deiasinb2=0
A[-ia(3b12-a2)sinhb1+b1(b12-3a2)coshb1]e-ia+B[b1(b12-3a2)e-iasinhb1
+ia(3b12-a2)coshb1e-ia]-[b2(3a2+b22)sinb2-ia(3b12+a2)cosb2]eia+D[-ia(3b22+a2)sinb2
-b2(3a2+b22)cosb2]eia=0 (18)

Substituting the first of eq. (18) into the remaining equations and arranging, the result is the following matrix equation:

$$\begin{bmatrix} b_1 & -2ia & b_2 \\ e^{-ia}\sinh b_1 & e^{-ia}\cosh b_1 - e^{ia}\cos b_2 & e^{ia}\sin b_2 \\ \alpha_1 & \alpha_2 & \alpha_3 \end{bmatrix} \begin{Bmatrix} A \\ B \\ D \end{Bmatrix} = 0 \quad (19)$$

where: α1= [ia(3b12-a2)sinhb1+b1(b12-3 a2)coshb1] e-ia, α2=[b1(b12-3a2)e-iasinhb1-ia(3b12-a2)coshb1e-ia]-[b2(3a2+b22)sinb2-ia(3b12+a2)cosb2]eia, α3=[-ia(3b22+a2)sinb2- b2(3a2+b22)cosb2] eia.

Eq. (19) can be restated in the following form:

[α]{A}=0 (20)

For nontrivial solution:

$|\alpha| = 0$ (21)

On expansion of the determinant of eq. (21) and isolating the real and imaginary terms can get the following equations, respectively.

[3a2(b22-b12)+4a4-b14-b24]sinhb1sinb2-[b1b2(6 a2+b22- b12)]coshb1cosb2
+[b1b2(6a2+b22-b12)]cos2a=0 (22)

ab1(4a2+b12+3b22)coshb1sinb2+ab2(4a2+b22-3b12)sinhb1cos(b2)+[b1b2(b12+b22)]sin2a=0 (23)

Where a, b1, b2 are as defined in eqs. (15) and (13).

Solution of eqs. (22) and (23) can give the critical speeds and the critical natural frequencies for flutter instability at any values of pressure γ and mass ratio β [13].

3. Results and discussions

At dimensionless pressure γ=0, the mass ratio β is taken as a variable parameter which is varied from 0 to 0.9, therefore at any value of β the solutions of eqs. (22) and (23) can give the critical fluid velocity of flutter instability U and the associated real frequency Ωr. When these velocities are plotted in U- β plain a map of the regions of stable, neutrally stable and flutter instability can be constructed. Such a map for zero pressure is shown in Figure 2. Figure 3 shows the corresponding real natural frequency at which flutter instability occurs for the same pipe parameters.

To check the validity of the present approach the results obtained in Figures 2 and 3 are compared with those reported in two other papers. The first is ref. [1] in which Galerkin method was used and the second is ref. [6] where the Finite Element method was employed. These results are presented in Table 1. Table 1 indicates two important points. Firstly, it shows that the present solution is in a very good agreement with the other two methods. Where the maximum errors in U and Ωr are not exceeded 5.5% for the worst cases. Secondly, it shows that the critical velocities and the real frequencies obtained by the present approach have the lowest values. This can be attributed to the fact that, the present technique is based on infinite DOF (continuous system) analysis. In contrast, the other approaches are based on descretize the continues system to finite DOF which leads to increasing stiffness of the original system and hence increasing the natural frequencies and the associated critical velocities.

To study the effect of the pressure on flutter instability, Figures 4 and 5 are constructed. In these figures the dimensionless pressures are assigned the values γ =1,2 and 3. It is clear from these figures that the effect of increasing the fluid pressure leads to a slight decreasing both the critical velocities and the natural frequencies. This can be interpreted from observing eq. (5). Evidently the pressure has small effect on the axial force (second term) and has no effect on the Coriolis force (third term) which is represent the dynamic mechanism of the flutter.

