INTERNATIONAL JOURNAL OF
ENERGY AND ENVIRONMENT

Volume 9, Issue 5, 2018 pp.473-480
Journal homepage: www.IJEE.IEEFoundation.org

Evaluation of crest length effect on piano key weir discharge coefficient

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Received 22 May 2018; Received in revised form 26 July 2018; Accepted 4 Aug. 2018; Available online 1 Sep. 2018

Abstract
This paper aims to investigate and quantify the effect of changing the crest length of a piano key weir on its discharge coefficient. Empirical equation relating the discharge coefficient with the crest length has been developed based on dimensional analysis by means of statistical computer software. Data were gathered by physical modeling of five 2-unit piano key weir models in an experimental flume 15 m long, 0.3 m wide and 0.45 m deep. Results showed that increasing the crest length has a major influence on the discharge capacity that may reach as much as 100%. The empirical equation estimates the value of discharge coefficient in terms of the dimensionless ratio of (crest length to the weir width), as well as the ratio of (total head to the weir height). It has showed good agreement with experimental results.

Keywords: Piano key weir; Discharge coefficient; Physical modeling; Crest length; Empirical equation; Statistical software.

1. Introduction

Piano Key Weir (abbreviated PKW) is a special type of labyrinth weirs developed about 18 years ago as a solution to the problem of traditional labyrinth weir which is the inadequacy to construct on dams due to the large base area [1]. PKW geometry is similar to the labyrinth one, i.e. consisting of successive repetitions of cycles, with new features introduced to form the particular shape of PKW, namely: rectangular layout, sloped floors, overhangs and reduced footprint area. These features make the PKW more economical and enable its construction on gravity dam sections.
The main characteristic that raises the attention to this type of weirs is the increased crest length (the same concept of labyrinth weirs) which makes the PKW 3 to 4 times more efficient than the ordinary linear weir. This is done by multiplying or "folding" the crest of the linear weir into non-linear zigzag layout [2].
While many researchers have studied the effect of increasing the crest length on the PKW discharge capacity, this paper aims to quantify this effect by proposing an empirical equation based on laboratory experiments. Statistical software has been utilized to develop a formula that predicts the discharge coefficient of PKW for a range of crest length variations.

2. PKW geometry description

A group of researchers have made an important contribution by developing a convention for the terminology used in the description of PKW geometry. The reader is referred to their paper [3] for full details. Generally, piano key weir is consisted of repetitions of "units" or "cycles" in a simple rectangular layout. Sloped floors are arranged alternatively in both upstream and downstream directions, thus forming a collection of alveoli (or chambers) known as inlet keys and outlet keys depending on whether they are open to the upstream or downstream direction [4]. Each PKW unit represents the "smallest extent of a complete structure and is composed of an entire inlet key with a sidewall and half an outlet key on both sides" [3]. As the unit represents a complete structure, studying only one unit is sufficient to identify the whole structure behaviour. A general view of the PKW is shown in Figure 1 and the notations are defined in Table 1.

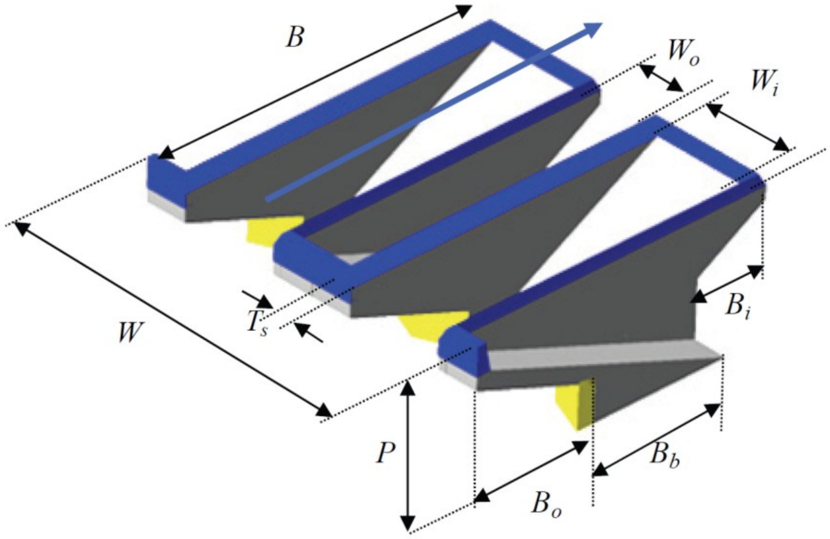


Figure 1. Sketch of PKW geometry [3].