To investigate the behavior of the natural frequencies at the critical fluid velocity a selected point from Figures 2 and 3 are chosen as an example. In this example β= 0.13 is taken and the complex natural frequencies for the lowest four modes are calculated. The fluid velocity U is varied from 4 to 7. In calculating the natural frequencies the procedure of ref [6] is followed where a 5DOF Galerkin analysis is used. The aim of plotting of Figures 2 and 3 is to evaluate the boundaries which separate the stable and unstable regions for a given pipe parameters as the mass ratio increased from 0 to 1. For any mass ratio the dimensionless velocity and frequency can be evaluated. The resulted complex frequencies are plotted in Figures (6-9). As it can be seen from these figures that the imaginary part of the natural frequency becomes zero at the second mode, Figure 7 where U=5. This is a sufficient condition for initiation the flutter instability hence U=5 is the critical velocity. The corresponding real natural frequency from the same figure is Ωr=13.5. These values are coincide with those given in Figures 2 and 3 for β= 0.13.

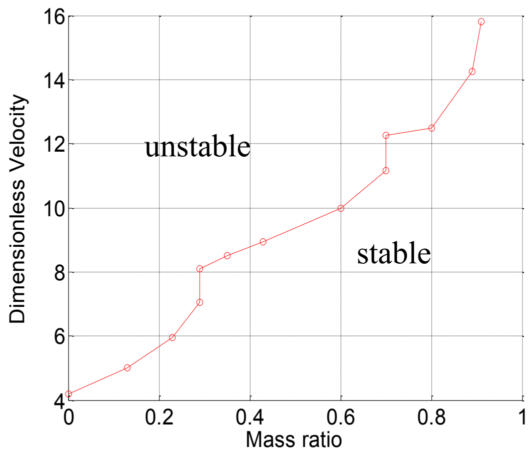

Figure 2. Stability map for cantilever pipe, γ = 0.

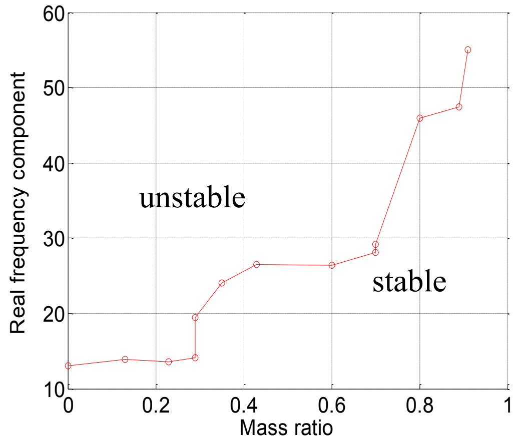

Figure 3. Real frequencies at critical velocities, γ = 0.

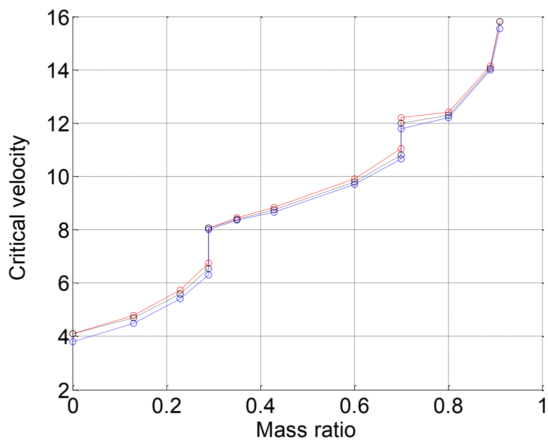

Figure 4. Stability map of cantilever pipe at different values of γ.

(——) γ=1,(----) γ=2,(- -) γ=3

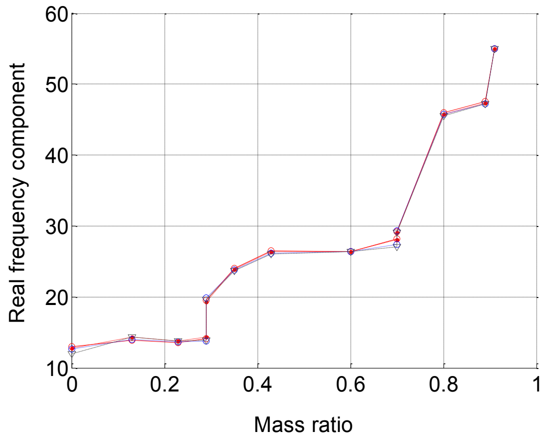

Figure 5. Real frequencies at critical velocities at different values of γ.

(——) γ=1,(----) γ=2,(- -) γ=3

Table 1. Comparis of the theoretical results of the present work with the numerical results of ref. [6] and ref. [1].

β	Present solution		Finite element ref [6]				Galerkin method ref [1]			
			U	Ω_r	%Error		U	Ω_r	%Error	
	U	Ω_r			In U	In Ω_r			In U	In Ω_r
0.00	4.20	13.00	4.300	13.25	2.38	1.92	4.37	13.70	4.05	5.34
0.13	4.95	13.90	5.100	13.95	3.03	0.36	5.22	14.00	5.45	0.72
0.23	5.95	13.55	6.050	13.66	1.68	0.81	6.16	13.65	3.53	0.74
0.29	7.05	14.15	7.120	14.25	0.99	0.71	7.18	13.99	1.92	-1.13
0.29	8.10	19.50	8.150	19.66	0.62	0.82	8.20	20.20	1.23	3.60
0.35	8.50	24.00	8.550	24.13	0.59	0.54	8.75	23.80	2.94	0.83
0.43	8.95	26.55	9.050	26.66	1.11	0.41	9.20	26.98	2.79	1.62
0.60	10.00	26.35	10.10	26.39	1.00	0.15	10.23	26.85	2.30	1.89
0.70	11.15	28.15	11.20	28.27	0.45	0.42	11.80	28.35	5.82	0.71
0.70	12.25	29.15	12.35	29.18	0.82	0.10	12.58	29.35	2.69	0.69
0.80	12.50	46.00	12.61	46.89	0.88	1.93	12.82	45.78	2.56	-4.47
0.89	14.25	47.50	14.31	47.83	0.45	0.69	14.55	47.25	2.11	-0.53
0.91	15.80	55.00	15.89	55.55	0.57	1.00	16.12	55.60	2.02	1.10

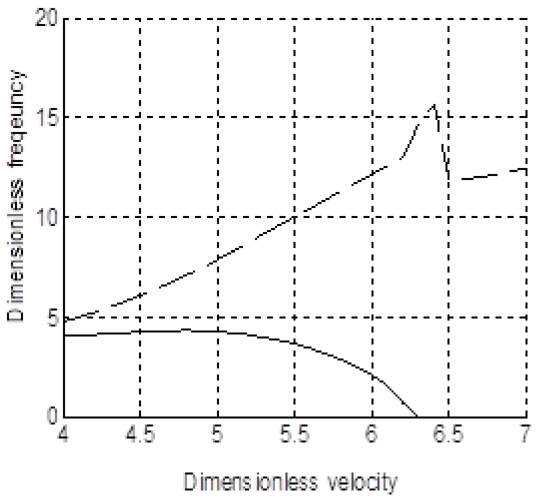

Figure 6. First natural frequencies at $\beta = 0.13$. (——) real (- - -) imag.

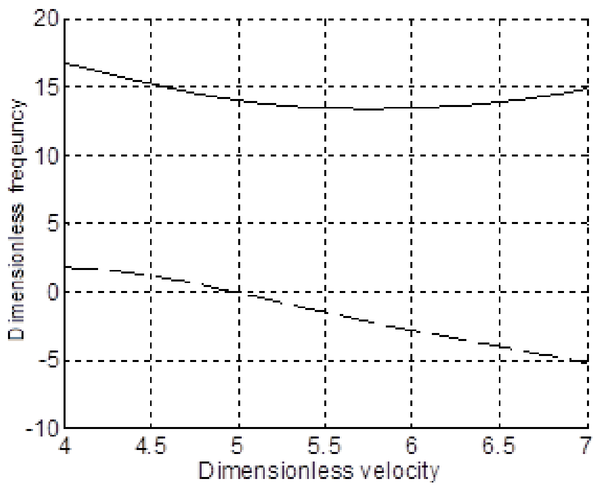

Figure 7. Second natural frequencies at $\beta = 0.13$. (——) real (- - -) imag.