It is important to notice that the weir may be constructed on a dam (or any height); therefore, in addition to the height P, another notation that represents the dam height was utilized, namely P_d (not shown in Figure 1) [3].

Table 1. Terminology of PKW geometrical parameters [3].

Parameter symbol	Meaning
B	Upstream-downstream length of the PKW, $B=B_b+B_i+B_o$
B_o	Upstream (outlet key) overhang length
B_i	Downstream (inlet key) overhang length
B_b	Base length
P	Height of PKW measured from the crest (including possible parapet walls)
P_d	Dam height (or any platform under the PKW)
W	Total width of the PKW
W_i	Inlet key width (sidewall to sidewall)
W_o	Outlet key width (sidewall to sidewall)
T_s	Sidewall thickness
T_i	Horizontal crest thickness at inlet key extremity
T_o	Horizontal crest thickness at outlet key extremity
L	Total developed length along the overflowing crest axis

3. Determination of PKW discharge capacity

The standard rectangular weir equation is used to determine the discharge over PKW in free flow conditions [5]:

$$Q = C_{dW} \frac{2}{3} \sqrt{2g} W H_o^{1.5} \tag{1}$$

Where Q is the discharge, C_{dW} is discharge coefficient, g is the gravitational acceleration, W is the channel width and H_o is the total head which is the sum of the piezometric head h_o and the velocity head $V_o^2/2g$. Figure 2 illustrates the flow over PKW.

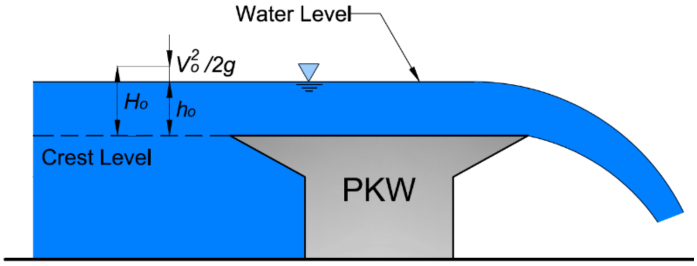


Figure 2. Sketch of the flow over PKW.

In order to perform a parametric study on the PKW discharge coefficient, dimensional analysis technique is used to find a functional relationship between discharge coefficient, C_{dW}, and the geometrical parameters. According to π-theorem, the relationship in Equation 2 has been developed. Further details about its derivation are given in [6].

$$C_{dW} = f\left(\frac{H_o}{P}, \frac{L}{W}, \frac{W_i}{W_o}, \frac{B}{P}, \frac{B_i}{B}, \frac{B_o}{B}, \frac{P_d}{P}\right) \tag{2}$$

As we are interested in the crest length of the weir, L, the focus of this paper will be on the parameter L/W. Several variations of L/W are going to be considered while the other parameters are kept constant.

4. Literature review on the influence of *L/W* on PKW discharge capacity

This parameter represents the ratio between the developed crest length, L, and the weir width, W. All the available literature consider it as the main parameter influencing discharge capacity with the remaining parameters considered minor relative to it. In fact, increasing the crest length is the distinction between linear and non-linear weirs. When L/W=1, the weir is linear and no capacity increase is obtained. But when using a value of L/W>1, the gain in discharge capacity starts to occur.

The effect of L/W has been studied by Lempérière and Jun [7] and they reported that values ranging between 4 and 7 are recommended in design. Barcouda et al. [8] recommended a value of 6 as optimum design value. However, Lempérière [9] stated that L/W=5 is near optimal according to hydraulic and economical points of view. Similar results were reported by Hien et al. [10] stating that although a value of L/W=7 is more efficient for small H_o/P ratios, an L/W value that ranges from 5-6 is more cost effective.