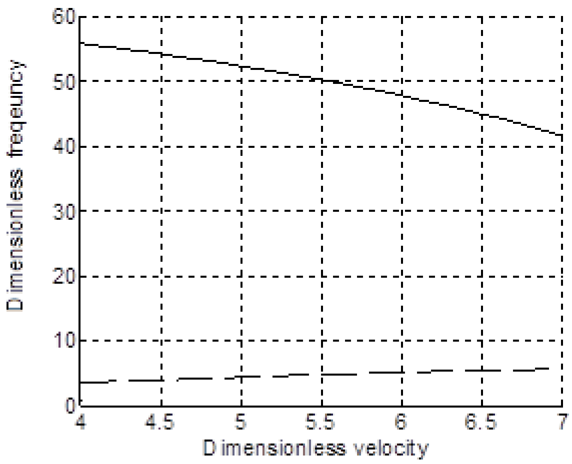

Figure 8. Third natural frequencies at $\beta = 0.13$. (——) real (- - -) imag.

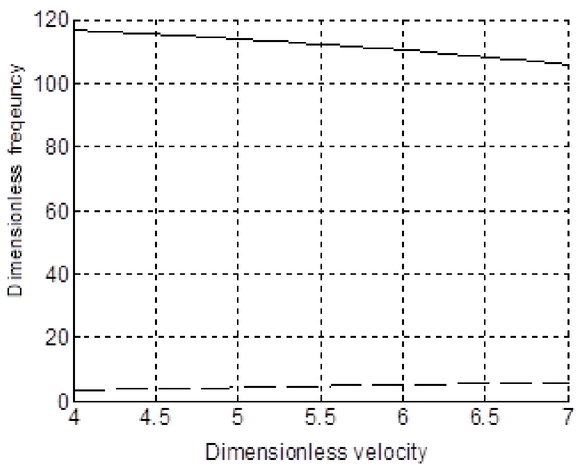

Figure 9. Fourth natural frequencies at $\beta = 0.13$. (——) real (- - -) imag.

4. Conclusions

The following main conclusions can be derived;

1- The present theoretical approach provides an accurate and simple analytical method for evaluating the critical fluid velocities and the real natural frequencies for flutter instability of cantilever pipes conveying fluid. Instead of using the Numerical methods which are attempted in the literature.

2- This approach is checked with other two approaches available in literature and the results showed a very good agreement where the error is not exceeded 5.82% for the worst cases.

3- The damping has predominated effect on critical speeds of flutter instability since it may become negative and amplify the oscillation.

4- The effect of the fluid pressure on flutter instability is studied. The results showed that increasing the pressure slightly reduces the critical velocities and the real frequencies of flutter instability.

Nomenclature

A_f, A_p: Fluid and pipe cross sectional area, respectively (m^2)
E: Modulus of elasticity (N/m^2)
L: Pipe length (m)
m_f, m_p: Fluid and pipe mass per unit length, respectively (kg/m)
P: Fluid pressure (N/m^2)
U: Dimensionless fluid velocity
V: Fluid velocity (m/s)
η, ζ: Dimensionless coordinates
U, β, γ: Dimensionless velocity, mass ratio, and pressure, respectively
Ω: Dimensionless frequency = $\omega L^2[(m_f + m_p) / E\ I)^{1/2}$
Ω_r, Ω_i: The real and imaginary components of the dimensionless frequency
ω: Circular frequency (rad/sec)
τ: Dimensionless time
ρ_f, ρ_p: Fluid and pipe material density, respectively (kg/m^3)

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