Noui and Ouamane [11] noted that raising the ratio of L/W from 4 to 6 increases the discharge coefficient significantly. This increase is about 15% for H_o/P=0.2, and 8% for H_o/P=0.4. Leite Ribeiro et al. [12] found that raising L/W from 3 to 7 made a gain of about 50% in discharge coefficient for low heads (low H_o/P ratios). But with increasing the ratio of H_o/P, this gain tends to decrease. However, there is an upper limit at which the increase of L/W does not have an effect on the discharge coefficient. As stated by Kabiri-Samaini and Javaheri [13], this limit is L/W=7 and values more than 7 does not increase the weir capacity; while Ouamane and Lempérière [14] found that a value of L/W=8.5 has a gain in the discharge coefficient but for small H_o/P values only. For higher values of H_o/P, the large values of L/W do not contribute significant increase to the discharge capacity. Furthermore, El-Katib et al. [15] found that increasing L/W more than 5 in type-B PKW contributes to the capacity in low heads only (H_o/P<0.25) due to local submergence at the outlet key. This is not the case for type-C PKW.

5. Experimental setup and testing procedure

Five physical models were prepared in this investigation. Firstly, a standard PKW model that agrees with the limitations of [9] was selected for purpose of comparison. The limitations are: (L/W=5, W_i/W_o=1.25, B/P=2.4, B_i/B=0.25, B_o/B=0.25). Then, four other models where prepared having L/W values of (3, 4, 6 and 7). The value of P_d/P for all models was 0.6. Table 2 shows the parameters used in each model.

Table 2. Geometrical parameters of the tested models.

Model	L/W	W_i/W_o	B/P	B_i/B	B_o/B	P_d/P
L5	5	1.25	2.4	0.25	0.25	0.6
L3	3	1.25	2.4	0.25	0.25	0.6
L4	4	1.25	2.4	0.25	0.25	0.6
L6	6	1.25	2.4	0.25	0.25	0.6
L7	7	1.25	2.4	0.25	0.25	0.6

The models where made using acrylic glass sheets of 2.5 mm cut by CNC machine and assembled by pasting materials. The crest shape was “flat-top”. According to the flume limitations, the models where consisting of 2 units.

The experimental flume was 15 m long having a rectangular section of 0.3 m wide by 0.45 m deep. The discharge is measured by means of a pre-calibrated sharp-crested rectangular weir. The flume is equipped with a rolling point gauge apparatus with accuracy of ±0.5 mm. A main tank, of 4.5 m^3 capacity, is located at the downstream end of the flume. Water is conveyed from the main tank to an inlet tank, of 0.5 m^3 capacity, at the upstream end by means of a pump having maximum discharge of 36 ℓ/s.

Models were located at the mid-section of the flume to guarantee that uniform flow is developed and to avoid the downstream effects. Each model was fixed to the stainless-steel flume bed by screws and enough quantity of silicon rubber to prevent movement and ensure water tightness.

Testing procedure may be summarized as follows:

- Start by operating the pump and wait for the flow to stabilize and uniform condition to occur.
- Measure the flume discharge.
- Measure the flow depth at a suitable distance to the upstream of the PKW model. The suitable distance recommended by the USBR manual [16] is four times the maximum head over the weir (32 cm in this study).
- Change the discharge value and repeat the steps.

The head-discharge relationship is then constructed and the associated discharge coefficient is determined using Equation 1. Any reading of water head that is below 3 cm was avoided. This is because readings below this value are influenced by the scale effects and would not reflect the behaviour of real structures [17].

6. Results of experimental work

Test results showed important influence for the parameter L/W on the discharge coefficient. Figure 3 shows the behaviour of the five models. Note that the data are under different ranges of H_o/P. This is explained by their different heights P. Therefore, one should pay attention that a fixed value of H_o/P does not represent the same absolute head H_o for the different models. It rather represents a dimensionless relation between the head and the weir geometry, so that it can be linked to the discharge coefficient according to dimensional analysis. Regression equation for the curves in Figure 3 may be presented in the following form:

$$C_{dW} = a\left(\frac{H_o}{P}\right)^b \tag{3}$$

The constants a and b are given in Table 3.

Table 3. Constants of discharge coefficient regression equations (curves in Figure 3).

Model	*a*	*b*	R^2
L3	1.1197	−0.300	0.9908
L4	1.2566	−0.433	0.9982
L5	1.3042	−0.479	0.9986
L6	1.4038	−0.496	0.9972
L7	1.5263	−0.469	0.9883

However, the comparison of different models in Figure 3 is unclear due to the data shift resulted from different model heights. Therefore, a relationship between C_{dW} and the absolute head H_o is presented in Figure 4. It is obvious from this perspective how much the discharge coefficient increase when using higher values of L/W for a fixed upstream head H_o. All the five models lose their capacity as H_o increases. The percentage change of C_{dW} is calculated for the tested models relative to the model (L5) and presented in Figure 5, where:

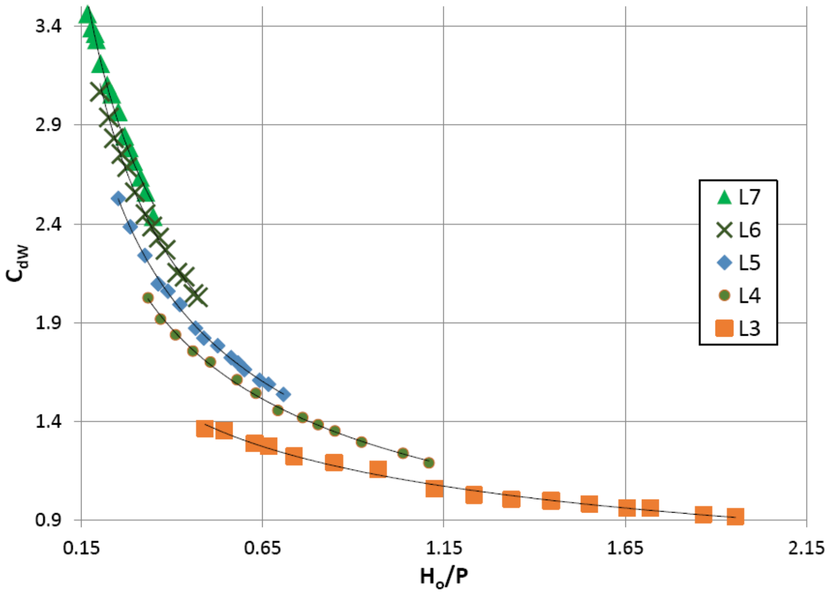


Figure 3. Variation of C_{dW} vs. H_o/P for five ratios of L/W.

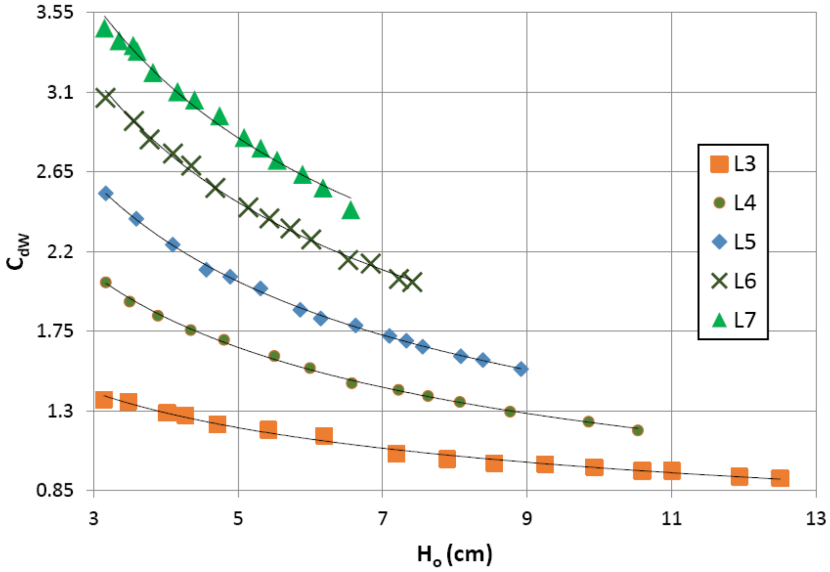


Figure 4. Variation of C_{dW} vs. H_o for five ratios of L/W.

$$(\%Change) = \frac{(C_{dW})_{Tested} - (C_{dW})_{L5}}{(C_{dW})_{L5}} \times 100\% \tag{4}$$

It seems that the model (L7) is about 40% more efficient than (L5), while the gain of (L6) is nearly 22%. The two models (L4) and (L3) show a loss in capacity relative to (L5) by about 20% and 40% respectively.

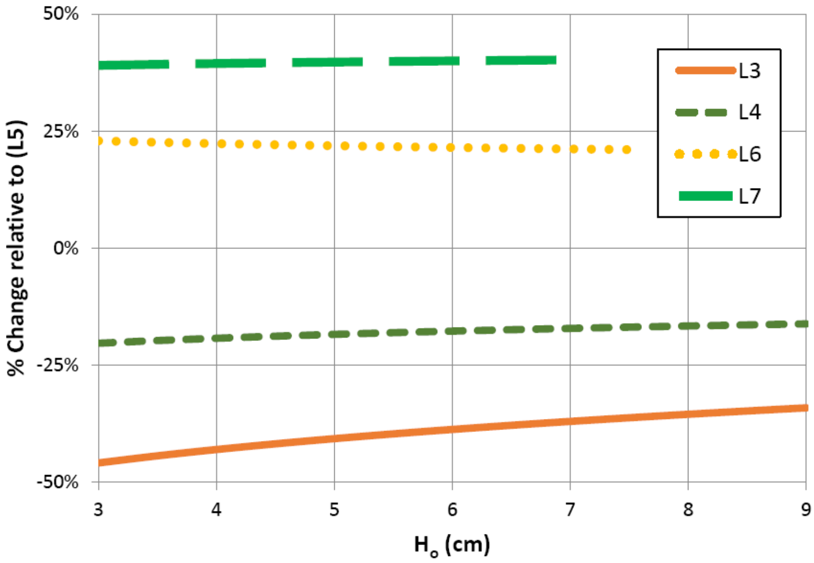


Figure 5. Percentage change of C_{dW} vs. H_o relative to model L5.

When we observe the above values of gains and losses, a conclusion can be made: for a constant value of H_o, every time the ratio L/W is changed by a value of 1, the discharge capacity decreases or increases correspondingly by approximately 20%.

For more clarification to the relationship between L/W and C_{dW}, Figure 4 is re-drawn by reversing the data representation; L/W being the x-axis, C_{dW} still on the y-axis, while several curves are drawn for selected H_o values. This presentation of data illustrates how the L/W increase can influence the discharge capacity; see Figure 6. The increase of L/W has always a positive effect on C_{dW}.

Finally, it may be understood that L/W has an important effect on the PKW discharge capacity that can reach up to 100% in case of increasing L/W from 3 to 7.

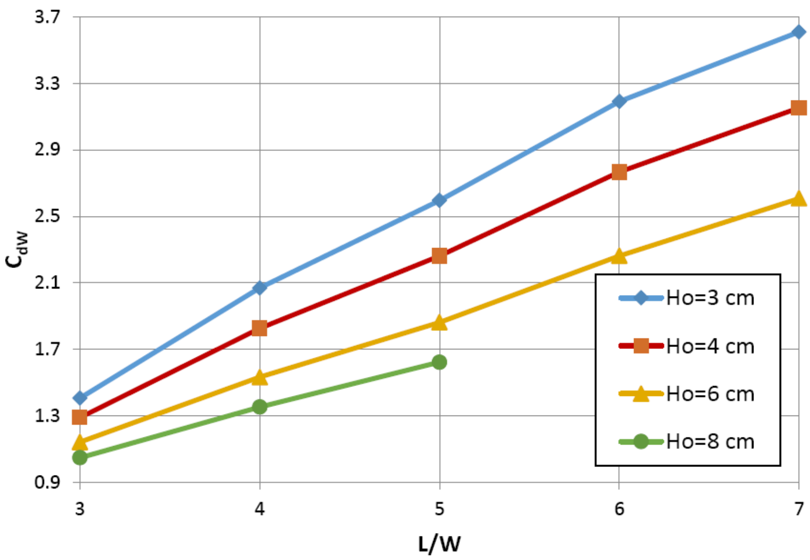


Figure 6. Variation of L/W vs. C_{dW} for different values of the head H_o.

7. Empirical relationship

In an attempt to estimate the discharge coefficient over PKWs with different values of *L/W*, an empirical relationship has been developed by means of the statistical computer software *SPSS*. It is based on the form of Equation 2 but with the exclusion of every parameter other than H_o/P and L/W:

$$C_{dW} = f\left(\frac{H_o}{P}, \frac{L}{W}\right) \quad (5)$$

The total number of data was divided into two parts: 1) 80% of the data used to develop the equation, and 2) 20% of the data used for checking the developed equation (randomly selected so that they represent all models).

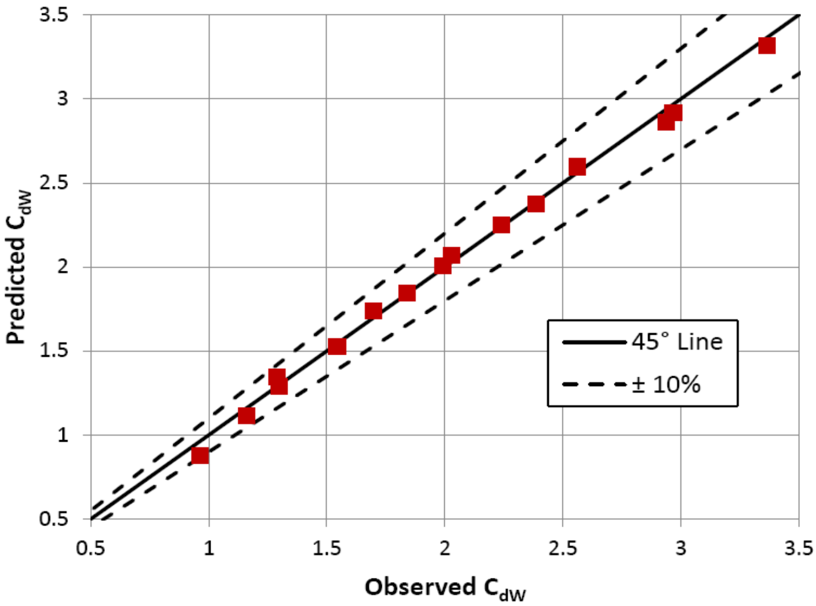
According to modeling limitations shown in Table 2, it should be realized that the equation can only be used when the value of each parameter is within the limitations; that is $3.0 \le L/W \le 7.0$, $W_i/W_o=1.25$, $B/P=2.4$, $B_i/B=0.25$, $B_o/B=0.25$ and $P_d/P=0.6$. Tested range of the hydraulic parameter H_o/P is from about 0.15 to 1.95 as shown in Figure 3. However, it is more conservative to constrain H_o/P between 0.15 and 1.0.

The equation that resulted after performing a non-linear regression analysis is:

$$C_{dW} = 0.6793\left(\frac{H_o}{P}\right)^{-0.4421}\left(\frac{L}{W}\right)^{0.4354} \quad (6)$$

The coefficient of determination (R^2) of this formula is (0.996). To check this equation, the 20% part of the data is used to show the convergence of the experimental results with those predicted by Equation 6. Figure 7 shows the comparison between the observed and predicted values of the discharge coefficient C_{dW}. The two dotted lines illustrate the range in which ±10% deviation of the data occurs.

The normalized root-mean-square deviation (NRMSD) was calculated as 0.0184. Data convergence in Figure 7 illustrates good agreement between observed and predicted results.



Figrue 7. Comparison of Equation 6 with experimental results.

7. Conclusions

- Experimental investigation was performed to estimate the effect of PKW crest length on its discharge coefficient using five physical models. It was found that increasing the crest length has a major contribution to the gain in discharge capacity of piano key weirs.

- For a given value of the head H_o, if the parameter *L/W* is changed by ±1, the discharge coefficient C_{dW} will change consequently by ±20% (direct proportion).
- An empirical equation with sufficient accuracy has been developed using statistical software. The discharge coefficient can be estimated using this equation for different values of *L/W*.

Acknowledgement
Experimental work has been conducted in the Laboratory of Hydraulics, Structures and Water Resources Engineering Department, Faculty of Engineering, University of Kufa, Iraq.

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